Power Allocation In Cooperative Relay Channels

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***Abstract*— This paper concerns power allocation in relay-assisted wireless channels for two-hop transmission. First, the transmitter sends the information to both the relay and receiver parts. Next, in the second hop, the transmitter cooperates with the relay to increase the received signal to noise ratio (SNR), assuming the relay makes use of the Amplify and Forward (AF) strategy. Moreover, it is assumed a linear combiner is employed at the receiver to effectively combine the received signals of two hops. In this regard, under total transmit power constraint, an iterative power allocation strategy together with a proper combining method is proposed, showing the proposed approach achieves a comparable SNR as compared to the best known methods addressed in the literature, while having much lower complexity.**

***Index Terms*—** Power Allocation, relay channel, cooperative strategy

I. INTRODUCTION

Relay networks are mainly justified due to the need to increase the coverage area and the system throughput, while having an acceptable transmit power [1]-[3]. Moreover, deploying multiple relays makes it possible to send the information through several independent wireless links, thereby increasing the system reliability. Using relays can be thought as a variation of transmit diversity, called spatial diversity, which can effectively alleviate the impact of fading in wireless channels [4]. In spatial diversity methods, some replicas of signal are sent from various locations, thereby having independently faded versions of the transmitted signal at the receiver part [5], [6].

In relay channels, a variation of spatial diversity, called cooperative diversity, can be used [6]-[9], in which the transmitter as well as the relay(s) cooperate to jointly increase the received signal power effectively. In this case, it is shown the received signal power outperforms that of the previous non-cooperative schemes. Accordingly, there have been some attempts to investigate the impact of various cooperative schemes in relay channels, among them, the following strategies are thoroughly investigated in the literatures: (i) Amplify-and-Forward (AF), (ii) Decode-and-Forward (DF), and finally (iii) Compressed-and-Forward (CF) relaying [6], [10], [11].

In AF strategy, each relay receives a noisy version of the transmitted signal, amplifies and retransmits this signal to the affiliated receiver. In DF, the relay first decodes the received signal and

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transmits the re-encoded version to the destination. Finally, in CF strategy, a quantized version of the received signal at the relay is sent to the receiver. It is widely recognized that based on channel condition, one of the aforementioned coding strategies may outperform the others. On the other hand, AF [2], [3], [10], [12], [13] has attracted more attentions due to some practical issues, like its inherent simplicity and the ability to impose less delay to the system. Thus, AF is more likely to find some practical implications. This motivated us to rely on AF strategy.

In this regard, a plethora of works are devoted to the power allocation of AF strategy to achieve a specific criterion. For instance, in [12], an upper bound for the SNR is derived and accordingly a simple power allocation strategy for this upper bound is deduced, assuming the mean of channel strengths are merely available. Also, in [13], the received SNR in the presence of certain number of AF relays and under various assumptions (with and without having direct link) is derived and the optimum power allocation strategy for an approximation of the received SNR is derived.

In [14], a relay activation strategy for both AF and DF strategies is proposed, wherein each relay is incorporated as far as certain condition holds, otherwise, the relay goes to an idle state. Accordingly, in an attempt to increase the received SNR, a distributed power allocation strategy for the selected relays is proposed. In [15], the optimum power allocation strategy to improve the outage performance in AF relay assisted networks is investigated. In [16], a collaborative beamforming strategy in the presence of perfect channel state information for AF relays with both total and individual power constraint is proposed, assuming there is not a direct link from the transmitter to the affiliated receiver.

In [17], the issue of distributed beamforming strategy for AF relays is investigated, assuming the second order statistics of the channel corresponding to the source-to-relays and relays-to-destination are available and also there is not any direct link from the source to the destination. Accordingly, for the case of having stringent Quality of Service (QoS) constraint, the total power of relays is minimized. Then, for either total or individual power constraint, two distributed beamforming strategies are developed. In [18], the optimum beamforming strategy in terms of maximizing the received SNR for two-hop AF relays under individual power constraint is proposed, assuming there is not any direct link from the transmitter to the affiliated receiver. Moreover, in the presence of a direct link between the transmitter and the respected receiver, an iterative numerical power allocation strategy is proposed to effectively allocate the transmit power, assuming there is a constraint for the total transmit power of two hops. Accordingly, considering relays are subject to individual power constraint, a proper power allocation at each relay is computed. Moreover, it is demonstrated that when there is only one relay, this relay should communicate at full power [18].

In this paper, we consider there is only one relay and the transmitter aims at sending signals by the help of this relay to the affiliated receiver in two hops. Moreover, it is assumed there is a direct link

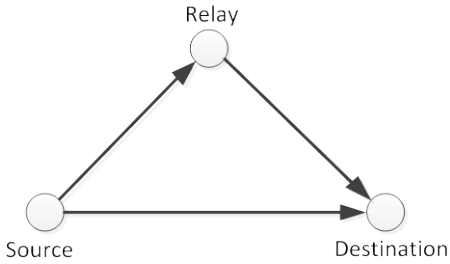

Fig. 1. The block diagram of a relay-assisted wireless channel

between the transmitter to the respected receiver, and the relaying protocol is AF. In this case, similar to what is argued in [18], we simply assume the relay operates at full power. Accordingly, a proper transmit power allocation strategy for two hops is proposed, showing the proposed method can achieve the same SNR as compared to the method of [18], while having much lower complexity.

The rest of this paper is organized as follows: In section II, we present the system model of a relay-assisted wireless channel. The problem formulation is given in section III. Accordingly, the proposed power allocation strategy is discussed in section IV. Section V presents the simulation results and gives some discussions regarding the superiority of the proposed method as compared to others, and finally section VI wraps up the paper with findings and future directions.

Throughout the paper, boldface letters indicate vectors (lower case) or matrices (upper case). The x^T notation denotes the transpose of vector x . In addition, x^H indicates the transpose conjugate of vector x .

II. SYSTEM MODEL

We consider a relay channel in which the transmitter sends the information in two hops (Fig. 1). First, it sends the information to both the relay and receiver sides. Then, the relay cooperates with the transmitter, assuming the relay makes use of AF strategy. It is assumed that each node is equipped with single antenna. Moreover, the relay operates in half duplex mode, thus, it cannot receive and transmit simultaneously.

Let's g_{sd} , g_{sr} and g_{rd} denote, respectively, the corresponding channel gains from source-to-destination $S \rightarrow D$, source-to-relay $S \rightarrow R$ and relay-to-destination $R \rightarrow D$ links. Also, both the destination and relay are affected by zero-mean additive white complex Gaussian noise with power N_0 . Furthermore, all channels are assumed to be quasi-static block fading, thus, the channel coefficients g_{sd} , g_{sr} and g_{rd} remain constant throughout one transmission block, and vary for the next block. Also, it is assumed the channel gains are drawn from independent complex Gaussian distribution with zero mean and unit variance. Thus, assuming $h_{sr} = |g_{sr}|$, $h_{rd} = |g_{rd}|$, $h_{sd} = |g_{sd}|$, it follows $E\left[|h_{sr}|^2\right] = E\left[|h_{rd}|^2\right] = E\left[|h_{sd}|^2\right] = 1$, where $E[.]$ denotes expectation. Finally, it is assumed the channel coefficient associated with each link is perfectly available at the corresponding transmitter and receiver parts.

As is mentioned earlier, in the first hop, the transmitter sends the information signal x to both the relay and receiver parts. We also assume that y_R and y_{D1} denote, respectively, the received signals at the relay and destination. Thus, assuming the phase of channel gains are simply compensated (this is done by applying simple phase rotation at each link), it follows,

$$\begin{aligned} y_R &= w_1 h_{sr} x + n_R, \\ y_{D1} &= w_1 h_{sd} x + n_{D1}, \end{aligned} \tag{1}$$

where it is assumed n_R, n_{D1} are unit variance additive noises. Thus, the received signal power at the relay becomes,

$$E_{x,n}[|y_R|^2] = E_x\left[|w_1 h_{sr} x|^2\right] + E_n\left[|n_R|^2\right] = |w_1|^2 |h_{sr}|^2 + 1, \tag{2}$$

where it is assumed $E_x\left[|x|^2\right] = 1$, and w_1 is the transmit weighting factor of the first hop. Assuming the AF relay is subject to power p_R, the relay scales and retransmits the received signal y_R by the following scaling factor,

$$\beta = \sqrt{\frac{p_R}{E_{x,n}\left[|y_R|^2\right]}} = \sqrt{\frac{p_R}{|\omega_1|^2 |h_{sr}|^2 + 1}}. \tag{3}$$

In the second time slot, the source sends out the signal x by the weighting factor w_2 and the AF relay transmits βy_R, thus, the received signal at the destination would be,

$$y_{D2} = w_2 h_{sd} x + \beta h_{rd} y_R + n_{D2}, \tag{4}$$

where n_{D2} is the received noise term of the second time slot and is assumed to be of unit power, i.e., $E_n\left[|n_{D2}|^2\right] = 1$. As a result, substituting (1) in (4), it follows,

$$\begin{aligned} y_{D2} &= w_2 h_{sd} x + \beta h_{rd}\left(w_1 h_{sr} x + n_R\right) + n_{D2} \\ &= \left(w_2 h_{sd} + \beta w_1 h_{sr} h_{rd}\right) x + \left(\beta h_{rd} n_R + n_{D2}\right). \end{aligned} \tag{5}$$

Assuming the total transmit power of two time slots is subject to p_T, i.e., $E\left[|w_1 x|^2\right] + E\left[|w_2 x|^2\right] = |w_1|^2 + |w_2|^2 \le p_T$, the problem is to find a proper combining method at the receiver as well as the best power allocation strategy in which the received SNR is maximized. In the next section, we address the aforementioned issues in details.

III. PROBLEM FORMULATION

We assume the receiver makes use of linear combiner to effectively combine the received signals of two hops. Assuming $a=\left[\alpha_1 \quad \alpha_2\right]^T$ entails the corresponding weighting factors of two hops and considering y as the received vector whose elements are the received signals of two hops, i.e. $y=\left[y_{D1} \quad y_{D2}\right]^T$, thus the output of combiner (r) can be written as,

$$r=a^T y. \tag{6}$$

Noting (1) and (4), we have,

$$y=\begin{bmatrix} w_1 h_{sd} \\ w_2 h_{sd}+\beta w_1 h_{rd} h_{sr} \end{bmatrix} x+\begin{bmatrix} n_{D1} \\ \beta h_{rd} n_R+n_{D2} \end{bmatrix}, \tag{7}$$

which can also be written as,

$$y=\begin{bmatrix} h_{sd} & 0 \\ \beta h_{rd} h_{sr} & h_{sd} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} x+\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \beta h_{rd} \end{bmatrix}\begin{bmatrix} n_{D1} \\ n_{D2} \\ n_R \end{bmatrix}. \tag{8}$$

Thus, referring to (6), r can be computed as,

$$\begin{aligned} r&=\left(a^T HW\right)x+a^T Gn \\ &=\left(a^T HW\right)x+a^T \tilde{n}, \end{aligned} \tag{9}$$

where, for the sake of notational simplicity, it is assumed,

$$\begin{gathered} H=\begin{bmatrix} h_{sd} & 0 \\ \beta h_{rd} h_{sr} & h_{sd} \end{bmatrix}, \; W=\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ G=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \beta h_{rd} \end{bmatrix}, \; n=\begin{bmatrix} n_{D1} \\ n_{D2} \\ n_R \end{bmatrix}, \; \tilde{n}=Gn. \end{gathered} \tag{10}$$

Assuming the covariance matrix of $\tilde{n}$ is $R_{\tilde{n}\tilde{n}}$ it follows,

$$\begin{aligned} R_{\tilde{n}\tilde{n}}=E\left[\tilde{n}\tilde{n}^H\right]&=\sigma^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \beta h_{rd} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & \beta h_{rd}^* \end{bmatrix} \\ &=\sigma^2\begin{bmatrix} 1 & 0 \\ 0 & 1+\beta^2\left|h_{rd}\right|^2 \end{bmatrix}. \end{aligned} \tag{11}$$

Thus, referring to (9) and noting (11), one can arrive at the following,

$$SNR = \frac{E\left[\left|a^T HWx\right|^2\right]}{E\left[\left|a^T \tilde{n}\right|^2\right]}. \tag{12}$$

Recall that in (12), it is assumed $E_x\left[|x|^2\right] = 1$. Thus, the problem is to find the best transmit power allocation, W , in the feasible regime of $|W|^2 \le p_T$ as well as the optimum receive combining a for which the received SNR defined in (12) is maximized. Note that the covariance matrix $R_{\tilde{n}\tilde{n}}$, introduced in (11) depends on β, and β itself is related to w_1 (refer to equation (3)). Consequently $R_{\tilde{n}\tilde{n}}$ depends on W . As a result, one can readily verify that the problem of maximizing the received SNR is not convex with respect to W . Thus, the task of finding W_{opt} and a_{opt} does not yield trivial solution.

IV. The Proposed Method

This section aims at introducing an iterative solution to increase the received SNR defined in (12). First, assuming the transmit power allocation vector W is fixed, the optimum solution for a is derived. Then, W is deduced such that the received SNR is maximized. The algorithm precedes recursively as long as the SNR difference in two consecutive iterations exceeds a certain threshold. In what follows, the algorithm is described in more details. Assuming $W(n)$ is the computed W at the n^{th} iteration, it can be verified that to maximize (12), $a(n+1)$ should be computed as (see Section VIII),

$$a(n+1) = R_{\tilde{n}\tilde{n}}^{-1} H(n) W(n). \tag{13}$$

Thus, substituting (13) in (12) and after some manipulations, it follows,

$$SNR(n+1) = W^T(n) H^T(n) R_{\tilde{n}\tilde{n}}^{-1}(n) H(n) W(n). \tag{14}$$

According to the Rayleigh-Ritz theorem [19], and assuming $\Omega(n) = H^T(n) R_{\tilde{n}\tilde{n}}^{-1}(n) H(n)$, the best value of $W(n+1)$ for which the equation (14) is maximized can be computed as,

$$W(n+1) = \sqrt{p_T} V_{\max}(n) \tag{15}$$

where $V_{\max}(n)$ is the eigen vector corresponding to the maximum eigen value $(\lambda_{\max}(n))$ of matrix $\Omega(n)$. Also, the resulting SNR in (14) becomes,

$$\begin{aligned} SNR(n+1) &= p_T V_{\max}^T(n) \Omega(n) V_{\max}(n) \\ &= p_T \lambda_{\max}(n). \end{aligned} \tag{16}$$

The algorithm can be summarized in the following steps,

- Set $n = 0$ and $W(0) = \sqrt{\frac{p_T}{2}}[1 \quad 1]^T$ as the initial transmit power allocation vector (note that $|W(0)| = p_T$).
- Compute the singular value decomposition (SVD) [19] of matrix $\Omega(n) = H^T(n)R^{-1}_{\tilde{n}\tilde{n}}(n)H(n)$ and accordingly derive λ_{max} and V_{max}.
- Compute $W(n+1)$ using (15), and update $R_{\tilde{n}\tilde{n}}$ and H.
- Compute $SNR(n+1)$ from (16).
- If $\left|\frac{SNR(n+1) - SNR(n)}{SNR(n)}\right| \geq \eta$, go to step 2, otherwise, stop.

V. RESULTS

This section aims to provide some numerical results to evaluate our proposed approach through comparing with some of existing methods, including the iterative method proposed in [18]. First, we simply assume the transmitter sets the transmit power vector to $W = \sqrt{p_T/2}\,[1 \quad 1]^T$, meaning it sends an equal power during each transmission. We call this strategy as *Method A*. Also, it is assumed the transmitter sends at full power in the first hop to both the relay and receiver side. Then, for the second hop, the relay is the only active node and it sends at full power to the respected receiver. This zero-one strategy is called *Method B*. The method proposed in [18] where to the author's knowledge is the best known method is called *Method C*. Fig. 2 illustrates the comparison results for the Rayleigh fading channel and various relay's power, assuming the total transmit power is set to one. Fig. 2 shows the proposed approach performs like the *Method C*, while numerical results indicate that our proposed approach exhibits a fast convergence speed as compared to *Method C*. To get an indication, the average number of required iterations for our proposed method for 10000 channel realizations is less than 2, however, for *Method C*, it is around 25. It is worth mentioning that in our method and for most of channel realizations, either one or two iterations is adequate to get the favorable result.

CONCLUSION

This paper concerns transmit power allocation in relay assisted channels, when the transmission occurs in two hops. Also, it is assumed the relay amplifies the received signal of the first hop and cooperates with the transmitter during the second hop to increase the received SNR. In this regard, noting the task of finding the optimal transmit beamforming is not convex, an iterative solution to this is proposed, showing our approach can achieve the best known result, while having less complexity as compared to the existing methods.

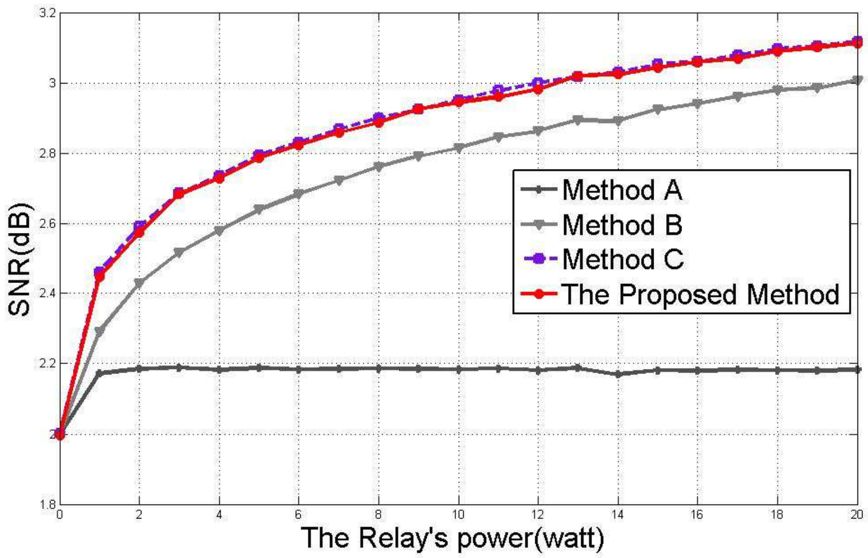

Fig. 2. Comparison results for the resulting SNR in the presence of a single-relay and assuming channel gains are Rayleigh distributed

Further research can extend the idea presented in this paper to the case of having certain number of AF relays. Despite of single-relay in which the relay is desired to send at full power, this is not the case happening for multi-relay networks [18]. As a result, it is desired to seek for a power allocation strategy at the relays as well as finding a proper transmit beamforming which increases the received SNR in such networks.

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APPENDIX

We consider the following optimization problem,

$$\max_{a} \frac{\left|a^H h\right|^2}{a^H R a}, \tag{17}$$

where R is a positive semi-definite matrix, and it is assumed h is a complex vector. We aim at taking derivation with respect to the vector $a = \left[a_1 \; a_2 \; \cdots \; a_n\right]^T$ to get the best value for a_{opt}, where we have,

$$\frac{\partial}{\partial a} = \begin{bmatrix} \frac{\partial}{\partial a_1} \\ \frac{\partial}{\partial a_2} \\ \vdots \\ \frac{\partial}{\partial a_n} \end{bmatrix}, \tag{18}$$

and considering $a = a_R + ja_I$, we have,

$$\frac{\partial}{\partial a} = \frac{1}{2}\left(\frac{\partial}{\partial a_R} - j\frac{\partial}{\partial a_I} \right). \tag{19}$$

Finally, taking derivation from (17) with respect to a , and noting (18) and (19), we arrive at the following,

$$a_{opt} = \eta R^{-1} h, \tag{20}$$

where η is any arbitrary non-zero value and is defined such that a_{opt} to have any desired norm.

REFERENCES

[1] Y. Li, B. Vucetic, Z. Zhou and M. Dohler, “Distributed adaptive power allocation for wireless relay networks,” IEEE Trans. on Communications , vol. 6, no. 3, pp. 948-958, 2007.

[2] I. Maric. and R.D.Yates,” Bandwidth and power allocation for cooperative strategies in Gaussian relay networks,” Proceedings of the 38th Asilomar Conference on Signals, Systems and Computers, vol.2, pp.1907-1911, Nov. 2004.

[3] Y. Zaho, R.S. Adve and T.J.Lim,” Improving amplify-and-forward relay networks: optimal power allocation versus selection,” IEEE Trans. on Communications , vol.6, pp.3114-3123, Aug. 2007.

[4] T.S. Rappaport, Wireless Communications:Principles and Practice, Second ed. Prentice Hall, 2002.

[5] T. David. and V. Pramod., Fundamentals of Wireless Communication, Cambridge University Press, 2005.

[6] A. Nosratinia, T.E. Hunter and A. Hedayat, ”Cooperative communication in wireless networks,” IEEE Communications Magazine, vol. 42, no.10, pp. 74-80, Oct. 2004.

[7] A. Sendonaris, E. Erkip, Q. Aazhang and C.A. Campbell, ”User cooperation diversity. Part I. System description,” *IEEE Trans. on Communications*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.

[8] A. Sendonaris, E. Erkip, Q. Aazhang and C.A. Campbell, ” User cooperation diversity. Part II. Implementation aspects and performance analysis,” *IEEE Trans. on Communications*, vol. 51, no. 11, pp. 1939-1948, Nov. 2003.

[9] A. Sendonaris, E. Erkip and Q. Aazhang, ”Increasing uplink capacity via user cooperation diversity,” Proc. International Symposuim on Information Theory (ISIT), Cambridge, MA. 1998.

[10] J.N. Laneman, D. Tse and G. W. Wornell, ”diversity in wireless networks: Efficient protocols and outage behaviour,” *IEEE Trans. on Information Theory,* vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[11] J.N. Laneman, G. W. Wornell and D. Tse, ”An Efficient Protocol for Realizing Cooperative Diversity in Wireless Networks,” Proc. International Symposuim on Information Theory (ISIT), Washington, DC. p. 294, June 2001.

[12] X. Deng and A.M. Haimovich,"Power Allocation for Cooperative Relaying in Wireless Networks," IEEE Commun. Letters, vol. 9, no. 11, Nov. 2005.

[13] S. Huan, D. Wang, S. Meng and X. You, "Power Allocation Algorithm in Amplify-and-Forward Relay Networks," Proc. Wireless Communications, Networking and Mobile Computing (WiCOM), Dalian, pp. 1-4, Oct. 2008.

[14] Y. Li, B. Vucetic, Z. Zhou and M. Dohler, "Distributed Adaptive Power Allocation for Wireless Relay Networks," IEEE Trans.on Communications, vol. 6, no. 3, pp. 948-958, March 2007.

[15] M. Badr, E. C. Strinati and J.C. Belfiore, "Optimal Power Allocation for Hybrid Amplify-and-Forward Cooperative Networks," Proc. IEEE Vehicular Technology Conference, Singapore, pp. 2111-2115, May 2008.

[16] G. Zheng and K.K. Wong and A. Paulraj and B. Ottersten," Collaborative-Relay Beamforming with Perfect CSI: Optimum and Distributed Implementation," *IEEE Signal Processing Letters*, vol.16, pp. 257-260, April 2009.

[17] V. Havary-Nassab, Shahram Shahbazpanahi, A. Grami and Zhi-Quan Luo, "Distributed Beamforming for Relay Networks Based on Second-Order Statistics of the Channel State Information," IEEE Trans.on Signal Processing, vol. 56, no. 9, pp. 4306-4316, Sep. 2008.

[18] Y. Jing and H. Jafarkhani, "Network Beamforming Using Relays With Perfect Channel Information," *IEEE Trans. on Information Theory*, vol. 55, no.6, pp. 2499-2517, June 2009.

[19] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.