Applying hierarchical hub location problem on perishable good distribution systems

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ABSTRACT

Freshness and delivery time limits are non-negligible features in perishable good distribution systems. Designing an appropriate distribution system can decrease the transportation cost besides of increasing goods freshness level. In order to get this purpose, developing a hierarchical hub network system with a refreshing operation in the network can be valuable. The first level consists of a complete network that connects the central hubs and the second one is in the form of star networks connecting the remaining hubs to central hubs. Demand nodes as the third level are connecting to hubs and central hubs in a star form. In this paper which is proposed for the perishable goods distribution systems, different level hubs provide refreshment operations for such goods that their delivery time exceeds the freshness time limit. During the proposed model locations of hubs in different levels and allocation structure of nodes to them are determined in order to minimize the total transportation and operational costs in the network considering of goods freshness. For illustration of the proposed method, some computational studies have been reported using the CAB dataset. The analysis shows that the proposed hierarchical operational hub location model performs better than classical approaches for perishable goods.

Keywords: Hhierarchical hub location problem, Perishable good, Freshness limit, Receive time limit, Operational hub feature

1 INTRODUCTION:

Proper transportation system has a major impact on the quality of the final products. Improper logistic systems not only increase the total costs but also affect on the freshness of delivered goods to customers. Improper systems such as the absence of cooling equipment can cause the spoilage losses up to 33 percent of agricultural productions [1]. In spite of the importance of food spoilage there has not much study in designing and developing of the logistic networks for perishable goods. To the best of our knowledge there is not any study in designing a proper hub location problem with equipping to cooling system in hub centres for perishable goods.

Hub location problem in transportation systems is about using hub facilities as a switching centre to efficiently route commodities between origin and destination (*OD*) pairs to

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minimize the total transportation cost. Typically, there is a routing cost reduction factor, α , between two hub facilities to reflect savings due to economies of scale because of using hub facilities [2]. In these centres, cargo from various origins and similar destinations are collected together and transhipped to their destinations. In these networks cargoes cannot go from their origins to their destinations directly and at least must pass one hub facility. Hub location problems can be divided in to the single and multiple allocations strategies [3]. In this study the network is designed based on the single allocation strategy and demand nodes should be allocated to only one hub facility.

O'Kelly [4] presented the first model of hub location problem, then Campbell [5] presented a linear model for the *p*-hub median location problem. Many studies have been done on the hub location problem and many new versions of the basic model have been presented. In the study of Kara et al. [6] travel and latest arrival time were added to previous models. In recent studies Rodriguez et al. [7] and Serra et al. [8] Added new features to HLP by using queuing system in the hub location problem. Alumur et al. [9] considered an uncertainty of the fixed cost and demands to reach more applicable conditions while Li et al. [10] developed a model for multimodal hub location problem with capacity constraint attempted to get an applicable model in real world problems. Kim et al. [11] proposed a robust hub network in telecommunication networks by applying reliability measure to HLP.

Also in 1990, a different version of hub design which was the mixture of the hierarchical facility design with hub location problem has been introduced by Chou[12]. In this study the first hierarchical hub covering problem with different hub levels and different hub links to minimize the total travelling time in a delivery time restriction was proposed. Moreover to add the establishing cost of hub nodes in different levels Lin et al.[13] Presented a model for hierarchical hub location problem. In following Yaman [2]presented a mixed integer and a binary model by time limitations in deliveries. In their model central hubs as higher level hubs, are fully connected while the second level hubs are connected to their central hubs by single allocation strategy and made a star form network.

Agricultural, medical and dairy products have different lifetime from the production time ranging from 1 week to 1 month [14]. Normally these products start deterioration from the beginning of the production. The improper logistic and maintenance systems accelerate this deteriorating process. As a result, goods may get wasted before reaching to their destination in long routs. Therefore the customer satisfaction level will decrease with increasing of the spoilage. It is worth to mention that freshness of the goods is very important for the customers of these kind of products [15].

There are some studies that show the importance of the perishable goods to be discussed. For example in finding the optimal number of storage facilities for deteriorating items Hwang[16] presented a model in a stochastic approach. In routing and scheduling problem Chen[17] added the perishability feature and proposed a new model. Ahumada[18] proposed an operational model for the harvest and distribution of perishable agricultural products. Amorim et.al[14] introduced a new model of production and distribution planning of perishable goods. To the best of our knowledge there is not any work in transportation system of perishable goods in a hub network structure. The review of previous studies for the deteriorating items confirms that the design of a transportation network for such products is essential.

Using the hierarchical hub location model for transportation system will decrease the transportation cost because of the larger discount factor in central hubs and decreasing the number of total hub links. Besides this reduction, the delivery time of goods will increase, which can increase goods spoilage rate. Moreover by considering of both aforementioned studies it is realized that hierarchical hub networks with special equipments in the hub nodes can be useful for such deteriorating products. If the traveling time exceeds the fresh time limit, some extra cooling operation should be performed during the transportation inside of the hub facilities to keep the freshness requirement. Hence in this study a hierarchical operational hub network is proposed for perishable goods.

In this study mentioned operation are decided for the designed network based on the hierarchical hub. Therefore by equipping these hubs to the cooling system we could decrease the spoilage rate besides the transportation cost.

The rest of this paper is structured as follows: in section 2, the proposed model for the hierarchical hub location problem on perishable goods distribution system is provided with some numerical examples. Finally conclusions and remarks are presented in Section 3.

2 PROPOSED MODEL

In this section the mathematical model of the perishable goods distribution network with hierarchical approach is proposed.

2.1 Model formulation

Here we focus on the formulation of the model by considering problem assumptions. This model will decide the location and allocation of hub nodes and necessity of refreshing operations in selected hubs considering the length of products delivery time.

2.1.1 Variables and Parameters

Variables:

$$X_{ijh}=\begin{cases}1 & \text{if node } i \text{ is assigned to hub } j \text{ while hub } j \text{ is assigned to central hub } h, \\ 0 & \text{otherwise}\end{cases} \quad \forall i\in I, j\in H/\{i\}, h\in C$$

$f1_{ijh}$ the traffic amount originated at node *i* through hub *j* and central hub *h*

$$\forall i\in I, j\in H/\{i\}, h\in C$$

$f2_{ihl}$ the traffic amount originated at node *i* passing central hubs *h* and *l* $(h\neq l)$

$$\forall i\in I, h,l\in C$$

$$Z_{is}=\begin{cases}1 & \text{If delivery time of a product between nodes } i \text{ and } s \text{ exceeds the fresh time limit} \\ 0 & \text{Otherwise}\end{cases} \quad \forall i,s\in I, i\neq s$$

$$V_{isj}=\begin{cases}1 & \text{If a refreshing operation is done on traffic between } i \text{ and } s \text{ in hub } j \\ 0 & \text{Otherwise}\end{cases} \quad \forall i,s\in I, i\neq s, j\in H$$

dr_{is} the good delivery time originated from node *i* to its destination node *s*

AT_h The arrival time that all traffics from nodes and lower level hubs arrive to the hub *h*

RT_h The release time which all traffic of nodes and hub nodes that has been allocated to *h* can be leaved to their destination.

Sr_{is} The satisfaction rate obtained by refreshing operation between each pair nodes *i* and *s*

Parameters:

P Number of hub nodes

Pc Number of central hub nodes

W_{is} Flow originated in node $i \in I$ that is destined at node $s \in I$

T_{is} Travel time between node $i \in I$ and $s \in I$

α_h Cost discount factor between lower level hub and central hub node

α_c Cost discount factor between two central hub nodes

α'_h Time discount factor between lower level hub and central hub node *c*

α'_c Time discount factor between two central hub nodes

B Delivery time limit

Ft Freshness time restriction

RS The artificial satisfaction rate for refreshing operation

M A large value

LT_i Loading time in demand node *i*

UT_i Unloading time in demand node *i*

r_i The processing time at node *i*

C_{ij} The unit transportation cost from node *i* to hub node *j* which node *i* has been allocated to hub *j*

P_1 The unit transportation time cost of delivery time in the objective function

P_2 The unit artificial satisfaction rate income because of goods freshness

cr_j^f The fixed cost of refreshing operation in hub $j \in H$

2.1.2 Mathematic formulation

The objective function of the model contains transportation cost, operational cost, and delivery time cost and weighted satisfaction rate income. Hence the model will minimize the total cost and increases customers' satisfaction rate.

And the objective function of the proposed model can be stated as following:

$$Min \quad \sum_{i\in I}\sum_{s\in I}(w_{is}+w_{si})\sum_{j\in H}c_{ij}\sum_{h\in C}X_{ijh}+\sum_{i\in I}\sum_{s\in I}\sum_{h\in C/\{j\}}\alpha_h c_{jh} f1_{ijh}+\sum_{i\in I}\sum_{h\in C}\sum_{l\in C/\{h\}}\alpha_c c_{hl} f2_{ihl}$$
$$+P_1\sum_{i\in I}\sum_{s\in I}dr_{is}-P_2\sum_{i\in I}\sum_{s\in I}(Sr_{is})+P_3\sum_{i\in I}\sum_{s\in I}\sum_{j\in H}V_{is}cr_j^f \quad (1)$$

Subjected to

$$\sum_{j\in H}\sum_{h\in C}X_{ijh}=1 \quad \forall i\in I \quad (2)$$

$$X_{ijh}\leq X_{jjh} \quad \forall i\in I, j\in H/\{i\}, h\in C \quad (3)$$

$$\sum_{k\in H}X_{jkh}\leq X_{hhh} \quad \forall j\in H, h\in C/\{j\} \quad (4)$$

$$\sum_{j\in H}\sum_{h\in C}X_{jjh}=P \quad (5)$$

$$\sum_{h\in C}X_{hhh}=P_c \quad (6)$$

$$f1_{ijh}\geq \sum_{s\in I/\{j\}}(w_{is}+w_{si})(X_{ijh}-X_{sjl}) \quad \forall i\in I, j\in H, h\in C/\{j\} \quad (7)$$

$$\sum_{l\in C/\{h\}}f2_{ihl}-\sum_{l\in C/\{h\}}f2_{ilh}=\sum_{s\in C}w_{is}\sum_{j\in H}(X_{ijh}-X_{sjh}) \quad \forall i\in I, h\in C \quad (8)$$

$$AT_h\geq \sum_{j\in H}(LT_i+t_{ij}+\alpha_h t_{jh})X_{ijh} \quad \forall i\in I, h\in C \quad (9)$$

$$RT_l\geq AT_h+(\alpha_c t_{hl})X_{hhh} \quad \forall l\in C, h\in C \quad (10)$$

$$RT_l+\sum_{j\in H}(\alpha_l t_{lj}+t_{ji}+UT_i)X_{ijh}\leq \beta \quad \forall i\in I, l\in C \quad (11)$$

$$dr_{is}\geq r_i+X_{ijh}t_{ij}+X_{sjh}t_{sj} \quad \forall i\in I, s\in I/\{i\}, j\in H, h\in C \quad (12)$$

$$dr_{is}\geq r_i+X_{ijh}t_{ij}+\alpha_h(X_{ijh}+X_{skh}-1)(t_{jh}+t_{ks})+X_{skh}t_{sk} \quad (13)$$
$$\forall i,s\in I, i\neq s. \quad j,k\in H, j\neq k. \quad h\in C$$

$$dr_{is} \geq r_i + \sum_j \left(\left(t_{ij} + \alpha_h' t_{jh} \right) X_{ijh} \right) + \alpha_c' t_{h,l} \left(\sum_j X_{ijh} + \sum_k X_{skl} - 1 \right) + \sum_k \left(\left(t_{sk} + \alpha_l' t_{kl} \right) X_{skl} \right) \tag{14}$$
$$\forall i,s \in I, i \neq s. \quad j,k \in H, j \neq k. \quad l,h \in C, l \neq h$$

$$dr_{is} - ft \geq M(Z_{is} - 1) \quad \forall i,s \in I \tag{15}$$

$$dr_{is} - ft \leq M(Z_{is}) \quad \forall i,s \in I \tag{16}$$

$$V_{isj} \leq \sum_{h \in C} X_{ijh} + \sum_{h \in C} X_{skh} \quad \forall i,s \in I \quad j,k \in H \tag{17}$$

$$\sum_{j \in H} V_{isj} = Z_{is} \quad \forall i,s \in I \tag{18}$$

$$Sr_{is} = RS \sum_{j \in H} V_{isj} \quad \forall i,s \in I \tag{19}$$

$$X_{hjh} = 0 \quad \forall j \in H, h \in C / \{j\} \tag{20}$$

$$X_{ijh} = \{0,1\} \quad \forall i \in I, j \in H, h \in C \tag{21}$$

$$f1_{ijh} \geq 0 \quad \forall i \in I, j \in H, h \in C \tag{22}$$

$$f2_{ihl} \geq 0 \quad \forall i \in I, h \in C, l \in C / \{h\} \tag{23}$$

$$AT_h \geq 0 \quad \forall h \in C \tag{24}$$

$$RT_l \geq 0 \quad \forall l \in C \tag{25}$$

Constraint (2) satisfies the single allocation feature of this model. Constraints (3) and (4) ensure valid allocations for lower and higher levels hubs. In constraint (5) and (6) the number of hub nodes and central ones are determined. Traffic amount that originated in node i and passed through hub node j and central hub l is calculated in constraint (7). Constraint (8) is used to assess the flow balance between central hubs.

For each central hub nodes constraint (9) states the latest arrival time. Constraint (10) determines the latest arrival time to the second central hub in delivery path. Constraint (11) defines the delivery time limit between each pair nodes, where α_h' is time discount factor between lower level hub and central hub node c, α_c' is time discount factor between two central hub nodes and ß is the delivery time limit.

Constraints (12), (13) and (14) calculate the transportation time between pair nodes according to their allocated hubs. Whenever the transportation time between pair nodes i and s exceeds its freshness time limit (ft) constraints (15) and (16) assure that extra operation should be done on transferring of goods. Constraint (17) determines the hub that the refreshing operation should be done in lower or higher level hubs.

Constraint (18) ensures that only one refreshing operation is allowable passing different hubs. When an extra operation is needed, Sr_{is} shows the total artificial satisfaction rate because of good freshness using refreshing operation on a flow of traffic between pair node *(i,s)* which is determined by constraint (19). Constraint (20) states that a central hub cannot be assigned to a lower level hub as a demand node. And the rest of constraints show types of the different variables in the model.

2.2 Numerical examples

To get the better vision of the proposed model we solve some numerical examples with different sizes based on the CAB dataset and results of one of them with three states are reported. In the reported example, we have 10 demand nodes 3 hub nodes and three states of different number of central hub nodes (1,2 and 3) with $(\alpha_c = 0.7, \alpha_h = 0.9, \alpha'_c = 0.6, \alpha'_h = 0.8)$. In Table 1 the results of this example have been reported. At Table 2 we compare the results of the proposed model with the classical hierarchical hub model for perishable goods.

Table 1: Results of selected example with β=2760, *ft*=1500 by the proposed model

Number of central hubs	Hubs/central hubs	Objective value	Number of refreshing operations	Solution time
1	4,6,7/4	845045868	12	244 sec
2	4,6,7/4,7	805842738	6	846 sec
3	4,7,9/4,7,9	764205945	4	40 sec

Table 2: The comparison results obtained by the proposed and classic models for the selected example

	Total Cost	Spoilage Cost	Transportation Cost	Operational cost
Classic hierarchical hub	850732868	10287000	840445868	0
Proposed model	845045868	0	840445868	4600000

In this model to get the spoilage cost we use the difference of the delivery time of each pair nodes with the freshness time limit according to the equation 26.

Spoilage Cost = $P_{sp} * \sum_{i\in I}\sum_{s\in I}(dr_{i,s} - ft)$, **(26)**

where P_{sp} shows the good spoilage penalty cost per time unit. For examples for essential goods like drugs and blood get the larger value than the agricultural ones.

As shown in table 1 by increasing of the number of central hubs because of the discount factor of central hubs, the delivery time and the transportation cost will be decreased and the number of the refreshing operations is reduced. Also in table 2, it is shown that by using the classic model we have the same transportation cost, but because of existing of the spoilage cost we have the larger total cost comparing to the proposed model.

The transportation system of the first state of selected example (with one central hub) has been depicted in Figure 1. In this example, node 4 has been selected as central hub with refreshing feature. And nodes 6 and 7 are the lower level hubs while hub 7 is equipped with refreshing feature. Optimal position of refreshing operations in established hubs between each pair of nodes have been reported in Table 3.

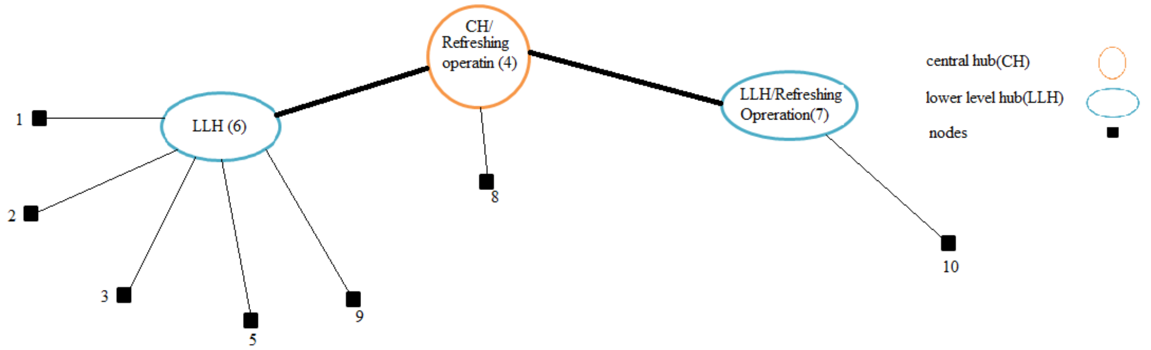

Figure 1: The optimal network structure of the first example with 10 nodes, 3 hubs and 1 central hub

Table 3: Required refreshing operations in established hubs between each pair of nodes

Pair nodes	Refreshing centre
(1,8)	4
(1,10)	7
(3,8)	4
(3,10)	7
(7,8)	7
Other pairs	-

3 CONCLUSION

In this study, a hierarchical hub location problem for perishable goods with refreshing feature in hub centres is proposed and a mixed integer model is presented. A complete network for central hub nodes has been used in this problem and the hub nodes are connected to their central hub with single allocation strategy. In the proposed model, hubs have an additional cooling operation called refreshing compared to the classic hub nodes. Whenever the delivery time of each pair nodes exceeds the freshness time limit, the proposed model decides to have an extra refreshing operation in corresponding hub or central hubs on the established path. Hence by using this network as transportation system for perishable goods we can decrease the spoilage rate with a little operational cost that could increase the satisfaction rate for the freshness of goods. The numerical study confirms that by the proposed model an efficient distribution network can be designed to deliver deteriorating products. As a direction for future works, different technologies in operational hubs can be studied in the proposed model. As another future study, extending of this model for the ameliorating products, which the longer delivery time will increase the users satisfaction rate.

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