Partial and total fortification consideration for reliable supply chains

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Abstract: Risks play an important role in distribution network management. In this paper we analyze the reliable capacitated facility design model under disruption in supply chain networks. Aim of this study is determining of the optimal location and allocation that minimizes the total expected costs of customers' travels by assuming a limited budget for facility fortification. In reality if a facility fails, its assigned customers are deflected to other facilities which lead to an excessive transportation cost. In this study partial fortification is considered for improving the network reliability while there is a limited budget, so it can be used for real applications. Then we formulate this problem as a mixed integer programming model and develop a lagrangian relaxation-based solution algorithm to solve the problem. So the exact formulation and an efficient heuristic are presented. Finally a numerical example is conducted to assess the performance of the solution approach.

Keywords: Disruption, Reliability, Partial and total fortification, Lagrangian relaxation.

1. Introduction

Supply systems are subject to enlargement risks of disruption causing important economic penalties. To mitigate the influence of supply chain risks, facility location problems have been extremely studied in the past few years. Snyder et al.[1]presented a perfect review of the literature on reliability. In continuing Berman et al. [2] and Shen et al. [3] extended new facility-location models with disruption and Cui et al. [4]removed the assumption of homogeneous failure probability, also in the field Li et al. [5] proposed a new modeling which transforms an infrastructure system with correlated facility failures into an analogous one with a supporting structure. Lim et al. [6] work on network design with fortification. On other hand several authors surveyed the uncapacited fixed-charge models with two types of facilities, unreliable and totally reliable or ''hardened'', and don't have assumed partial reliability, in reality they assumed one primary supplier and one totally reliable backup supplier for each customer. Although many authors incorporated the fixed cost of locating a reliable facility in the objective function, since fortification budget in formulation assumed unlimited, it may be in real world unfeasible. Hence Qingwei et al. [7] considered budget for fortification but without partially fortification, and capacity for facility that aren't in real world never, so we extend it in a finite budget fortification and capacity for facilities, moreover in this paper totally and partially fortification are considered to repel from further cost. Remainder of this paper is organized as follows: The model formulation is presented in the next section. Later an efficient heuristic algorithm (LR) is presented to solve this problem, and their efficiency is analyzed and the main results are summarized in the section3. Finally some concluding directions for future studies are presented in the last section.

2. Problem statement and formulation:

The goal of the proposed model is minimizing the expected transportation cost by optimally locating P facilities, allocating a finite fortification budget (B) for totally and partially reliability, and considering capacity for any facility. Is formulated this problem as a nonlinear integer programming model NP-hard, therefore we develop a lagrangian relaxation-based heuristic algorithm. If defined I and J the set of customers and potential facility locations respectively and other parameters be similar to[7], in addition Let r_j be percent of change in probability of failure of facility j, each customer should be assigned to a primary supplier and a different backup supplier. C_j, c_j, are capacity of facility and cost of partial fortification, respectively. Usually stipulated for any customer, backup supplier is available always our model incorporates the following extra decision variable: $a_j=1$ if facility j partially fortified otherwise 0.This model can be formulated follow:

$$\text{minimize} \sum_{i=1}^{n}\sum_{j=1}^{m}(y_{0ij}h_i d_{ij}(1+a_j r_j)(1-q_j(1-z_j))$$
$$+h_i d_{ij} y_{1ij}\sum_{k\neq j}^{m} y_{0ij}(1-z_k)(q_k-a_k r_k(1-q_k))) \quad (1)$$

$$z_j \leq 1-a_j \qquad \forall j \qquad (2)$$

$$\sum_{i=1}^{n} h_i(y_{0ij}+y_{1ij}) \leq C_j \qquad \forall j \qquad (3)$$

$$y_{0ij}+y_{1ij} \leq x_j \qquad \forall i,j \qquad (4)$$

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$$\sum_{j=1}^{m} (s_j + k_j q_j) z_j + a_j c_j \le B \qquad (5)$$

(5)

The objective function is the expected transportation cost for satisfying the demands of customers. The first term, represents the part of the expected transportation cost associated with customer i served by its primary supplier, where in partial fortification by $(1+a_j r_j)$ lead to larger availability, and reminder of model is the cost of customer i served by its backup supplier that term $q_k - a_k r_k (1-q_k)$ decrease failure probability of primary facility, constraints (2) demonstrate facilities can be only totally or partially fortified, also constraints (3) considered capacity for facilities and constraints (4) assure that only an open facility can serve as a supplier, also let different facilities for each customer, its primary and backup suppliers. Finally constraints (5) limits available budget.

2.1 Lower bound

In this reliability facility location model we can relax the set of constraints (2) or (4) or both of them by using Lagrange multipliers λ_{ij}. Generally the performance of lagrangian relaxation algorithms can be susceptible to the choice of first multipliers. In order to obtain it, we understand that the formula $\lambda_{ij}^0 = \frac{h_i}{|I|}$ generated efficient initial multipliers. When the algorithm starts running, at each iteration k, we utilize modified subgradient optimization [8] for updating multiplier as follows:

$$\lambda_{ij}^k = \max\{0, \lambda_{ij}^{k-1} + \gamma^k (s_{ij}^k)^2\} \qquad (6)$$

Where $s_{ij}^k = y_{0ij}^k + y_{1ij}^k - x_j^k \qquad (7)$

and $\gamma^k = \dfrac{\mu^k (z^* - z(\lambda^k))}{\sum_{i=1}^{n}\sum_{j=1}^{m} (s_{ij}^k)^2} \qquad (8)$

Wherein: z^* is the value of upper bound, and μ^k is a decreasing adapting parameter with $0 < \mu^0 \le 2$ and:

$$\mu^k = \begin{cases} \alpha\mu^{k-1} : if & \text{Z didn't increase in the last T iteration,} \\ \mu^{k-1} : o.w & \end{cases}$$

with parameters $0 < \alpha \le 1,\ T > 1$.

The algorithm finishes when $\dfrac{z^* - z(\lambda^k)}{z^*} \le \varepsilon$, for some optimality tolerance ε is specified by the user.

2.2 Upper bound

At each iteration of the lagrangian relaxation algorithm, a lower bound and an upper bound are obtained. We suggest an upper bound for a solution, by using the displacement answer of the solution to provide a lower bound, in relaxed model.

3. Numerical example

Suppose that there are 5 customers and 5 potential locations that should 2 facilities be opened, according parameters are considered for this problem, and α=μ=0.5 we relax constraints (4) and solve model by LR. Obtained lower bound results from Matlab and Cplex version (12.4) have been illustrated in table 1, that adjacent to original problem. Also in the example, fast convergence algorithm showed schematic below:

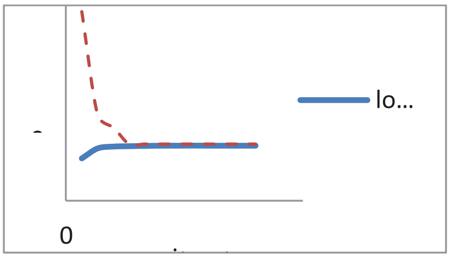

Figure 1: Convergence of lower and upper bounds of the algorithm for the numerical example.

TABLE 1

COMPARISON RESULTS OF OBJECTIVE FUNCTION VALUE BY DIFFERENT APPROACHES

3. CONCLUSIONS AND FUTURE RESEARCH

Solution (by GAMS)	Upper bound (LR)	Lower bound (LR)	relative gap
2003	2015	1985.481	0.01

In this paper we presented model for design of distribution networks exposed to risk of disruptions. We assumed both of partially and totally fortification and limited fortification budget and limited capacity for facilities. Results showed that the developed lagrangian relaxation-based heuristic algorithm is efficient for the presented model. This work can be extended for a dynamic environment in different periods. Moreover considering of dependent facilities failure probabilities can be as another future study direction.

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