Multi-objective Markov-based economic-statistical design of EWMA control chart using NSGA-II and MOGA algorithms

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Abstract: The exponentially weighted moving average (EWMA) control charts are useful for detecting small shifts in the process mean. In this paper, we investigate multi-objective economic-statistical design of the EWMA control charts and propose two evolutionary algorithms including non-dominated sorting genetic algorithm (NSGA-II) and multi-objective genetic algorithm (MOGA) to determine the optimal chart parameters. The cost function used in this paper is Lorenzen and Vance cost function. We also used quadratic Taguchi loss function to determine the costs of producing non-conforming items under both in-control and out-of-control situations. The average run length values in both in-control and out-of-control states are computed by using Markov chain approach. A numerical example is applied to compare the results of proposed algorithms in finding the Pareto optimal solution of the multi-objective economic-statistical model. Finally, a sensitivity analysis on the economic and the statistical criteria of the EWMA control chart under both proposed algorithms is conducted.

Keywords: statistical process control; economic-statistical design; average run length; ARL; non-dominated sorting genetic algorithm; NSGA-II; multi-objective genetic algorithm; MOGA; Markov chain; Taguchi loss function.

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1 Introduction

Statistical process control (SPC) aims to improve the quality of the product in a continuous manner. One of the most important tools of SPC is control charts first proposed by Shewhart. The most popular control charts is $\bar{X}$ control chart that is used for monitoring the process mean. However, the recent studies prove that the EWMA control chart has better performance in detecting shifts in the process mean than $\bar{X}$ control chart, especially when the magnitude of the shifts is small.

In traditional design of control charts, only the statistical properties were noticed by researchers. In statistical design, the sample size and control limit coefficient were obtained so that the chart's power in detecting a given shift in the quality characteristic as well as the probability of type I error were equal to predetermined values. However, the results of statistical design of control charts were not satisfactory in terms of economic criteria. Hence, incorporating economics criteria in control charts design were noticed by the researchers. Duncan (1956) proposed an economic model to determine the parameters of $\bar{X}$ control chart including sample size, sampling interval and control limit coefficient when a single assignable cause exists. Duncan (1971) also proposed the economic design of $\bar{X}$ control chart under multiple assignable causes. Lorenzen and Vance (1986) extended Duncan's model and proposed a generic approach in economic design of various control charts. In the proposed approach, the process could be continued during the searching for assignable cause and repairing the process. Another difference between the Duncan and Lorenzen and Vance models is using in-control and out-of-control average run length (ARL) criteria in Lorenzen and Vance instead of probabilities of type I and II errors. The Lorenzen and Vance cost function consists of three cost elements including the costs of production under in-control and out-of-control states, cost of sampling as well as the costs due to repair and false alarms. Torng et al. (1994) investigated the economic design of EWMA control chart based on Lorenzen and Vance

cost function. Park et al. (2004) extended the economic design of EWMA control chart in the case that the sampling interval as well as the sample size could be changed. Serel and Moskowitz (2008) presented the joint economic design of EWMA control charts for mean and variance based on Lorenzen and Vance cost function. Serel (2009) investigated economic design of EWMA control chart for monitoring the process mean and process variability based on Lorenzen and Vance cost function. He used Linear, quadratic and exponential loss functions in computing of the costs arising from producing non-conforming items. For more detailed information about economic design of control charts, refer to Ho and Case (1994) and Montgomery (1980).

The main weakness of economic design of control charts is neglecting the statistical properties such as probability of type I and II errors. On the other hand, the statistical design of control charts does not consider the economic criteria. Hence, the economic-statistical design of control charts has been suggested by some researchers to overcome the limitations of economic and statistical designs. In the economic-statistical design of control charts, both economic and statistical properties are considered, simultaneously. Saniga (1989) incorporated some statistical constrains to Duncan cost function and proposed the economic-statistical design of $\bar{X}$ and R control charts. Montgomery et al. (1995) proposed the economic-statistical design of EWMA control chart. Yu et al. (2010) studied the economic-statistical design of $\bar{X}$ control chart in the case that multiple assignable causes exist. Safaei et al. (2012) proposed a multi-objective model of the economic-statistical design of $\bar{X}$ control chart considering Taguchi loss function. For more information about economic-statistical design of control charts refer to review paper by Celano (2011).

In this paper multi-objective economic-statistical design of EWMA control chart based on Lorenzen and Vance cost function using non-dominated sorting genetic algorithm (NSGA-II) and multi-objective genetic algorithm (MOGA) is investigated. The Taguchi loss function and Markov chain approach are used for computing the costs of producing non-conformities and the ARL values, respectively. The structure of this study is organised as follows: in Section 2, the EWMA control chart is summarised. Then, the Lorenzen and Vance cost function is explained. Afterwards, the Markov chain approach as well as the quadratic loss function is discussed. In Section 3, the proposed model for multi-objective economic-statistical design of EWMA control chart is expressed. Then, the MOGA and NSGA-II algorithms are described. In Section 4, a numerical example is given to illustrate the performance of the MOGA and NSGA-II algorithms in optimising the economic-statistical proposed model. In Section 5, a sensitivity analysis on the economic and statistical criteria of the EWMA control chart under both proposed algorithms is performed. Finally, Section 6 is devoted to the conclusion and future researches.

2 EWMA control charts and the economic cost function

2.1 EWMA control charts

Roberts (1959) proposed the exponentially weighted moving average (EWMA) control charts as an alternative to the traditional Shewhart control charts. The EWMA control charts were then developed by Lucas and Saccucci (1990). The EWMA control charts

take into account both the current and previous samples of the process. Hence, they are more sensitive to small shifts than $\bar{X}$ control charts.

Assume that the variable X follows a normal distribution with mean μ_0 and variance σ_0^2. The EWMA control chart statistic can be computed as equation (1):

$$Z_t = \lambda \bar{X}_t + (1-\lambda) Z_{t-1}, \tag{1}$$

where $\bar{X}_t$ is the sample average at time t, λ is the smoothing constant, $0 < \lambda \leq 1$ and $Z_0 = \mu_0$. The lower and upper control limits of the EWMA control chart are computed according to equations (2) and (3), respectively:

$$LCL_{EWMA} = \mu_0 - L\sigma, \tag{2}$$

$$UCL_{EWMA} = \mu_0 + L\sigma, \tag{3}$$

where $\sigma = \sigma_0 \sqrt{\dfrac{\lambda}{(2-\lambda)n}}$ and L is the coefficient of control limits. An out-of-control signal is issued when the chart statistic Z_t falls outside the control limits.

2.2 The economic cost function

Determining the EWMA control chart's parameters including sample size (n), sampling interval (h), control limit coefficient (L) and smoothing constant (λ) by minimising a specific cost function is called the economic design. In the Lorenzen and Vance cost function that is used in this paper, it is assumed that the process starts with an in-control state and follows by an out-of-control state. We also assume that the time that the process remains in an in-control state follows an exponential distribution with mean $1/\theta$. When the process is out-of-control, the mean of the quality characteristic X becomes $\mu_1 = \mu_0 + \delta\sigma_0$, where μ_0 and σ_0 are the in-control mean and standard deviation of the quality characteristic X, respectively and δ is the magnitude of shift in the process mean. The Lorenzen and Vance function consists of three major cost elements including in-control and out-of-control production costs, sampling costs as well as repair and false alarms costs. Equation (4) represents the Lorenzen and Vance function that is derived by dividing the expected total cost per cycle to the expected cycle time:

$$C = \frac{\left\{\dfrac{C_0}{\theta} + C_1\left(-\tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right) + \dfrac{sF}{ARL_0} + w\right\}}{\left\{\dfrac{1}{\theta} + \left(1-\gamma_1\right)\dfrac{sT_0}{ARL_0} - \tau + nE + h\left(ARL_1\right) + T_1 + T_2\right\}} + \frac{\left\{\left[\dfrac{a+bn}{h}\right]\left[\dfrac{1}{\theta} - \tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right]\right\}}{\left\{\dfrac{1}{\theta} + \left(1-\gamma_1\right)\dfrac{sT_0}{ARL_0} - \tau + nE + h\left(ARL_1\right) + T_1 + T_2\right\}} \tag{4}$$

where the parameters are defined as follows:

h sampling interval

C_0 hourly cost of producing non-conformities while the process is in-control

C_1 hourly cost of producing non-conformities while the process is out-of-control,

τ expected time between an assignable cause and the time of the last sample taken before the assignable cause which is computed as equation (5):

$$\tau = \frac{1-(1+\theta h)e^{(-\theta h)}}{\theta\left(1-e^{(-\theta h)}\right)}, \tag{5}$$

E time of sampling and charting one item

ARL_0 ARL while the process is in-control

ARL_1 ARL while the process is out-of-control

T_0 expected search time when the signal is false

T_1 expected time required to detect the assignable cause

T_2 expected time to eliminate the assignable cause (repair the process)

γ_1 1 if the production continues during searches, 0 otherwise

γ_2 1 if the production continues during repair, 0 otherwise

s expected number of samples taken while the process is in control which is obtained by: $s = \dfrac{e^{(-\theta h)}}{1-e^{(-\theta h)}}$

F cost per false alarm

W cost of locating and repairing an assignable cause

a fixed cost per sample

b variable cost per item sampled.

The Lorenzen and Vance function is a general cost function and can be used in various control charts. When a statistic falls outside of control limits, the chart signals that the process is out-of-control and the search for assignable cause is initiated. Hence, in order to restore the process into the in-control state, a corrective action is done. As noted, in order to compute the costs due to producing non-conformities, the quadratic Taguchi loss function is used. The quadratic loss function is described in sub-Section 2.4.

2.3 Markov chain approach

The ARL criterion is defined as the expected number of consecutive samples taken until the control chart signals that the process is out-of-control. To reduce the total cost, the value of ARL under in-control and out-of-control states should be large and small, respectively. In this paper the values of ARL_0 and ARL_1 are calculated using Markov chain approach. This approach has been used in many researches such as Serel (2009). In

the Markov chain approach, the control limits interval are divided into the m (an odd integer) equal subintervals. The ith subinterval in EWMA control chart is obtained as follows:

$$R_i = [u_{i-1}, u_i], \quad i = 1, \ldots, m, \tag{6}$$

where $u_i = LCL_{EWMA} + i\Delta u$ and $\Delta u = (UCL_{EWMA} - LCL_{EWMA}) / m$. In order to calculate the transition probability of p_{ij}, we assume that when the EWMA statistic Z_t falls into i^{th} subinterval, it is at the midpoint of this subinterval. Hence, the value of p_{ij} is calculated as equation (7):

$$p_{ij} = P(Z_t \in R_j \mid Z_{t-1} \in R_i) = P(u_{j-1} < Z_t \le u_j \mid Z_{t-1} = (u_{i-1} + u_i)/2). \tag{7}$$

We assume that $u_{(m+1)/2}$ is equal to μ_0. The transition probabilities are determined as equation (8):

$$p_{ij} = f_{ij} - f_{i,j-1}, \quad i, j = 1, 2, \ldots, m, \tag{8}$$

where $f_{ij} = \Phi\left(\dfrac{2L(j-(1-\lambda)(i-0.5)-0.5\lambda m}{m\sqrt{\lambda(2-\lambda)}} - \delta\sqrt{n}\right) \quad i = 1, 2, \ldots, m,\ j = 0, 1, \ldots, m.$

Note that $\Phi(.)$ is the cumulative distribution function for the standardised normal distribution. The run length distribution of the EWMA control chart is determined by using the initial probability vector and transition probability matrix. The initial probability vector consists of the probabilities of the EWMA statistic starting in each state of the Markov chain. In this paper, we assume that the starting state of the Z_t statistic is the in-control mean with the probability equals to one.

The ARL values of the EWMA control chart if the process mean changes to the new value of $\mu_0 + \delta\sigma_0$ is calculated as follows:

$$\text{ARL} = \mathbf{e}_i^T (\mathbf{I} - \mathbf{P})^{-1} \mathbf{1} \tag{9}$$

where $\mathbf{P} = [p_{ij}]$ is a $m \times m$ matrix of p_{ij} values, $\mathbf{e}_i$ is a column vector with zero elements except the ith element which is equal to one, $\mathbf{I}$ is a $m \times m$ identity matrix, and $\mathbf{1}$ is a column vector of ones. Note that the value of δ is considered equal to zero in computation of ARL_0 value.

The value of ARL in the EWMA control chart depends on control limits which, in turn, depend on sample size, coefficient of control limits and smoothing constant. Hence, the design parameters of EWMA control chart are n, h, L and λ.

2.4 Quadratic Taguchi loss function

Traditionally, the costs due to producing non-conformities in both in-control state (C_0) and out-of-control states (C_1) have been considered as the constant values. In the other words, the quality loss was considered to the products that were outside the specification limits. However, in the Taguchi loss function approach, the cost of poor quality only depends on the value of deviation between quality characteristic and its corresponding target. Hence, the quality loss is considered for the quality characteristic that is not equal to its corresponding target. In this approach, increasing in deviation between the quality characteristic and the target leads to increasing in the value of quality loss. In recent

studies different types of loss function including linear, quadratic and exponential types have been used. One of the most common loss function used in designing various control charts is the symmetric quadratic loss function.

Consider a process with quality characteristic X with mean μ_0 and variance σ_0^2. Suppose that T and $f(x)$ are the target value and probability density function (pdf) of quality characteristic X, respectively. The expected quality cost per product under in-control and out-of-control states are calculated according to equations (10) and (11), respectively.

$$\begin{aligned} J_0 &= \int_{-\infty}^{\infty} K(x-T)^2 f(x)dx \\ &= \int_{-\infty}^{\infty} K\left(x-\mu_0+\mu_0-T\right)^2 f(x)dx \\ &= K\left(\sigma_0^2+\left(\mu_0-T\right)^2\right) \end{aligned} \tag{10}$$

$$\begin{aligned} J_1 &= \int_{-\infty}^{\infty} K\left(x-\mu_0-\delta\sigma_0+\mu_0+\delta\sigma_0-T\right)^2 f(x)dx \\ &= K\left(\sigma_0^2+\left(\mu_0-T\right)^2+\delta^2\sigma_0^2-2\delta\sigma_0\left(\mu_0-T\right)\right), \end{aligned} \tag{11}$$

where K is loss coefficient used for calculation of costs due to non-conformities. The hourly cost of producing non-conforming items in in-control and out-of-control states can be calculated by $C_0 = J_0p$ and $C_1 = J_1p$, respectively. Note that p is the number of items produced per hour.

3 Economic-statistical design of EWMA control chart using MOGA and NSGA-II algorithms

3.1 Multi-objective model of economic-statistical design of EWMA control chart

In the economic-statistical design of control charts, not only minimising the cost function but also achieving the proper statistical properties of control chart is considered. The basic idea in economic-statistical design is increasing a little in the value of cost function by improving the statistical properties of control chart. In single objective approach, the cost function is minimised by adding some statistical constraints. In this paper, optimisation of two objective functions including minimising the cost function as well as maximising the power of EWMA control chart in detecting different shifts in the process mean are considered. Moreover, two statistical constraints such as lower bound of in-control ARL (ARL_0^L) and upper bound of out-of-control ARL (ARL_1^U) are added to the model. In order to have only integer values for the sample size (n), we also add a constraint to the model and consider it in all parts of both algorithms including in generating initial population as well as in crossover and mutation operators. Note that implementing this constraint in the generation of initial population leads to obtain just integer values for n, and in crossover and mutation operators leads the generated values of n to remain integer.

Hence, the multi-objective economic-statistical model of the EWMA control chart for obtaining the chart parameters is as follows:

$$\begin{aligned} &Min\ C(n, h, L, \lambda) \\ &Min\ ARL_1 \\ &\text{subject to:} \begin{cases} ARL_0 \geq ARL_0^L \\ ARL_1 \leq ARL_1^U \\ n, L \text{ and } \lambda > 0 \text{ and } n \text{ is integer} \end{cases} \end{aligned} \tag{12}$$

In order to have solutions with small values of cost function, the values of ARL_0 and ARL_1 should be large and small, respectively. We use the penalty function approach for both algorithms in order to evaluate whether the solutions exceed from both constraints of the model. To do that, we add the value of 10,000 to the cost function in constraint violation situations. In the other words, if the value of ARL_0 is smaller than its corresponding lower bound or the value of ARL_1 is more than its corresponding upper bound, this penalty function is performed. Consequently, the solutions that violate at least one constraint are removed.

3.2 Optimisation algorithm

The objective functions in multi-objective optimisation problems are usually in conflict. Hence, the problem is rarely converged to the solutions that optimise all the objective functions, simultaneously. For example in the proposed model, as the value of cost function increase the ARL_1 function will also increase. To overcome such problems, some multi-criteria algorithms are presented. These evolutionary algorithms solve the model by searching various decision variables. In this paper, two algorithms including MOGA and NSGA-II algorithms are developed and applied for solving the multi-objective economic-statistical model of EWMA control chart.

3.2.1 Multi-objective genetic algorithm

To solve the non-linear programming models such as expected value of cost function in the economic and economic-statistical design of the EWMA control chart, some different algorithms are proposed by researchers. MOGA is a suitable method to solve multi-objective optimisation problems, because it works with a population of solutions. Hence, the MOGA generates a representation of many solutions and converges to the best solutions by evaluating different values of decision variables. The MOGA algorithm is also applied by some researchers such as Bashiri et al. (2013) for multi-objective economic-statistical design of $\bar{X}$ control chart. In the proposed algorithm, the population consists of ten chromosomes. Each chromosome is represented by a numerical string that is separated into different genes. For example in the proposed model each chromosome consists of four genes including sample size (n), sampling interval (h), control limit coefficient (L) and smoothing constant (λ). Figure 1 shows the representation of each chromosome:

Figure 1 Representation of a chromosome in MOGA algorithm

n	h	L	λ

The chromosomes are compared in terms of their corresponding objective functions and the chromosomes of optimal Pareto solution are selected.

The crossover is the operation in which two chromosomes are selected randomly and their selected genes are replaced with each other. In the proposed MOGA algorithm, the chromosomes with better objective functions have more chance to be selected for crossover operation. To crossover the selected chromosomes, the single-point crossover is used in the proposed method. The single-point crossover involves selecting random integer i between 1 and $k - 1$ uniformly (k is the number of genes in each chromosome.). Then, the variables (genes) in positions $i + 1$ to k are swapped between the selected parent chromosomes and generate the offspring chromosomes. For example, if the $i = 1$, the single-point crossover is as follows:

Figure 2 Representation of single-point crossover

Parent 1	n_1	h_1	L_1	λ_1
Parent 2	n_2	h_2	L_2	λ_2
Offspring 1	n_1	h_2	L_2	λ_2
Offspring 2	n_2	h_1	L_1	λ_1

Note that the crossover operation in MOGA algorithm is performed with the probability of 0.8. The mutation operator changes the selected chromosomes randomly in order to increase the variety of generated solutions as well as to prevent the solutions falling into local optimum. In the proposed MOGA algorithm, 20% of the population will be mutated through the mutation operator (the mutation operator is applied with the probability of $p_m = 0.2$). Note that the chromosomes with better objective functions have more chance to be selected for mutation operation.

Suppose that the chromosome that is selected for mutation operation is $x_t = (n_t, h_t, L_t, \lambda_t)$ and the lower and upper bound of decision variables are defined as $u = (u_n, u_h, u_L, u_\lambda)$ and $l = (l_n, l_h, l_L, l_\lambda)$. We also define the random variation vector of ε as follows:

$$\begin{aligned}\boldsymbol{\varepsilon} &= \left(\varepsilon_n, \varepsilon_h, \varepsilon_L, \varepsilon_\lambda\right) \\ &= \boldsymbol{\varepsilon}_\mathbf{l} + \left\{\text{uniform}[0,1].\left(\varepsilon_{u_n} - \varepsilon_{l_n}\right), \text{uniform}[0,1].\left(\varepsilon_{u_h} - \varepsilon_{l_h}\right),\right. \\ &\qquad \left.\text{uniform}[0,1].\left(\varepsilon_{u_L} - \varepsilon_{l_L}\right), \text{uniform}[0,1].\left(\varepsilon_{u_\lambda} - \varepsilon_{l_\lambda}\right)\right\}\end{aligned} \tag{13}$$

where the $\boldsymbol{\varepsilon}_u = (\varepsilon_{u_n}, \varepsilon_{u_h}, \varepsilon_{u_L}, \varepsilon_{u_\lambda})$ and $\boldsymbol{\varepsilon}_l = (\varepsilon_{l_n}, \varepsilon_{l_h}, \varepsilon_{l_L}, \varepsilon_{l_\lambda})$ are the upper and lower bound of variation vector (ε), respectively. Then, the genes of the mutated chromosome are as follows:

$$n_t = round\left[\min\left\{\max\left\{l_n, n_t + \varepsilon_n\right\}, u_n\right\}\right] \tag{14}$$

$$h_t = \min\left\{\max\left\{l_h, h_t + \varepsilon_h\right\}, u_h\right\} \tag{15}$$

$$L_t = \min\left\{\max\left\{l_L, L_t + \varepsilon_L\right\}, u_L\right\} \tag{16}$$

$$\lambda_t = \min\left\{\max\left\{l_\lambda, \lambda_t + \varepsilon_\lambda\right\}, u_\lambda\right\} \tag{17}$$

Note that, considering that the first decision variable (n) is integer and the others are continuous, we generate only integer values for decision variable n. Then, applying the explained crossover and mutation operators leads the values of decision variable n remain integer.

In order to obtain the Pareto optimal solution by applying of the MOGA algorithm, we use an iterative approach. In this approach, first the cost objective function is minimised with considering of the both constraints. Then, the obtained chromosomes by optimising cost function are considered as the initial population for optimising the ARL_1 objective function. At this step, the ARL_1 is minimised with considering of both constraints using the new initial population. We continue this process ten times where in each step the quality of solutions will improve and solutions will converge to the best solutions. Note that in each objective function optimisation, the MOGA algorithm is continued until the number of iterations is satisfied.

3.2.2 Non-dominated sorting genetic algorithm

One of the most effective algorithms for determining the optimal Pareto solutions is NSGA-II algorithm.

In the proposed NSGA-II algorithm, each chromosome consists of four genes and each gene shows a decision variable. The chromosomes are compared in terms of their corresponding fitness function. The representation of each chromosome is similar to the one discussed in the MOGA algorithm (see Figure 1):

For adapting the NSGA-II algorithm in order to obtain the optimal solution of multi-objective economic-statistical model of EWMA control chart the following steps are performed:

1 Generate initial population of size n-pop, randomly.

2 Calculate out-of-control ARL (ARL_1), in-control ARL (ARL_0) and cost function (C) for each chromosome.

3 Rank the initial population by using non-domination criteria.

4 Compute crowding distance for the initial population.

5 Use the crossover and mutation operator to generate intermediate population of size n-pop.

6 Evaluate objectives (C and ARL_1) and constraints for this created intermediate population.

7 Combine the parent and intermediate populations, then rank them and compute the crowding distance.

8 Select population that has best individuals based on the rank and crowding distance criteria as a new population of size n-pop.

9 Go to Step 3 and repeat the steps until the stopping rule (number of generations) satisfies.

In the proposed NSGA-II algorithm, the population size is considered equal to ten (n-pop = 10). The crossover operation is performed with the probability of 0.8. In the mutation operation, first we choose one chromosome and select one gene from the

selected chromosome randomly. Then, the selected gene is taken a value from the defined interval for the corresponding decision variable. Note that, since the values for n is generated by an integer command in Matlab software, hence, the value of n remains integer after the mutation operation. The mutation operator that is used in the NSGA-II algorithm creates the mutated offspring chromosomes using adaptive mutation of the genes with the probability of 0.2. Also, the crowding distance means the relative closeness of a solution to other solutions in the population and is applied for the solutions of each front with the same rank. Eventually, chromosomes with Pareto optimal values are reported as the Pareto solution set for multi-objective economic-statistical design of the EWMA control chart. For more detailed information about NSGA-II algorithm refer to Deb (2001) as well as Deb et al. (2002).

4 Numerical example

In this section a numerical example is presented to illustrate the performance of the proposed method. Consider a process with the parameters as the follow: the fixed cost per sample is 5, the variable cost per item sampled is 1, the cost of locating and repairing an assignable cause is 150, the cost per false alarm is 900, the time of sampling and charting one item is 0.5 hour, the expected search time when a false alarm is occurred is 0.5, the expected time required to detect the assignable cause is 0.5 hour, the expected time to eliminate the assignable cause is 0.75, the production rate is 120 items per hour.

The time that the process remains in an in-control state follows an exponential distribution with mean $1/100$. The process continues during the search and ceases during repair, i.e., $\gamma_1 = 1$ and $\gamma_2 = 0$. The quality characteristic's mean is increased by a shift with the magnitude of 0.86 standard deviations when an assignable cause occurs. A quadratic loss function with parameters $K = 4$, $\mu_0 = T$ and $\sigma_0^2 = 1$ is used for computing the costs of non-conformities. See Table 1:

Table 1 The parameters of the process and control charting

$\gamma_1 = 1$	$a = 5$	$E = 0.5$	$p = 120$	$\delta = 0.86$	$W = 150$	$T_1 = 0.5$	$\sigma_0^2 = 1$
$\gamma_2 = 0$	$b = 1$	$T_0 = 0.5$	$K = 4$	$\theta = 0.01$	$F = 900$	$T_2 = 0.75$	$\mu_0 = T$

As the statistical constraints, the lower bound of in-control ARL (ARL_0^L) and upper bound of out-of-control ARL (ARL_1^U) equals to 320 and 5 are considered, respectively. The values of ARL in in-control and out-of-control states are determined by Markov chain approach in which the interval between upper and lower control limits are divided into 15 equally subintervals. The decision variables are assumed to be limited as follows: $n \leq 20$, $L \leq 4$ and $0.01 \leq \lambda \leq 0.99$.

We solve the proposed model with MOGA and NSGA-II algorithms to obtain the optimal parameters of EWMA control chart. The optimal parameters obtained from solving the model via MOGA and NSGA-II algorithms are summarised in Tables 2 and 3, respectively. Moreover, the corresponding ARL_0 values of these parameters are given. Note that in this paper, all simulations by both MOGA and NSGA-II algorithms are implemented in single run by using MATLAB computer package.

Table 2 Optimal parameters resulted from the MOGA algorithm

n	L	h	λ	ARL_0	ARL_1	C
5	3.12	1.31	0.32	640.88	3.92	511.19
6	3.15	1.25	0.52	622.36	3.50	511.70
7	3.12	1.24	0.63	556.53	3.12	512.71
8	3.10	1.15	0.71	529.14	2.81	514.21
8	3.16	2.45	0.57	640.38	2.75	515.97
11	3.22	1.80	0.51	810.96	2.20	516.79
12	3.03	1.29	0.78	417.57	1.86	519.58
14	3.02	1.80	0.58	412.21	1.66	520.53
16	3.09	1.77	0.53	515.00	1.59	524.11
17	3.28	2.13	0.64	959.94	1.58	524.56
18	3.12	2.30	0.75	515.10	1.39	525.21
19	3.15	1.75	0.98	604.58	1.37	528.93
20	1.91	3.25	0.95	857.12	1.36	529.80

Table 3 Optimal parameters from the NSGA-II algorithm

n	L	h	λ	ARL_0	ARL_1	C
2	2.03	1.70	0.67	536.29	1.39	493.30
7	1.10	2.30	0.56	556.59	1.30	503.31
8	2.63	2.34	0.54	867.54	1.26	504.83
8	3.58	2.12	0.47	868.75	1.26	504.91
9	3.02	2.73	0.56	382.35	1.19	506.66
10	1.22	2.78	0.33	437.26	1.13	507.95
10	1.31	2.77	0.33	420.63	1.13	507.98
10	1.78	2.76	0.56	415.46	1.13	507.99
11	1.96	2.92	0.46	504.14	1.12	509.66
11	2.37	2.93	0.50	342.59	1.12	509.92
12	2.33	3.16	0.23	574.78	1.11	511.28
12	2.56	3.15	0.22	561.51	1.11	511.29
13	1.06	3.36	0.52	722.35	1.09	512.81
14	1.72	3.41	0.56	818.43	1.07	514.35
14	2.32	3.41	0.66	783.01	1.07	514.36
15	2.06	3.61	0.63	540.79	1.05	516.10
15	1.51	3.61	0.48	430.47	1.05	516.20
16	2.87	3.75	0.30	670.94	1.03	517.55
17	1.01	3.92	0.40	512.15	1.02	519.23
17	0.87	3.63	0.42	507.71	1.02	519.27
18	1.49	4.05	0.41	627.87	1.01	520.71
18	1.65	4.29	0.36	627.94	1.01	520.72
19	1.97	4.49	0.57	459.60	1.00	522.35
19	1.34	4.49	0.38	397.64	1.00	522.41

The optimal Pareto solution of Table 2 consists of 15 chromosomes by average computational time of 40.15 seconds for each solution which is obtained by a single run. We define the vector of (n, L, h, λ) and (C, ARL_1) as the vectors of decision variables and objective functions, respectively. Table 2 shows that the optimal decision variables vector from economic view is (5, 3.12, 1.31, 0.32) with objective functions vector of (511.19, 3.92). Table 2 also shows that increasing the value of cost function from 511.19 to 511.70 leads to decreasing the value of ARL_1 function 3.92 to 3.50. In the other words, the value of ARL_1 function decreases 10.77% if the value of cost function increases 0.1%. Moreover, as the sample size increases in the vectors of Pareto solutions, the values of cost functions increase whereas the value of ARL_1 functions decrease.

The optimal Pareto solution of Table 3 consists of 24 chromosomes by average computational time of 31.67 seconds for each solution which is obtained by a single run. Table 3 shows that the optimal decision variables vector from economic view is (7, 1.10, 2.30, 0.56) with objective functions vector of (503.31, 1.30). Table 3 also shows that increasing the value of cost function from 503.31 to 504.83 leads to decreasing the value of ARL_1 function from 1.30 to 1.26. In the other words, the value of ARL_1 function decreases 3.08% if the value of cost function increases 0.3%. Note that similar to the MOGA algorithm, as the sample size increases in the vectors of Pareto solutions, the values of cost functions increase whereas the value of ARL_1 functions decrease.

5 Sensitivity analysis

In this section, first a sensitivity analysis on the economic and statistical criteria of the EWMA control chart under both proposed algorithms is conducted. Then, another sensitivity analysis with respect to tuning parameters of both algorithms is also performed. The economic and statistical parameters are as follow:

The time of sampling and charting one item (E) with values of 0.05 and 0.5, cost per false alarm (F) with values of 300 and 900 and the process mean shift with values of 0.86 and 1 standard deviation. The solutions corresponding to minimum value of cost function within the Pareto optimal solution of each eight states from MOGA and NSGA-II algorithms are given in Tables 4 and 5, respectively.

Table 4 The results of sensitivity analysis on process parameters using MOGA algorithm

No.	E	F	δ	n	h	L	λ	ARL_0	ARL_1	C
1	0.05	300	0.86	12	1.70	3.02	0.60	410.99	1.85	499.90
2			1	11	1.77	3.10	0.71	529.14	1.60	501.14
3		900	0.86	12	1.58	3.16	0.69	644.17	1.69	499.72
4			1	14	2.20	3.23	0.93	795.23	1.41	501.81
5	0.5	300	0.86	7	1.77	3.23	0.54	828.18	3.21	513.81
6			1	7	1.31	3.03	0.58	421.11	2.23	515.75
7		900	0.86	5	1.31	3.12	0.32	640.88	3.92	511.19
8			1	5	1.23	3.01	0.20	529.55	3.33	516.45

Table 5 The results of sensitivity analysis on process parameters using NSGA-II algorithm

No.	*E*	*F*	δ	*n*	*h*	*L*	λ	ARL_0	ARL_1	*C*
1	0.05	300	0.86	5	2.36	3.38	0.49	550.75	1.05	489.46
2			1	4	1.94	1.96	0.76	612.40	1.14	491.94
3		900	0.86	2	1.79	2.11	0.42	441.54	1.41	490.61
4			1	4	2.39	2.23	0.57	425.52	1.08	492.27
5	0.5	300	0.86	3	2.05	1.69	0.69	394.00	1.12	493.37
6			1	3	1.59	1.92	0.42	388.38	1.26	498.27
7		900	0.86	2	1.70	2.03	0.67	536.29	1.39	493.30
8			1	2	1.56	1.73	0.64	334.23	1.53	498.45

The average computational time of the eight chromosomes of Table 4 is equal to 40.21 seconds for each solution. The results of Table 4 show that if the magnitude of mean shift increases from 0.86 to 1, the value of cost function will increase whereas the value of ARL_1 objective function decreases. The results of Table 4 also confirm that increasing in the value of the time of sampling and charting one item (E) leads to increase in the value of both cost and ARL_1 objective functions.

The average computational time of the eight chromosomes of Table 5 is equal to 31.62 seconds for each solution. We can conclude from Table 5 that as the magnitude of mean shift increases from 0.86 to 1 standard deviation, the value of cost objective function will increase. The results of Table 5 also confirm that as the value of the time of sampling and charting one item (E) increases, the value of cost function will increase and the value of ARL_1 increases most of the time.

We also investigate the effect of tuning parameters including probabilities of crossover and mutation operators on the optimal chart parameters as well as economic and statistical criteria. For each algorithm, we consider three situations for probability of crossover (p_c) and probability of mutation (p_m) in which the sum of these probabilities are equal to one. The solutions corresponding to minimum value of cost objective function within the Pareto optimal solution of MOGA and NSGA-II algorithms are given in Tables 6 and 7, respectively. Note that these simulations are done with the same parameters of the numerical example in Section 4 (see also Table 1).

Table 6 Results of sensitivity analysis on tuning parameters of MOGA algorithm

No.	p_c	p_m	*n*	*h*	*L*	λ	ARL_0	ARL_1	*C*
1	0.75	0.25	6	1.34	3.23	0.25	944.72	3.71	512.40
2	0.8	0.2	5	1.31	3.12	0. 32	640.88	3.92	511.19
3	0.85	0.15	5	1.55	2.99	0.39	404.97	3.64	511.50

Table 7 Results of sensitivity analysis on tuning parameters of NSGA-II algorithm

No.	p_c	p_m	*n*	*h*	*L*	λ	ARL_0	ARL_1	*C*
1	0.75	0.25	5	2.17	1.78	0.48	536.27	1.36	499.70
2	0.8	0.2	2	2.03	1.70	0.67	536.29	1.39	493.30
3	0.85	0.15	3	1.51	1.29	0.48	559.49	1.77	497.54

The average computational time of three solutions of Tables 6 and 7 is equal to 40.33 and 31.30 respectively for each solution. The results in Tables 6 and 7 show that the best solution from the economic view is obtained in the second state of tuning parameters where the probabilities of crossover and mutation are equal to 08 and 0.2, respectively in the both algorithms.

6 Conclusions and future researches

In this paper, multi-objective economic-statistical design of EWMA control chart based on Lorenzen and Vance cost function using two evolutionary algorithms including MOGA and NSGA-II was investigated. The quadratic Taguchi loss function and Markov chain approach was also used for computing the costs of producing non-conformities and ARL values, respectively. The results showed that the NSGA-II algorithm has a better performance than the MOGA algorithm in economic-statistical design of the EWMA control chart. The results also confirmed that a little increase in the value of cost function will increase the power of control chart in detecting different shifts in the process mean. After that a sensitivity analysis on chart parameters including time of sampling and charting one item (E), cost per false alarm (F) and magnitude of the mean shift was conducted. The results of sensitivity analysis in both algorithms revealed that when the magnitude of mean shift increases, the value of cost function will increase. These results also confirmed that the value of both objective functions will increase if the value of E increases. The sensitivity analysis on tuning parameters of both MOGA and NSGA-II algorithms was also performed. The results showed that the best performance of the EWMA control chart from economic view is obtained when the probabilities of the crossover and the mutation are equal to 0.8 and 0.2, respectively.

The algorithms applied in this paper can be compared with strength Pareto evolutionary algorithm 2 (SPEA2) as a future research. In addition, designing other control charts such as cumulative sum (CUSUM) control chart or multivariate control charts such as multivariate cumulative sum (MCUSUM) using multi-objective approach can be considered as fruitful issues for studies.

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