

# Preferred Robust Response Surface Design with Missing Observations Based on Integrated TOPSIS-AHP Method: An Application for a Nanolubrication Industry

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**Abstract**—Missing observations appear in experimental designs as a result of insufficient sampling, machine breakdown and high costs and errors in measurements. In many real applications in nanomanufacturing, experiments deal with a design with missing observations because the factors combination or molecular structure selected by a designer cannot be experimented simply. In a nano-scale environment, effects of design parameters on product characteristics cannot be ignored. So, selection of preferred robust response surface designs with missing observations is a critical decision for economic performance of nanomanufacturing because the design is insensitive to risk of missed values. In this paper, Box-behnken and Face-centered composite designs are studied because of their widely applications. For this purpose, eight robustness criteria including, D-efficiency,  $t_{max}$ ,  $t_{max} (1-\alpha)$  and their related sub-criteria are considered for evaluating of the robustness of mentioned designs. Finally, the integrated TOPSIS-AHP methodology is applied to select the most suitable robust design. Numerical example illustrates applicability of the proposed approach.

**Keywords**—Robustness criteria; Risk of missed values; preferred robust response surface design; TOPSIS-AHP methodology; Nanomanufacturing

## I. INTRODUCTION

Missing observation has attracted a lot of attention in the recent decades in experimental designs [1], [2], [3]. Missing observations appear in experimental designs as a result of insufficient sampling, high costs, and errors in measurements or during data acquisition [4]. Also, machine breakdown, illegible recording of response and damaged experimental resource are the common reasons, too [2]. Moreover, missing observations can be appeared in the nanomanufacturing industries while only some combinations of factors can be experimented successfully. As an example, the study of gas phase nano-scale lubrication experiments can be an application of missing observations in experimental designs, so it confirms the necessity of this study [2]. Hence, in many circumstances manufacturing industries face with this problem. To overcome with this problem, experimenters have used robust criteria to investigate the robustness of design. Some authors have investigated the robustness of design experiments where we deal with missing observations [4], [5], [6], and [7]. Ghosh investigated robustness of BIBD in the presence of missing observations [8]. MacEachern et al. studied a number of techniques for finding  $t_{max}$  and applied it for evaluating the robustness of central composite designs and factorial experiments against

missing data [9]. Evaluating the robustness of Box-Behnken design in the presence of missing observations has been done by Whitting [10]. For some recent researches, Navinchandra et al. proposed three new Bayesian algorithms based on predictive ability and minimization of the residual sum of squares in the presence of missing observations for factorial design [2]. Tanco et al. extended the new robustness criterion that provides a better assessment of the robustness of each design than previous criteria and evaluated robustness of some design experiments by three robust criteria including D-efficiency,  $t_{\max}$  and  $t_{\max}(1-\alpha)$  [3].

On the other hand, multiple criteria decision making problems can be divided to two main categories including multiple objective decision making (MODM) and multiple attribute decision making (MADM). Choice of preferred robust response design with various robustness criteria is a MADM problem including some methods such as, technique for order preference by similarity to ideal solution (TOPSIS)[11], analytic hierarchy process (AHP)[12], elimination and choice translating reality (ELECTRE)[13], grey relational analysis (GRA)[14], vjse kriterijumska optimizacija kompromiseno resenje (VIKOR)[13] and etc. To determine the weights of each criterion in TOPSIS method, we should apply an efficient tool. For this purpose, we consider the AHP method which is based on the decision maker judgments developed by Saaty [15]. Some researches have been done based on integrated two decision making methods such as AHP-VIKOR for plant location problem considered by Tavakkoli and Mousavi [16]. Also, Rao and Davim applied a combined TOPSIS and AHP method for material selection [17].

For Analyzing robustness of designs at first, we consider Box-behnken and Face-centered composite designs. We use eight robustness criteria for evaluation of the robustness of Box-behnken and Face-centered composite designs including worst D-efficiency, average D-efficiency,  $t_{\max}$ ,  $t_{\max}(0.99)$ ,  $t_{\max}(0.95)$ ,  $t_{\max}/n$ ,  $t_{\max}(0.99)/n$  and  $t_{\max}(0.95)/n$  that are explained in the next section.

The structure of the paper is as follows: In section 2, we introduce the robustness criteria applied for investigated designs. Explanations of combined TOPSIS-AHP methodology is presented in section 3. Then, the proposed methodology is applied in a case study to distinguish more preferred robust designs by ranking designs in section 4. Finally, concluding remarks and some future directions are presented in the last section.

## II. ROBUSTNESS CRITERIA FOR EVALUATING OF EXPERIMENTAL DESIGNS

Factorial designs are widely used in experimental design. These designs include several factors and it is essential to investigate the significance of the main and interaction effect of the factors on a response. The most crucial of these designs is including  $k$  factors with two

levels. The factors can be quantitative such as temperature, pressure or qualitative such as machines and operators [18]. However, this design cannot estimate quadratic relationships. However, it is necessary to estimate quadratic relationships in some experimental designs. For this purpose, we consider Box-behnken and Face-centered composite design that can estimate a full second-order polynomial model. The robustness in the presence of missing observations in the mentioned designs was studied for three to six factors with four center points using the three robustness criteria [3], [19]. In this section, we introduce three of most common robust criteria and the value of robust criteria that obtained for Box-behnken and Face-centered composite designs.

### A. Explanation of robustness criteria for experimental designs

The first criterion is D-efficiency that proposed by Ghosh [20]. Lal et al. use A-efficiency for evaluating robustness. The A-efficiency leads to minimizing the trace of  $(X^T X)$  and investigates the effect of missing observation on the sum of variances of the regression coefficients [21]. Hence, we prefer to apply D-efficiency instead of A-efficiency to investigate the robustness of design. There is a fact that in most cases in experimental design some observations are more importance than the others. It is obvious that if the most important observation is missing, the overall loss in efficiency is greater than when the least important observation is missing. Hence, the D-efficiency of the remaining design after missing some row of design matrix ( $X_1$ ) compared to the original design matrix ( $X$ ) is defined as

$$D\text{-efficiency} = \frac{\left| \begin{matrix} X_1^T & X_1 \\ X^T & X \end{matrix} \right|}{\left| X^T X \right|} \quad (1)$$

This criterion minimizes the volume of the joint confidence region on the vector of regression coefficients. We assumed two criteria including the average of D-efficiency and the minimum value (worst case) when we have only one arbitrary row of design as missing. Table I shows the D-efficiency values related to Box-behnken and Face-centered composite designs. In Table I,  $k$  is the number of factors of designs and  $n$  is the number of runs.

TABLE I. AVERAGE AND WORST VALUES OF D-EFFICIENCY IN FCC AND BB DESIGNS

| $k$ | $n$ | FCC     |        | BB |         |        |
|-----|-----|---------|--------|----|---------|--------|
|     |     | average | worst  | n  | average | worst  |
| 3   | 18  | 0.4440  | 0.2060 | 16 | 0.3750  | 0.2500 |
| 4   | 28  | 0.4643  | 0.3412 | 28 | 0.4643  | 0.4167 |
| 5   | 30  | 0.3000  | 0.0353 | 44 | 0.5227  | 0.5000 |
| 6   | 48  | 0.4167  | 0.3236 | 52 | 0.4615  | 0.4375 |

The next considered criterion is  $t_{\max}$  that proposed by Ghosh [20]. Assume the following ordinary linear model:

$$y = X\beta + \varepsilon, \quad (2)$$

where  $X$  is  $n \times p$  matrix that  $n$  rows are of the form  $(1, x_{1i}, \dots, x_{ki}; x_{1i}, x_{2i}, \dots, x_{k-1,i}, x_{ki}; x_{1i}^2, \dots, x_{ki}^2)$ . For the second-order polynomial model,  $p$  is defined as  $p = (k+1)(k+2)/2$  that  $k$  is the number of factors.

If the remaining design matrix  $(n-t) \times p$  that is obtained after omitting  $t$  observations is able to estimate all the parameters, we can conclude that design matrix  $X$  is robust against missing observations. To check it, we apply the following Ghosh definition:

$$t_{\max} = \max \left\{ t \mid 1 \leq t \leq n - p, \text{ and each } (n-t) \times p \text{ matrix yields } |X_t^T X_t| \neq 0 \right\}.$$

To evaluate the robustness of design, we consider two criteria  $t_{\max}$  and  $t_{\max}/n$  where  $n$  is the number of runs. The obtained results are shown in Table II.

TABLE II.  $T_{\max}$  VALUES FOR BB AND FCC DESIGNS

| k | FCC |            | BB |              |
|---|-----|------------|----|--------------|
|   | n   | $t_{\max}$ | n  | $t_{\max}/n$ |
| 3 | 18  | 3          | 16 | 6.3          |
| 4 | 28  | 3          | 28 | 10.7         |
| 5 | 30  | 3          | 44 | 6.8          |
| 6 | 48  | 3          | 52 | 5.8          |

The third criterion is  $t_{\max}(1-\alpha)$  proposed by Tanco et al. [3]. This criterion is defined as the maximum number of observations that can be missing and we can estimate parameters of model with a high probability. This criterion is defined as:

$$t_{\max}(1-\alpha) = \max \left\{ t \mid 1 \leq t \leq n - p, \text{ and } P(\text{Model is not estimable} \mid x_t) \leq \alpha \right\},$$

where the probability of model is not estimable computed as

$$P(\text{Model is not estimable} \mid x_t) = \frac{\sum_{i=1}^{\binom{n}{t}} I[|X_{t,i}^T X_{t,i}| = 0]}{\binom{n}{t}},$$

where  $\binom{n}{t}$  is the total number of combinations that observation can be missed and indicator  $I$  counts the total number of combinations that  $I[|X_{t,i}^T X_{t,i}| = 0]$  and the model is not estimable.

To compare robustness of designs using this criterion,  $t_{\max}(0.95)$  and  $t_{\max}(0.99)$  were computed which are defined as the maximum number of observations which are allowed to be missing and we can estimate the model with probability of 95% and 99%, respectively. The obtained results are summarized in the following table.

TABLE III.  $T_{\max}(0.99)$  VALUES FOR BB AND FCC DESIGNS

| k | FCC |                  | BB |                    |
|---|-----|------------------|----|--------------------|
|   | n   | $t_{\max}(0.99)$ | n  | $t_{\max}(0.99)/n$ |
| 3 | 18  | 3                | 16 | 6.20               |
| 4 | 28  | 5                | 28 | 17.8               |
| 5 | 30  | 4                | 44 | 20.4               |
| 6 | 48  | 8                | 52 | 17.3               |

TABLE IV.  $T_{\max}(0.95)$  VALUES FOR BB AND FCC DESIGNS

| k | FCC |                  | BB |                    |
|---|-----|------------------|----|--------------------|
|   | n   | $t_{\max}(0.95)$ | n  | $t_{\max}(0.95)/n$ |
| 3 | 18  | 4                | 16 | 12.5               |
| 4 | 28  | 7                | 28 | 25.0               |
| 5 | 30  | 5                | 44 | 27.2               |
| 6 | 48  | 10               | 52 | 25.0               |

### III. INTEGRATED TOPSIS-AHP METHOD

In this section, we introduce TOPSIS method applied to distinguish more preferred robust designs. To select the best design by TOPSIS method, we should determine the relative importance of different robust criteria. AHP provides such a procedure. The TOPSIS-AHP method includes the following steps:

Step1. Assume a decision matrix having  $n$  criteria and  $m$  alternatives. The decision matrix is shown as

$$D = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & \dots & x_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & x_{m3} & \dots & \dots & x_{mn} \end{bmatrix}$$

where element  $x_{ij}$  indicates the performance of the  $i$ th alternative with respect to  $j$ th criterion.

Step2. Compute the normalized decision matrix. The normalized value  $r_{ij}$  can be given as

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, \quad i = 1, \dots, m \quad j = 1, \dots, n$$

Step3. Determine the relative importance of alternative. In the AHP method, the pair comparisons are done and values from 1 to 9 are assigned to introduce the relative importance of the alternatives. Table V indicates the comparison scale used in the weighting of two criteria [15].

TABLE V. DEFINITION OF SCALE VALUES IN PAIR-WISE COMPARISON MATRIX

| Relative importance | description            |
|---------------------|------------------------|
| 1                   | Equal importance       |
| 3                   | Weak importance        |
| 5                   | Strong importance      |
| 7                   | Very strong importance |
| 9                   | Absolute importance    |
| 2, 4, 6, 8          | Intermediate values    |

Then, the matrix of the comparison of criteria is as follows:

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{21} & 1 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & 1 \end{bmatrix}$$

The every element represents the relative importance of criterion  $i$  with respect to criterion  $j$ . So, we conclude that  $a_{ji} = 1/a_{ij}$ .

$W_j$  is importance degree for each criterion and is calculate as

$$W_j = \frac{\left( \prod_{i=1}^n a_{ij} \right)^{1/n}}{\sum_{i=1}^n \left( \prod_{j=1}^n a_{ij} \right)^{1/n}}, \quad i, j = 1, 2, \dots, n$$

A consistency ratio (CR) is used to determine inconsistency and expressed as

$$CR = \frac{CI}{RI},$$

where the consistency index (CI) can be calculated by

$$CI = \frac{\lambda_{\max} - n}{n - 1},$$

where  $\lambda_{\max}$  is the largest eigenvalue [21] and the RI is obtained based on Table VI that shows the random index used in decision making. If the value of CR is less than 0.1, we conclude that this model is validated.

TABLE VI. RANDOM CONSISTENCY INDEX

| n | 1 | 2 | 3    | 4   | 5    | 6    | 7    | 8    | 9    |
|---|---|---|------|-----|------|------|------|------|------|
| k | 0 | 0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 |

Step4: The weighted normalized matrix  $V_{ij}$  is computed as

$$V_{ij} = W_j r_{ij}$$

where  $W_j$  is obtained from AHP method in the previous step.

Step5. Obtain positive ideal solutions  $V^+$  and negative ideal solutions  $V^-$  that are calculated as

$$V^+ = \left\{ \left( \sum_i^{\max} V_{ij} / j \in J \right), \left( \sum_i^{\min} V_{ij} / j \in J' \right) / i = 1, 2, \dots, m \right\} = \{v_1^+, v_2^+, \dots, v_n^+\}$$

$$V^- = \left\{ \left( \sum_i^{\min} V_{ij} / j \in J \right), \left( \sum_i^{\max} V_{ij} / j \in J' \right) / i = 1, 2, \dots, m \right\} = \{v_1^-, v_2^-, \dots, v_n^-\},$$

where  $J = (j=1, 2, \dots, n) / j$  is related to the beneficial criteria and  $J' = (j=1, 2, \dots, n) / j$  is related to the non-beneficial criteria.

Step6. Calculate the separation distance of each alternative. The separation of each alternative from the positive ideal solution is defined as

$$S_i^+ = \sqrt{\sum_{j=1}^n (V_{ij} - V_j^+)^2}, \quad i = 1, 2, \dots, m$$

similarly, calculate the separation distance of each alternative from the negative ideal solution as follows:

$$S_i^- = \sqrt{\sum_{j=1}^n (V_{ij} - V_j^-)^2}, \quad i = 1, 2, \dots, m$$

Step7. Calculate the relative closeness to ideal solutions as

$$C_i = \frac{S_i^-}{S_i^- + S_i^+},$$

Step8. Rank alternatives based on the value of  $C_i$  in descending order.

#### IV. EVALUATION OF ROBUST DEIGNS BY THE PROPOSED METHOD

We use the combined TOPSIS-AHP method for selecting the most suitable robustness design. In order to select preferred design in terms of robustness against missing observations which is very important in nanolubrication industries, we consider eight efficient criteria including average D-efficiency, worst D-efficiency,  $t_{\max}$ ,  $t_{\max} / n$ ,  $t_{\max} (0.99)$ ,  $t_{\max} (0.95)$ ,  $t_{\max} / n$ ,  $t_{\max} (0.99) / n$

and  $t_{\max}(0.95)/n$ . These criteria are explained in details in section 2. The hierarchical structure of decision making is demonstrated in the following figure.

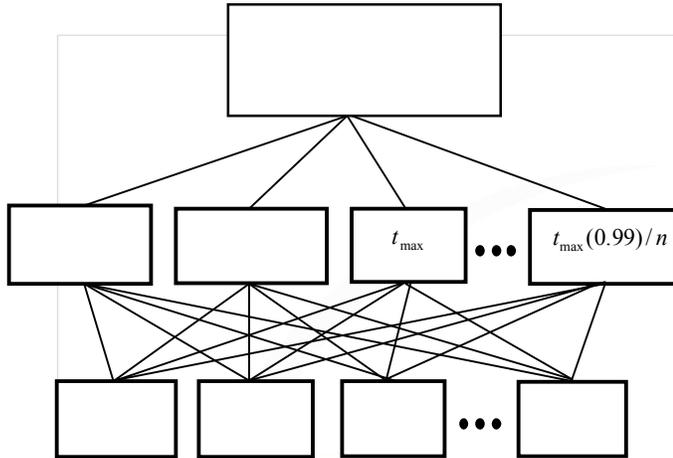


Figure I. The hierarchical structure of decision making

At first, we construct decision matrix where each element expresses the performance of considered alternative with respect to available criterion.

TABLE VII. DECISION MATRIX OF ROBUSTNESS CRITERIA

| $k$ | AVE-D  | $t_{\max}$   | $t_m(0.99)$   | $t_m(0.95)$   |
|-----|--------|--------------|---------------|---------------|
| 3   | 0.4440 | 3            | 3             | 4             |
| 4   | 0.4643 | 3            | 5             | 7             |
| 5   | 0.3000 | 3            | 4             | 5             |
| 6   | 0.4167 | 3            | 8             | 10            |
| 3   | 0.3750 | 1            | 1             | 2             |
| 4   | 0.4643 | 3            | 5             | 7             |
| 5   | 0.5227 | 3            | 9             | 12            |
| 6   | 0.4615 | 3            | 9             | 13            |
| $k$ | W-D    | $t_{\max}/n$ | $t_m(0.99)/n$ | $t_m(0.95)/n$ |
| 3   | 0.206  | 16.7         | 16.6          | 22.2          |
| 4   | 0.3412 | 10.7         | 17.8          | 25.0          |
| 5   | 0.0353 | 10.0         | 13.3          | 16.6          |
| 6   | 0.3236 | 6.3          | 16.6          | 20.8          |
| 3   | 0.2500 | 6.3          | 6.25          | 12.5          |
| 4   | 0.4167 | 10.7         | 17.8          | 25.0          |
| 5   | 0.5000 | 6.8          | 20.4          | 27.2          |
| 6   | 0.4375 | 5.8          | 17.3          | 25.0          |

Then, the normalization of decision matrix is obtained based on Equation (7) and expressed as:

TABLE VIII. NORMALIZATION OF DECISION MATRIX

| $k$ | AVE-D  | $t_{\max}$   | $t_m(0.99)$   | $t_m(0.95)$   |
|-----|--------|--------------|---------------|---------------|
| 3   | 0.3602 | 0.2122       | 0.375         | 0.6031        |
| 4   | 0.3767 | 0.3514       | 0.375         | 0.3864        |
| 5   | 0.2434 | 0.0363       | 0.375         | 0.3611        |
| 6   | 0.3381 | 0.3333       | 0.125         | 0.2275        |
| 3   | 0.3043 | 0.2575       | 0.375         | 0.2275        |
| 4   | 0.3767 | 0.4292       | 0.375         | 0.3864        |
| 5   | 0.4241 | 0.5150       | 0.375         | 0.2455        |
| 6   | 0.3745 | 0.4507       | 0.375         | 0.2094        |
| $k$ | W-D    | $t_{\max}/n$ | $t_m(0.99)/n$ | $t_m(0.95)/n$ |
| 3   | 0.1727 | 0.3608       | 0.1697        | 0.3522        |
| 4   | 0.2878 | 0.3869       | 0.2969        | 0.3966        |
| 5   | 0.2302 | 0.2891       | 0.2121        | 0.2634        |
| 6   | 0.4605 | 0.3608       | 0.4242        | 0.3300        |
| 3   | 0.0575 | 0.1358       | 0.0848        | 0.1983        |
| 4   | 0.2878 | 0.3869       | 0.2969        | 0.3966        |
| 5   | 0.5181 | 0.4434       | 0.5091        | 0.4316        |
| 6   | 0.5181 | 0.3760       | 0.5515        | 0.3966        |

The pair-wise comparison matrix is demonstrated in Table IX to show the relative importance of each criterion.

TABLE IX. PAIR-WISE COMPARISON MATRIX

|     |     |      |      |     |     |     |     |
|-----|-----|------|------|-----|-----|-----|-----|
| 1   | 1   | 1/2  | 1/3  | 5   | 4   | 3   | 2   |
| 1   | 1   | 1/2  | 1/3  | 5   | 4   | 3   | 2   |
| 2   | 2   | 1    | 2/3  | 10  | 8   | 6   | 4   |
| 3   | 3   | 3/2  | 1    | 15  | 12  | 9   | 6   |
| 1/5 | 1/5 | 1/10 | 1/15 | 1   | 4/5 | 3/5 | 2/5 |
| 1/4 | 1/8 | 1/12 | 1/12 | 5/4 | 1   | 3/4 | 2/4 |
| 1/3 | 1/6 | 1/6  | 1/9  | 5/3 | 4/3 | 1   | 2/3 |

To obtain weights of the robust criteria, we use Equation (9) in the previous section. The final results are shown in the following table:

TABLE X. WEIGHTS OF ROBUST CRITERION

| $n$ | 1      | 2      | 3      | 4      |
|-----|--------|--------|--------|--------|
| $w$ | 0.1207 | 0.1207 | 0.2414 | 0.3622 |
| $n$ | 5      | 6      | 7      | 8      |
| $w$ | 0.0241 | 0.0302 | 0.0402 | 0.0604 |

The largest eigenvalue of the pairwise comparison matrix is equal to 8. The value of random consistency index is obtained from table and it is equal to 1.48. Inconsistency ratio is computed near to zero. Because the value of RC is less than 0.1, we can conclude comparisons are consistent.

In the next step, the weighted normalized matrix is calculated and summarized in Table XI

TABLE XI. NORMALIZED WEIGHTED MATRIX

| $k$ | AVE-D   | $t_{\max}$     | $t_m(0.99)$     | $t_m(0.95)$     |
|-----|---------|----------------|-----------------|-----------------|
| 3   | 0.0435  | 0.0256         | 0.0905          | 0.2184          |
| 4   | 0.0455  | 0.0424         | 0.0905          | 0.1399          |
| 5   | 0.0294  | 0.0044         | 0.0905          | 0.1308          |
| 6   | 0.0408  | 0.0402         | 0.0905          | 0.0824          |
| 3   | 0.0367  | 0.0311         | 0.0302          | 0.0824          |
| 4   | 0.0455  | 0.0518         | 0.0905          | 0.1399          |
| 5   | 0.0512  | 0.0622         | 0.0905          | 0.0888          |
| 6   | 0.0452  | 0.0544         | 0.0905          | 0.0754          |
| $k$ | WORST-D | $t_{\max} / n$ | $t_m(0.99) / n$ | $t_m(0.95) / n$ |
| 3   | 0.0042  | 0.0109         | 0.0068          | 0.0213          |
| 4   | 0.0069  | 0.0117         | 0.0119          | 0.0239          |
| 5   | 0.0056  | 0.0087         | 0.0085          | 0.0159          |
| 6   | 0.0111  | 0.0109         | 0.0171          | 0.0199          |
| 3   | 0.0014  | 0.0041         | 0.0034          | 0.0120          |
| 4   | 0.0069  | 0.0117         | 0.0119          | 0.0239          |
| 5   | 0.0125  | 0.0134         | 0.0205          | 0.0261          |
| 6   | 0.0125  | 0.0113         | 0.0222          | 0.0239          |

Then, positive ideal  $V^+$  and negative ideal  $V^-$  solutions are calculated and given in Table XII.

TABLE XII. POSITIVE AND NEGATIVE IDEAL SOLUTIONS

| $V^+$  | $V^-$  |
|--------|--------|
| 0.0512 | 0.0294 |

|        |        |
|--------|--------|
| 0.0622 | 0.0044 |
| 0.0905 | 0.0302 |
| 0.2184 | 0.0758 |
| 0.0125 | 0.0014 |
| 0.0134 | 0.0041 |
| 0.0222 | 0.0034 |
| 0.0239 | 0.0120 |

Also, the separation distances between the alternatives and ideal solutions are calculated and illustrated in the following table:

TABLE XIII. SEPRATION DEISTANCE

| $S^+$  | $S^-$  |
|--------|--------|
| 0.7891 | 0.9217 |
| 0.8534 | 0.9586 |
| 0.5964 | 0.6982 |
| 0.9019 | 0.9806 |
| 0.3935 | 0.4832 |
| 0.8829 | 0.9895 |
| 0.1268 | 1.2066 |
| 0.0689 | 1.440  |

The relative closeness to ideal solution is calculated as follows:

TABLE XIV. RELATIVE CLOSENESS TO IDEAL SOLUTION

| $n$ | 1      | 2      | 3      | 4      |
|-----|--------|--------|--------|--------|
| $C$ | 0.5387 | 0.5290 | 0.5393 | 0.5209 |
| $n$ | 5      | 6      | 7      | 8      |
| $C$ | 0.4488 | 0.4715 | 0.9049 | 0.9413 |

Based on the relative closeness values, the nanolubricant processes can select the robustness design for experimental design with the following priorities:

$$C_8 > C_7 > C_3 > C_1 > C_2 > C_4 > C_6 > C_5$$

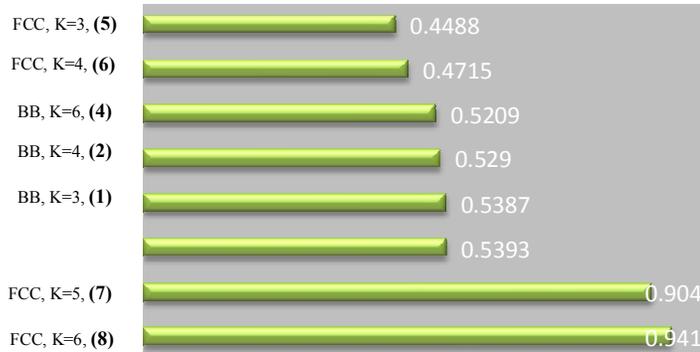


Figure II. The relative closeness ratio for different robust experimental designs

#### V. CONCLUSION AND FUTURE RESEARCH

As mentioned before missing observations in experimentations can be appeared in many industrial applications such as nanomanufacturing. Hence, the selection of a preferred design with missing observations in nanomanufacturing leads to the effective economic decision. We considered an integrated TPOSI-AHP methodology to select robust design. We considered Box-behnken and Face-centered composite designs due to their several applications. To evaluate the robustness of the Box-behnken and Face-centered composite designs, eight efficient robustness criteria were considered in the proposed decision making approach. The results show that the preferred design can be selected by the proposed approach. The analysis confirmed that the Face-centered composite design with the factor number of  $k=6$  can be selected as preferred robust design with missing observations. Proposing the other robust criteria for evaluating the other experimental designs in the presence of missing data is proposed as a future research.

#### REFERENCES

- [1] K. Lal, V.K. Gupa, and L. Bar, "Robustness of designed experiments against missing data. Journal of Applied Statistics" Journal of Applied Statistics, vol.28, pp.63-79, 2001.
- [2] N.N. Acharya and H.Black Nembhard, "Bayesian algorithms for missing observations in experimental designs for a nanolubrication process", IIE Transactions, vol. 41, pp. 969-978, 2009.
- [3] M. Tanco, E. del Castillo, E. Viles, "robustness of three-level response surface designs against missing data", IIE Transactions, vol. 45, pp. 544-553, 2013.

- [4] P.W.M.John, "Missing points in  $2n$  and  $2n - k$  factorial designs", Technometrics, vol. 21, 225-228, 1979.
- [5] A.M. Herzberg, P. Prescott, and M. Akhtar, "Equi-information robust designs: which designs are possible?", Canadian Journal of Statistics, vol. 15, pp. 71-76, 1987.
- [6] A. Dey, "Robustness of block designs against missing data". Statistica Sinica, vol. 3, pp. 219-231, 1993.
- [7] M.J.Anderson, and P.J. Whitcomb, "Using graphical diagnostics to deal with bad data", Quality Engineering, vol. 19, pp. 111-118, 2007.
- [8] S.Ghosh, "on robustness of designs against incomplete data". Sankhya, Series B, vol. 40, pp. 204-208, 1979.
- [9] S.N., MacEachern, W., Notz, D.C., Whittinghill, and Y. Zhu, "Robustness to the unavailability of data in the linear model with applications", Journal of Statistical Planning and Inference, vol. 48, pp. 207-213, 1995.
- [10] D.C. Whittinghill, "A note on the robustness of Box-Behnken designs to the unavailability of data". Metrika, vol. 48, pp.49-52, 1998.
- [11] P.Sen and J-B. Yang, "Multiple Criteria Decision Support in Engineering Design". London, Great Britain: Springer-Verlag London Limited, 1998.
- [12] TL. Saaty, "Multi Criteria Decision Making: The Analytic Hierarchy Process", New York: Mc Graw-Hill Inc., 1988.
- [13] P Chatterjee, VM. Athawale, S Chakraborty, "Selection of industrial robots using compromise ranking and outranking methods", Robotics and Computer- Integrated Manufacturing, vol. 26, pp.483-489, 2010.
- [14] Y.Kuo, T.Yang and G.W. Huang, "The use of grey relational analysis in solving multiple attribute decision-making problems. Computers & Industrial Engineering", vol.55, pp. 80-93, 2008.
- [15] TL.Saaty, "The analytic hierarchy process", New York: McGraw-Hill, 1980.
- [16] R. tavakkoli-Moghaddam and S.M. mousavi, "an integrated AHP-VIKOR methodology for plant location selection", International Journal of Engineering, Vol. 24, pp. 127-137, 2011.
- [17] RV.Rao and JP.Davim, "Decision-making framework models for material selection using a combined multiple attribute decision-making method". Int J Adv Manuf Technol, vol. 35, pp.751-60, 2008.
- [18] D.C. Montgomery, "Design and Analysis of Experiments", sixth edition, Wiley, New York, NY, 2005.
- [19] R.H. Myers and D.C. Montgomery, "Response Surface Methodology", second edition, Wiley-Interscience, New York, NY.
- [20] S.Ghosh, "Robustness of designs against the unavailability of data", Sankhya, Series B, vol. 44, pp. 50-62, 1982.
- [21] J. Malczewski, "GIS and Multicriteria Decision Analysis", New York: John Willey and Sons, Inc, 1999.