Economic-statistical design of EWMA-3 control chart for monitoring simple linear profiles

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Abstract: In this paper, the EWMA-3 control chart for monitoring simple linear profile is economic-statistically designed. Average run length criterion of the EWMA-3 control chart is computed by using a Markov chain approach. In the proposed model, the Lorenzen and Vance cost function is applied and the parameters of non-conforming production cost are computed based on the Taguchi cost function. The proposed model is solved by a genetic algorithm. Finally, a sensitivity analysis is done on the parameters of the proposed model.

Keywords: economic-statistical design; simple linear profile; EWMA-3 control chart; Markov chain; average run length.

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1 Introduction

In some statistical process control applications the performance of a process or the quality of a product could be described by a distribution of a quality characteristic and monitored by univariate control charts, or described by a distribution of a multivariate quality characteristic and monitored by multivariable control charts. In some situations, the quality of a process or a product is described by a relation between the response variable and one or more independent variables referred to as profile. This relation in some applications such as calibration is described by using a simple linear regression model, while more complex models are required in other situations. The relations of the resolved amount in grams of artificial sweetener in a litre of water under different temperatures (Kang and Albin, 2000), viscosity and the depth of a chip-board (Walker and Wright, 2002) and torque of the automobile engine with revolution per minute (Amiri et al., 2010) are some examples of numerous applications of profiles. Different types of profiles, including simple linear profiles, multiple linear profiles, polynomial profiles, multivariate profiles, generalised linear-based profiles and nonlinear profiles are considered by researchers and various methods have been proposed for monitoring them in both Phases I and II. Kang and Albin (2000), Mahmoud and Woodall (2004), Kim et al. (2003), Mahmoud et al. (2007) and Saghaei et al. (2009) have proposed different methods for monitoring simple linear profiles. One of the most well-known methods for monitoring simple linear profiles in Phase II is the EWMA-3 control chart proposed by Kim et al. (2003). In this paper, we economic-statistically design the EWMA-3 control chart. Note that the EWMA-3 control chart by Kim et al. (2003) is designed statistically. However, we design the control chart from both economic and statistical properties. The average run length criterion of the EWMA-3 control chart is computed by using a Markov chain approach. The proposed economic–statistical model is solved by using a meta-heuristic genetic algorithm. Different cost models are proposed by researchers, among which Duncan (1956) model and Lorenzen and Vance (1986) are more popular. The Lorenzen and Vance (1986) cost model is used in this paper as the objective function of economic-statistical design model. The main point in this model is that this model uses average run length under in-control and out-of-control situations instead of the probabilities of Type I and II errors. The characteristic of the proposed model is that it optimises three EWMA control charts, simultaneously.

The structure of the paper is as follows: The EWMA-3 control chart is described in Section 2. The Markov chain approach is presented in Section 3 for computing the average run length (ARL). Economic-statistical model of the EWMA-3 control chart is described in Section 4. A genetic algorithm for solving the proposed economic-statistical model is explained in Section 5. In Section 6, a numerical example is presented. Sensitivity analysis is done on the model parameters in Section 7 and conclusion is given in the final section.

2 EWMA-3 control chart

Let y_{ij} show the response variable in the ith level in the jth sample, when the process is in control. A simple linear profile is then written as equation (1).

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij} \quad j = 1, 2, \ldots, N = m \times n \quad i = 1, 2, \ldots, m, \tag{1}$$

where ε_{ij} values are independent, normally distributed random variables with the average of zero and variance of σ_0^2, x_i values are constant and fixed from profile to profile, m is the number of levels in explanatory variable, and n is the number of replications in each level.

Kim et al. (2003) coded the x values such that the average of coded x-values becomes zero. This causes the least square estimates of the slope and the intercept to become independent from each other in each sample. After coding the x values, the transformed model of the main model in equation (1) is as follows:

$$y_{ij} = B_0 + B_1 x_i^* + \varepsilon_{ij} \quad i = 1, 2, \ldots, m \;\; j = 1, 2, \ldots, N = m \times n, \tag{2}$$

where $B_0 = A_0 + A_1\bar{x}$, $B_1 = A_1$, and $x_i^* = (x_i - \bar{x})$. The least squares estimate of B_0 for the jth sample is as $b_{0j} = \bar{y}_j$, while the least squares estimate of B_1 is $b_{1j} = s_{xy_j} / s_{xx}$ where

$$s_{xx} = \sum_{i=1}^{m} (x_i - \bar{x})^2,$$

$$s_{xy(j)} = \sum_{i=1}^{m} (x_i - \bar{x})^2 \, y_{ij},$$

b_{0j} and b_{1j} follow normal distribution with average values of B_0 and B_1, respectively, and variances of $\sigma_{b_0}^2 = \sigma_0^2 / N$ and $\sigma_{b_1}^2 = \sigma_0^2 / (n s_{xx})$, respectively. Three EWMA control charts are used for monitoring the intercept, slope and errors variance. The EWMA statistic for monitoring the intercept B_0 is described in equation (3).

$$EWMA_I(j) = \theta_I b_{0j} + (1 - \theta_I) EWMA_I(j-1) \qquad j = 1, 2, \ldots, \tag{3}$$

where $0 < \theta_I \leq 1$ is the smoothing parameter for EWMA control chart for monitoring the intercept and $EWMA_I(0) = B_0$.

The upper and lower control limits for the EWMA statistic in equation (3) regarding n response values in each level of x-variable is as follows:

$$UCL_I = B_0 + L_I \sigma_0 \sqrt{\frac{\theta_I}{(2 - \theta_I) N}} \;\; \& \;\; LCL_I = B_0 - L_I \sigma_0 \sqrt{\frac{\theta_I}{(2 - \theta_I) N}}, \tag{4}$$

where $L_I > 0$ is selected such that a determined in-control ARL is obtained.

Estimates of B_1, b_{1j}, is used in EWMA statistic for monitoring the slope. In this case, the EWMA statistic for monitoring the slope is defined as equation (5):

$$EWMA_S(j) = \theta_S b_{1j} + (1 - \theta_S) EWMA_S(j-1) \qquad j = 1, 2, .., \tag{5}$$

where $0 < \theta_S \leq 1$ is the smoothing parameter for the EWMA control chart for monitoring the slope and $EWMA_S(0) = B_1$.

The upper and lower control limits of EWMA chart for monitoring the slope are calculated as equation (6) considering n response values in each level of the explanatory variable.

$$UCL_S = B_1 + L_S \sigma_0 \sqrt{\frac{\theta_S}{(2 - \theta_S) n S_{xx}}} \;\; \& \;\; \mathrm{LCL}_S = B_1 - L_S \sigma_0 \sqrt{\frac{\theta_S}{(2 - \theta_S) n S_{xx}}}, \tag{6}$$

where $L_S > 0$ is selected such that a predetermined in-control ARL is achieved.

Kim et al. (2003) used the EWMA control chart based on the Crowder and Hamilton (1992) approach for monitoring the variance of errors. In this control chart, they used the value of MSE_j (σ^2 estimator by the use of the residuals of the fitted line on the dataset in the jth sample) for calculating the EWMA statistic for monitoring the variance of errors.

$$EWMA_E(j) = \max\left\{\theta_E \ln\left(MSE_j\right) + (1-\theta_E)\, EWMA_E(j-1), \ln\left(\sigma_0^2\right)\right\} \quad j = 1, 2, ..., \tag{7}$$

where $0 < \theta_E \leq 1$ is the smoothing parameter for the EWMA control chart in equation (7) and $EWMA_E(0) = ln(\sigma_0^2)$.

It is assumed that the value of σ_0^2 (the value of σ^2 under in-control situation) is equal to one. Hence, $EWMA_E(0) = 0$, The upper control limits of EWMA for monitoring the standard deviation is as follows:

$$UCL_E = L_E \sqrt{\theta_E Var\left[\ln\left(MSE_j\right)\right] / (2-\theta_E)}. \tag{8}$$

In equation (8), $Var[\ln(MSE_j)]$ for the $EWMA_E$ control chart is calculated by using equation (9) regarding the n response values in each level of the explanatory variable.

$$Var\left[ln\left(MSE_j\right)\right] = \frac{2}{N-2} + \frac{2}{(N-2)^2} + \frac{4}{3(N-2)^3} - \frac{16}{15(N-2)^5}, \tag{9}$$

where $L_E > 0$ is selected such that a specific in-control ARL is obtained.

3 Markov chain approach

To calculate the ARL of the EWMA-3 control chart, the Markov chain approach is used. This method divides the distance between the upper and lower control limits of each EWMA control chart to $t = 2k + 1$ equal distance with the width of $2f$. Note that t is an odd integer. If the value of the EWMA statistic is between $d_j - f$ and $d_j + f$, for $j = -k, -k + 1, \ldots, k$, it implies that the value of the EWMA statistic in the jth limit is in the centre of this distance. d_j shows the central point of the distance j. In other words, d_j represents the midpoint of the jth interval. With this definition, the state of the system shows the position for EWMA value on the chart that could be lays in the sub-distance $j = -k, -k + 1, \ldots, k$.

In the Markov chain approach, the distribution of the run length of EWMA is determined by the starting probability vector and transition probability matrix. In the proposed ARL method, each of the charts for EWMA control, for monitoring the y-intercept, slope and standard deviation are calculated separately and then the total ARL will be determined according to the ARL results of each of the charts.

In-control transition probabilities for y-intercept of the EWMA control chart are obtained according to the Lucas and Saccucci (1990) approach given by equation (10):

$$\begin{aligned} h_{ij} = {} & \phi\left((\theta_I.\sigma_{b0})^{-1}\left\{(d_i + f) - (1-\theta_I)\, d_j - \theta_I B_0\right\}\right) \\ & - \phi\left((\theta_I.\sigma_{b0})^{-1}\left\{(d_i - f) - (1-\theta_I)\, d_j - \theta_I B_0\right\}\right) \end{aligned} \tag{10}$$

where $\phi(.)$ is the cumulative function of standardised normal distribution, θ is the smoothing parameter, d_j is the midpoint of the jth distance, f is half of the divided distances, B_0 is the y-intercept of the transformed model in equation (2) and h_{ij} is the transition probability of the EWMA value for the y-intercept, form the midpoint of ith distance to the midpoint of the jth distance. It is to mention that the distance between the limits of EWMA control chart for monitoring the y-intercept [according to equation (4)] is divided into t parts for calculation f, to be then divided by 2.

If the transition probability values are placed in matrix '**H**', a $t \times t$ transition probability matrix will be obtained. In this case, ARL for EWMA control scheme for the y-intercept monitoring is calculated by equation (11):

$$ARL_{0\mathrm{I}} = \mathbf{p}(\mathbf{I} - \mathbf{H})^{-1}\mathbf{s}, \tag{11}$$

where s is a $t \times 1$ column vector including the numbers 1, p is $1 \times t$ row vector including the numbers 0 that its qth element is equal to one and $q = t/2 + 0.5$ and I is a $t \times t$ identity matrix. ARL_{0I} shows the in-control average run length of the EWMA control chart for monitoring the y-intercept.

To calculate the out-of-control ARL of EWMA control chart for monitoring the y-intercept, the value of B_0 is replaced by $B_0 + \lambda\sigma_0$ in equation (10).

To calculate the ARL_{0S} (the in-control average run length of the EWMA control chart for monitoring the slope) and ARL_{1S} (the out-of-control ARL of EWMA control chart for monitoring the slope), the same method as for calculation of in-control and out-of-control ARL criteria of the EWMA control chart for monitoring the y-intercept will be performed, however in this case, the elements of transition probability matrix is calculated by using equation (12):

$$\begin{aligned} q_{ij} &= \phi\left((\theta_S.\sigma_{b1})^{-1}\left\{(d_i + f) - (1-\theta_S)d_j - \theta_S B_1\right\}\right) \\ &\quad - \phi\left((\theta_S.\sigma_{b1})^{-1}\left\{(d_i - f) - (1-\theta_S)d_j - \theta_S B_1\right\}\right), \end{aligned} \tag{12}$$

where σ_{b_1} is the standard deviation of the estimate of parameter B_1, the other parameters are similar to the definitions related to equation (10).

Note that for calculating ARL_{1S}, one should replace the value of B_1 by $B_1 + \beta\sigma_0$ in equation (12).

For calculating the in-control and out-of-control ARL of the EWMA control chart for monitoring the standard deviation, first, the distance between the control limits are divided into t equal parts, where t is an odd integer. Again, similar to the presented method for the y-intercept and the slope, the position of the EWMA statistic for monitoring the variance in the midpoint of the jth distance is considered as the state of the system. According to Dugan and Herbert (2008), the transition matrix elements are calculated by equation (13):

$$r_{ij} = p_{ij} - p_{i(j-1)} \qquad i, j = 1, 2, \ldots, t. \tag{13}$$

p_{ij} in the above equation is calculated by equation (14) as follows:

$$p_{ij} = \chi^2_{N-2}\left\{\frac{(N-2)\exp\left(\left[(j-1)-(1-\theta_E)\left(i-\frac{1}{2}\right)\right]\left(\frac{(UCL_{\mathrm{E}}-LCL_{\mathrm{E}})}{\theta_E}\right)\right)}{W^2}\right\}$$
$$j = 1,2,\ldots,t, \tag{14}$$
$$i = 2,3,\ldots t.$$

In equation (14), $\chi^2_{(N-2)}\{.\}$ shows a chi-square cumulative distribution function with $(N-2)$ degree of freedom. W is computed by σ_1 / σ_0 that is equal to 1 for in-control state. It should be noted that since the two parameters of B_0 and B_1 are estimated in the simple linear regression, the degree of freedom for chi-square distribution will be $(N-2)$. Also, p_{1j} and p_{i0} are calculated by using equations (15) and (16), respectively.

$$p_{1j} = \chi^2_{N-2}\left\{\frac{(N-2)\exp\left([(j-1)]\left(\frac{(UCL_{\mathrm{E}}-LCL_{\mathrm{E}})}{\theta_E}\right)\right)}{W^2}\right\} \quad j = 1,2,\ldots t, \tag{15}$$

$$p_{i0} = 0 \qquad i = 2,3,\ldots t. \tag{16}$$

Finally, the ARL_0 of EWMA control chart for monitoring the error variance is calculated by equation (17).

$$ARL_{0E} = \mathbf{p}(\mathbf{I}-\mathbf{H})^{-1}\mathbf{s}. \tag{17}$$

p, s and I are similar to the definitions related to Equation (11). To calculate ARL_{1E}, the value of W is replaced by (σ_1 / σ_0). Up to this stage, ARL_0 and ARL_1 of each EWMA control chart for monitoring the y-intercept, the slope and the standard deviation are computed. Equation (18) is applied to calculate the total ARL_0.

$$ARL_0 = \left(1\Big/\left[1-\left(1-\frac{1}{\mathrm{ARL}_{0I}}\right)\left(1-\frac{1}{\mathrm{ARL}_{0S}}\right)\left(1-\frac{1}{\mathrm{ARL}_{0E}}\right)\right]\right). \tag{18}$$

Clearly, equation (18) is calculated on the basis of equation (19)

$$\alpha_{overall} = 1-(1-\alpha_I)(1-\alpha_s)(1-\alpha_E). \tag{19}$$

The probability that none of the three EWMA control charts signal in the case that the y-intercept has changed is calculated by using equation (20):

$$\beta_{overall\ under\ shift\ in\ intercept} = \beta_{\mathrm{I}}(1-\alpha_{\mathrm{s}})(1-\alpha_{\mathrm{E}}). \tag{20}$$

Hence,

$$\beta_{overall\ under\ shift\ in\ intercept} = \left(\frac{ARL_{1I}-1}{ARL_{1I}}\right)\left(1-\frac{1}{ARL_{0S}}\right)\left(1-\frac{1}{ARL_{0E}}\right). \tag{21}$$

The total ARL_1 of the modified EWMA-3 control chart is calculated by

$$ARL_{1overall} = 1/(1-\beta).$$

It is to note that $\beta_{overall}$ indicates that although the process regarding the y-intercept is out-of-control, all the three charts show the process is in-control. Since the y-intercept is actually out-of-control, the probability of in-control state is equal to 'β'. Therefore since the slope and standard deviation are in-control, the expressing in-control condition of the relevant charts will be $(1 - \alpha_S)$ and $(1 - \alpha_E)$, respectively.

With comparable argument, the probability of Type II error of the general EWMA-3 chart with a shift in the slope and standard deviation will be calculated by equations (22) to (25), respectively.

$$\beta_{overall\ under\ shift\ in\ slope} = \beta_S\,(1-\alpha_I)(1-\alpha_E), \tag{22}$$

$$\beta_{overall\ under\ shift\ in\ slope} = \left[\frac{ARL_{1S}-1}{ARL_{1S}}\right]\left(1-\frac{1}{ARL_{0I}}\right)\left(1-\frac{1}{ARL_{0E}}\right), \tag{23}$$

$$\beta_{\text{overall under shift in standard deviation}} = \beta_E\,(1-\alpha_I)(1-\alpha_S), \tag{24}$$

$$\beta_{\text{overall under shift in standard deviation}} = \left[\frac{ARL_{1E}-1}{ARL_{1E}}\right]\left(1-\frac{1}{ARL_{0S}}\right)\left(1-\frac{1}{ARL_{0I}}\right). \tag{25}$$

The total ARL_1 of the EWMA-3 chart for a shift in the slope and standard deviation is calculated via equation (26):

$$ARL_{1\text{overall}} = 1/(1-\beta_{\text{overall}}). \tag{26}$$

To calculate the out-of-control ARL under the simultaneous shifts (bilateral and trilateral) in the parameters, a similar reasoning is used and they are hence calculated; e.g., ARL_1 value under simultaneous shifts for all of the three parameters is given by equation (27):

$$ARL_{1overall} = \left(1\Big/\left[1-\left[\frac{ARL_{1I}-1}{ARL_{1I}}\right].\left[\frac{ARL_{1s}-1}{ARL_{1s}}\right].\left[\frac{ARL_{1E}-1}{ARL_{1E}}\right]\right]\right), \tag{27}$$

And if only one parameter or two parameters shift, the overall ARL_1 is changed and calculated with regards to any of the parameters to be going under the shift.

4 Economic-statistical design of EWMA-3 control chart

The EWMA-3 control chart is designed in this paper in an economic-statistical form. The characteristics of the proposed model lays in the fact that three EWMA control charts are simultaneously optimised and the control chart parameters are calculated simultaneously such that the costs are minimised while the statistical constraints are satisfied. The Lorenzen and Vance cost function is used in this model as the objective function. The parameters determined by the model include h (interval time between two successive samples), n (number of response variables in each level of independent variable), L_I, L_S and L_E (the coefficients of the control limits of the EWMA control charts for monitoring the y-intercept, the slope and the standard deviation, respectively), θ_I, θ_S and θ_E (the smoothing parameters of the EWMA control charts for monitoring y-intercept, the slope and the standard deviation, respectively) and the statistical constraints are determined based on ARL_0, ARL_1 and out-of-control average time to signal (ATS_1) criteria.

The Lorenzen and Vance cost function is defined in equation (28). It is assumed in the presented model that the time that the process is in-control follows an exponential distribution with the average of 1/α. It is also assumed that in case of occurring an assignable cause, all the parameters of *y*-intercept, the slope and the standard deviation will change.

$$C = \left(\frac{C_0/\alpha + C_1\left((-\tau) + (\mathrm{N}).E + h.\left(ARL_1\right) + \gamma_1.T_1 + \gamma_2.T_2\right) + S.F/ARL_0 + \mathrm{R}}{(1/\alpha) + \left(\left(1-\gamma_1\right).S.T_0/ARL_0\right) - \tau + (\mathrm{N}).E + h.\left(ARL_1\right) + T_1 + T_2} \right) + \left(\frac{\left[(a + b.(N))/h\right).\left[1/\alpha - \tau + (\mathrm{N}).E + h.\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right]}{1/\alpha + \left(1-\gamma_1\right).S.T_0/ARL_0 - \tau + (N).E + h.\left(ARL_1\right) + T_1 + T_2} \right), \tag{28}$$

The parameters in Equation (28) are defined as follows:

C Expected cost for each time unit.

C_0 cost per hour due to non-conformities produced while the process is in-control

C_1 cost per hour due to non-conformities produced while the process is out-of-control

τ expected time between the occurrence of the assignable cause and the time of the last sample taken before the assignable cause = $[1 - (1 + \alpha h)\exp(-\alpha h)] / [\alpha (1 - \exp(-\alpha h)]$

E time to sample and chart one item

ARL_0 average run length while the process is in-control

ARL_1 average run length while the process is out-of-control

T_1 expected time to detect the assignable cause

T_2 expected time to repair the process when assignable cause is detected

T_0 expected search time when the signal is a false alarm

γ_1 1 if production continues during searches, 0 if production ceases during searches

γ_2 1 if production continues during repair, 0 if production ceases during repair

S expected number of samples taken while in-control

$$S = e^{(-\alpha h)}/\left(\left[1 - e^{\alpha h}\right)\right]). \tag{29}$$

F cost per false alarm

R cost to locate and repair the assignable cause

a fixed cost of sampling

b cost per unit sampled

α parameter of the exponential distribution.

The production costs of non-conforming products per unit time are modified for in-control and out-of-control situations using Taguchi approach, such that C_0 and C_1 are

replaced by j_0p and j_1p in the Lorenzen and Vance cost function. The values of J_0 and J_1 are calculated respectively by using equation (30), and p indicates the rate of production per each hour.

$$\begin{aligned} J_0 &= K\left[\left(\sigma_0^2/(N)\right)+\left(\sigma_0^2/(n.S_{xx})\right)\right] \\ J_1 &= K\left[\left(\omega^2.\left(\sigma_0^2/(N)\right)+\lambda.\left(\sigma_0^2/(N)\right)+\omega^2.\left(\sigma_0^2/(n.S_{xx})\right)+\beta^2\left(\sigma_0^2/(n.S_{xx})\right)\right)\right], \end{aligned} \tag{30}$$

where λ, β and ω in equation (30) represent shifts in the y-intercept, the slope and the standard deviation parameters in unit of sigma, respectively. The economic-statistical model for simultaneous design of the three EWMA-3 control charts is indicated in equation (31).

Min C

$$\begin{aligned} & s.t: 0<\theta_I \le 1, 0<\theta_S \le 1, 0<\theta_E \le 1, \\ & ARL_0 > a, ARL_1 < b, ATS_1 < c \\ & n: integer, \end{aligned} \tag{31}$$

where a is lower bound of ARL_0 and b and c are upper bounds of ARL_I and ATS_1, respectively. The decision variables of the model are θ_I, θ_S, θ_E, L_I, L_S, L_E, n and h that are calculated after solving the nonlinear model in equation (31). A method will be given for solving the model in the next section, by using the genetic algorithm.

5 Proposed method for solving the economic-statistical model

Genetic algorithm is used in this paper to solve the proposed economic-statistical model. This algorithm provides a search technique for finding solution close to optimum in optimisation problems. Genetic algorithm starts with a primary population of solutions. Each solution is shown by a chromosome. There is a relevant value to the competence of the solution that referred to as fitness function for each chromosome. Genetic algorithm aims to maximise the value of fitness function such that statistical constraints are satisfied. However, if the objective function is in the form of minimising, updating the algorithm for minimisation can be easily done.

5.1 Components of genetic algorithm

1 *Chromosome:* a strip of genes as a possible solution for the considered problem. The chromosome in our problem is designed as Table 1.

2 *Population:* a set of chromosomes are called population. The population or generations of chromosomes include a size known as the population size. Population size indicates the number of existing chromosomes in a generation or a population. In this paper the population size is selected to be '20'.

3 *Value of fitness function:* the suitability or inappropriateness of the solution is measured by the criterion obtained from the objective function. The more suitable the solution, it has the higher fitness value. The chromosome that is more suitable is used in reproduction with higher probability. If the aim is maximising a function, the

value of fitness function will be considered as an ascending function of the objective function, and if the aim is finding the minimum of a function, the value of fitness function is considered as a descending function. The goal in this paper is minimising the Lorenzen and Vance cost function

4 *Selection:* the process of selecting a pair of parents from the population is for the crossover operation. The aim of selection is to select more desired parents to lead to giving birth to offspring with higher fitness. The chromosomes selected for reproduction from the primary population are called parents. Selection is a method that randomly takes chromosomes for reproduction from the population.

Table 1 A schematic representation of the proposed chromosome

n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h

5.2 The proposed procedure of the genetic algorithm

In this paper, we produce a primary population with the size of '20' randomly. We select 10 chromosomes randomly from this primary population and then we produce 10 chromosomes as the offspring by using one-point or two-point crossover operator. Then, we select one chromosome out of the ten, randomly and use mutation operator in order to prevent the algorithm from getting into the local optimum. Afterwards, we combine the generated offspring's with the primary population to generate the new generation. For this purpose, we choose 18 chromosomes from the best solution and two chromosomes from the worst ones, by using the fitness criterion of minimum cost in which penalty coefficients are used considering statistical constraints. We continue this process to the extent to reach the stop condition. In this paper, we have considered the number of repetitions equals to 10 as the stopping condition.

6 A numerical example

In this section a numerical example is used to evaluate the presented model and its solution. The used parameters of the numerical example are based on the Dugan and Herbert (2008) defined as follows:

$$\tau = 0, E = 0.5, T_1 = 20, T_2 = 0, T_0 = 0, \gamma_1 = 1, \gamma_2 = 0, F = 500, R = 250,$$
$$a = 5, b = 1, \alpha = 0.01, p = 200$$

And the value of S is calculated by using equation (29). In the presented model, $C_0 = J_0 p$ and $C_1 = J_1 p$ and the values of J_0 and J_1 are computed by using equation (30). Also, shifts in the y-intercept, the slope and the standard deviation are considered to be $\lambda = 1$, $\beta = 1$ and $\omega = 1.5$ in unit of sigma, respectively. The statistical constraints are $ARL_0 > 250$, $ARL_1 < 10$, $ATS_1 < 8$. The optimum values of decision variables and ATS_1, ARL_1 and ARL_0 are calculated by using the genetic algorithm explained in Section 5. The considered coefficients for the penalty function are 2, 4 and 8, respectively. The obtained values are summarised in Table 2. Also the minimum value of C is reported in this table. It is to note that the total ARL_0 and ARL_1 are designed by using Markov chain approach and the total amount of ATS_1 is obtained from multiplying ARL_1 and 'h' and shown in

Table 2. In addition, the optimum values of the decision variables and ATS_1, ARL_1 and ARL_0 under economic design model are obtained and summarised in Table 3. Note that in the economic model, the statistical constraints are omitted.

According to the economic-statistical and economic models, the optimum values for the decision variables and costs in Table 2 and Table 3 are obtained under the simultaneous shifts in the y-intercept, the slope and the standard deviation. By comparing the results of two tables, it can be concluded that the cost is increased in the economic-statistical model. Also, the optimal sampling intervals in both economic and economic-statistical models are roughly close to each other, the number of runs in each level of the explanatory variable is equal and the changes in the values of L_S, L_E and L_1 are not significant for the optimum values. Moreover, the highest difference is related to L_1. The range of changes in smoothing parameters of the control chart related to the y-intercept and standard deviation is tangible. However, the smoothing parameter of the EWMA control chart of the slope in both economic and economic-statistical models are the same.

Table 2 Optimum decision variables, the optimum value of objective function, Total ARL_0, Total ARL_1 and Total ATS_1 for the optimum solution in the numerical example for the economic-statistical model

λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991

Table 3 Optimum decision variables, the optimum value of objective function, Total ARL_0, Total ARL_1 and Total ATS_1 for the optimum response in the numerical example for the economic model

λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932

7 Sensitivity analysis

According to the changes we make in the parameters, we obtain the optimum values for the design parameters and minimum cost, by using the economic and economic-statistical models and compare the results.

As it can be observed in Table 4, the range of changes in L_I, L_S and L_E coefficients, the sampling interval and the number of repetitions are not too large under different simultaneous shifts in the y-intercept, the slope and the standard deviation, but the range of changes in smoothing parameters is tangible. It is while the cost values are close to each other, with respect to some shift values, and no prominent changes are viewed in the costs with respect to the shifts.

Also as shown in Table 5 under different simultaneous shifts in the y-intercept, the slope and standard deviation for providing optimum solutions for the decision variables, the ranges of changes in L_I, L_S and L_E coefficients, the sampling interval and the number of repetitions are not considerable, however, the range of changes in smoothing parameters is tangible. In addition, the cost values in Table 4 for the economic-statistical model are higher than the values in Table 5 for the economic model.

Table 4 Optimum decision variables, the optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum response, solutions under different shifts in the y-intercept, the slop and the standard deviation for the economic-statistical model

λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
1	1	2	1,823.6	2	0.8719	0.8037	0.7342	3.4609	3.4497	5.3246	4.4196	909.3947	1	4.4198
0.5	0.5	1.5	3,088.9	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1.0031	2.6072
0.5	0.5	2	3,090	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1.0023	2.6077
1	0.5	1.5	1,822.3	2	0.8719	0.8037	0.7342	3.4609	3.4497	5.3246	4.4196	909.3947	1	4.4198
1	0.5	2	1,822.7	2	0.8719	0.8037	0.7342	3.4609	3.4497	5.3246	4.4196	909.3947	1	4.4198
0.5	1	1.5	3,089	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
0.5	1	2	3,090.1	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991

Table 5 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions under different shifts in y-intercept, the slop and the standard deviation for the economic model

λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932
1	1	2	1,218.1	1	0.0308	0.3767	0.1655	2.5311	2.5499	2.5360	3.3361	93.2518	1.0038	3.3488
0.5	0.5	1.5	1,610.3	1	0.8913	0.7544	0.7998	2.9683	5.8014	1.1617	2.5113	58.6557	1.0008	2.5128
0.5	0.5	2	1,217.7	1	0.0308	0.3767	0.1655	2.5311	2.5499	2.5360	3.3361	93.2518	1.3843	4.6181
1	0.5	1.5	1,573.2	1	0.4312	0.9145	0.8567	4.8382	1.4517	5.5744	2.6050	6.8601	1.0233	2.6557
1	0.5	2	869.2	1	0.5403	0.6995	0.0624	2.8429	5.4116	3.9856	4.6958	237.40	1.0001	4.6961
0.5	1	1.5	848.1	1	0.1943	0.706	0.1415	5.7298	3.6666	1.5286	4.8018	1,201.7	1.0501	5.0474
0.5	1	2	1,046.7	1	0.4080	0.4271	0.4857	2.2918	3.9499	2.3099	3.8917	57.1475	1	3.8917

By changing the value of Taguchi coefficient in Table 6 and comparing it with the obtained values in the numerical example of the previous section, we conclude that by creating the change in Taguchi coefficient, the cost is reduced considerably. However, the range of changes in L_S, L_E and L_I coefficients, sampling interval and number of repetitions is not very high, but the range of changes in smoothing parameters; especially for the slope, is tangible and the ARL_0 is also reduced considerably. It is observed in Table 6 that cost has increased slightly in the economic model and the optimum solutions are remained unchanged. By comparing the two Tables 6 and 7, the cost in the economic-statistical model is larger than the cost in the economic model.

By increasing α and comparing it with the obtained values in the numerical example in Table 8, it is concluded that cost has increased slightly and no changes has been occurred in the optimum values of the decision variables. Cost has considerably decreased in the economic model in Table 9. In addition, the optimum values for the decision variables have changed respect to the numerical example and the most extensive change is related to the smoothing parameters. By comparing Table 8 with Table 9 we conclude that the cost in the economic-statistical model is larger than the cost in the economic model.

By changing R in Table 10 and comparing it with the numerical example, we conclude that by increasing R from 250 to 900, the cost increases and the optimum values of decision variables does not change. In Table 11 the cost in the economic model increases slightly by increasing R from 250 to 900 and no changes occur in the optimum values of the decision variables. By comparing Tables 10 and 11, the cost in the economic-statistical model is more than the cost in the economic model.

By changing F in Table 12 from 500 to 900 and comparing it with the numerical example, it is concluded that the cost of the economic-statistical model is reduced considerably and change in the optimum values of decision variables is mostly related to the smoothing parameters. It is while; the ARL_0 value reduced considerably. In Table 13 in the economic model, by changing the parameter F, no changes is observed in the optimum values of design parameters as compared to the numerical example and the cost value is only increased slightly. By comparing the economic-statistical model and the economic model, it is concluded that cost has increased in the economic-statistical design model and most of the change in the optimum values is related to the smoothing parameters and ARL_0 values.

By changing the value of E in Table 14 and comparing it with the numerical example, we concluded that by changing E from 0.5 to 0.05, the cost and the ARL_0 are reduced and the maximum change in the optimum values of decision variables is related to the smoothing parameters. In Table 15, by changing in E, the cost is reduced. Generally, by comparing the Tables 14 and 15 the cost in the economic-statistical model is larger than the cost in the economic model.

Table 6 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in Taguchi coefficient for the economic-statistical model

K	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
0.1	1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
0.7	1	1	1.5	2,925.6	2	0.9133	0.0046	0.6541	5.0865	5.5435	1.4222	2.7707	514.662	1	2.7707

Table 7 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in Taguchi coefficient for the economic model

K	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
0.1	1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932
0.7	1	1	1.5	2,809.5	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932

Table 8 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the exponential distribution parameter for the economic-statistical model

α	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
0.01	1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
0.05	1	1	1.5	3,095.2	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991

Table 9 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the exponential distribution parameter for the economic model

α	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
0.01	1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932
0.05	1	1	1.5	1,223.7	1	0.0308	0.3767	0.1655	2.5311	2.5499	2.5360	3.3361	93.2518	1.0041	3.3496

Table 10 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the 'R' parameter for the economic-statistical model

R	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
250	1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
900	1	1	1.5	3,095.5	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991

Table 11 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the 'R' parameter for the economic model

R	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
250	1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932
900	1	1	1.5	2,788.1	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932

Table 12 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the 'F' parameter for the economic-statistical model

R	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
500	1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
900	1	1	1.5	2,899.2	2	0.9133	0.0046	0.6541	5.0865	5.5435	1.4222	2.7707	514.662	1	2.7707

Table 13 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the 'F' parameter for the economic model

R	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
500	1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932
900	1	1	1.5	2,788.1	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932

Table 14 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the 'E' parameter for the economic-statistical model

E	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
0.5	1	1	1.5	3,089.3	2	0.9619	0.2630	0.6541	4.4461	4.9408	3.2527	2.5991	93,801	1	2.5991
0.05	1	1	1.5	2,898.9	2	0.9133	0.0046	0.6541	5.0865	5.5435	1.4222	2.7707	514.662	1	2.7707

Table 15 Optimum decision variables, optimum value of objective function, Total ARL_0, Total ARL_1, Total ATS_1 for the optimum solutions, by changing in the 'E' parameter for the economic model

E	λ	β	W	C	n	θ_I	θ_S	θ_E	L_I	L_S	L_E	h	ARL_0	ARL_1	ATS_1
0.5	1	1	1.5	2,786	2	0.6020	0.2630	0.3500	1.9830	2.6554	4.0802	2.8932	20.9144	1	2.8932
0.05	1	1	1.5	1,215.9	1	0.0308	0.3767	0.1655	2.5311	2.5499	2.5360	3.3361	93.2518	1.0041	3.3496

8 Conclusions

In this paper, we proposed an economic-statistical model to optimise the parameters of the EWMA-3 control chart from both economic and statistical properties. The applied cost function was the Lorenzen and Vance cost function design, where the cost parameters of producing the non-conforming products were calculated according to the Taguchi cost function under both in-control and out-of-control states. A Markov chain approach was used to compute the joint in-control and out-of-control ARL of three EWMA control charts for monitoring the y-intercept, the slope and the standard deviation. After that, Genetic meta-heuristic algorithm was used to solve the economic-statistical model, through a numerical example. In addition, a comparison with the results of economic model was done. Finally, a sensitivity analysis was done by creating changes in the input parameters in both economic and economic-statistical model and comparing the results of designed parameters as well as economic and statistical criteria.

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