An online estimator for rotor resistance in vector drives of induction machines based on Walsh functions

Hamidreza SHIRAZI, Jalal NAZARZADEH†

Engineering Faculty, Shahed University, Tehran, Iran

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Abstract:

In a modern electrical driver, rotor field oriented control (RFOC) method has been used to achieve a good performance and an appropriate transient response. In this method, the space vector of the rotor flux comes handy by the rotor resistance value. The rotor resistance is one of the important parameters which varies according to motor speed and room temperature alteration. In this paper, a new on-line estimation method is utilized to obtain the rotor resistance by using Walsh functions domain. The Walsh functions are one of the most applicable functions in piecewise constant basis functions (PCBF) to solve dynamic equations. On the other hand, an integral operational matrix is used to simplify the process and speed of the computation algorithm. The simulations results show that the proposed method is capable of solving the dynamic equations in an electrical machine on a time interval which robustly estimates the rotor resistance in contrast with injection noises.

Keywords: Vector control; Walsh functions; Operational matrix; Rotor resistance estimation

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1 Introduction

Induction motors (IMs) are widely in use in the industry with different applications such as public services and household electrical appliances [1, 2]. In the vector control drives of induction machines, decoupling control of magnetization flux and electromagnetic torque will be achieved by adjusting stator current vector. This characteristic has made the method receive a great attention in the industry [3]. Amplitude and position of the rotor flux phasor are two important variables which have to be observed simultaneously in rotor flux oriented control (RFOC) drives. To find these variables, the time constant of the rotor circuit (rotor resistance value) is an important parameter in this drive [4, 5]. Exact estimation of rotor resistance is one of the most interesting problems regarding vector control in induction machines. Since the rotor resistance is related to room temperature and the rotor speed, the process of estimation should be done online and in a constant manner.

†Corresponding author.
E-mail: nazarzadeh@shahed.ac.ir.

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Different methods for achieving rotor resistance estimation have been analyzed in some papers. Vaclavek et al. performed a careful induction machine observability analysis [6]. Douiri et al. investigates the performances of fuzzy logic and artificial neural networks approaches [7]. Chehimi et al. used an unknown input observer for the constant time estimation from only input and output measurements [8]. Xu et al. applied high-frequency signal injection method carefully in [9] as this method could normally be employed for very low speed and zero speed operations of induction motor [10]. Toliyat et al. presented a detailed review of the online and offline rotor resistance estimation methods in modern RFOC drives [11]. Spectral analysis, model reference adaptive system, observer-based method, fuzzy logic and artificial neural networks were major techniques classified in [12]. In the spectral techniques, an additional disturbance signal is needed when a RFOC drive is under no load condition [13]. On the other hand, the process of estimation by spectral techniques can be done by sampling input and output signals [14]. Due to noisy signals, this method is unreliable and loses its performance.

In model reference adaptive method, an error signal detects time constant variation effects in rotor circuit which can be achieved by the use of torque signal or reactive-power of the system [12]. This method is very sensitive to motor parameters and strongly depends on the operating points of the drive. Moreover, the input signal of the system should be able to satisfy the classical persistent excitation condition to converge the estimated rotor resistance to the real value [15]. Also, an extended Kalman filter is a well-known observer which can estimate the rotor resistance and speed of RFOC drives in a high speed range [16]. Although, Bogosyan et al. presented an improved extended Kalman filter for wide speed range [17], however high order observer and high computing effort are two main setbacks of this method. Moreover, intelligent and heuristic methods were proposed to estimate the rotor resistance in RFOC drive. For example, Karanayil et al. presented an artificial intelligent algorithm based on the back propagation training of the neural networks for rotor resistance estimation in a RFOC drive [18]. Low rate of convergence and complicated computing algorithm are two main constraints of these methods.

Recently, Walsh functions are introduced as an efficient instrument for studying differential equations and dynamic systems [19]. Walsh functions are one of the PCBF which are applied to various kinds of problems such as: harmonic elimination in electrical machines [20], designing controllers in linear time-variant systems [21], estimation of parameters and control of the synchronous machines by algebraic method [22] and etc. These papers showed that the Walsh functions not only cause simplicity in solving non-linear equations but also decrease noise effects in the ultimate answer because of their piecewise properties. In comparison to other PCBF functions, Walsh functions are more reliable; for instance, block-pulse is another PCBF function that is used widely in the study of dynamic systems [23], yet it is a sampling method indeed and data may be ignored on the signals border. Note that solving a differential equation based on Walsh and block-pulse functions, the Walsh functions will give out more accurate answers in the same order of functions arranged. Walsh functions can be arranged in different orders to make series. The operational matrix has been used to solve dynamic equations based on PCBF. This matrix transfers dynamic equations of the system into nonlinear algebraic equations. The matrix technique is a very important approach since it simplifies system analysis in high-order Walsh series [20].

This paper presents a new online method for estimating rotor resistance based on Walsh functions. Introducing stator current and voltage as input signals, this method benefits from Walsh functions and applies operational matrix to solve induction machine equations in order to estimate rotor resistance. Applying operational matrix to nonlinear dynamical model of induction machines causes the dynamical equations of the observer convert into simultaneous algebraical relations which are simpler. Based on pseudo-inverse method, a relation will be achieved to find the estimated value of the rotor resistance. The observer of the Walsh functions can be used in a drive with a wide speed range with no need for injection signals, no convergence and persistent excitation conditions. Results confirm that using this method decreases noise and switching harmonics by a PWM converter in a RFOC drive in a wide range of operations.

2 Closed-loop RFOC realization

By adjusting real and imaginary components of the stator currents, a decoupling control of magnetization flux and electromagnetic torque in RFOC of induction machines can be achieved. In closed-loop speed control
system, rotor position \( \rho_r(t) \) and value of the rotor flux \( \hat{\psi}_r(t) \) can be observed by solving the rotor and stator voltage equations. Statement of space phasors of the rotor \( \hat{\vec{v}}_r(t) \) and stator \( \hat{\vec{v}}_s(t) \) voltage relations for a symmetrical induction machine in the arbitrary reference frame can be mentioned as [4]

\[
\begin{align*}
\hat{\vec{v}}_s(t) &= R_s i_s(t) + \frac{d}{dt} \hat{\vec{v}}_s(t) + j \omega_s \hat{\vec{v}}_s(t), \\
\hat{\vec{v}}_r(t) &= R_r i_r(t) + \frac{d}{dt} \hat{\vec{v}}_r(t) + j(\omega_r - \omega_s) \hat{\vec{v}}_r(t),
\end{align*}
\]

where \( j \) represents the imaginary unit \( \sqrt{-1} \). Here, \( R_s \) and \( R_r \) are resistances of the stator and rotor windings, \( \omega_s(t) \) and \( \omega_r(t) \) show the angular speed of the arbitrary and rotor reference frames. In this equation, \( \hat{\vec{v}}_s(t) \) and \( \hat{\vec{v}}_r(t) \) are space phasors of the rotor and stator currents, which can be written as

\[
\begin{align*}
\hat{\vec{i}}_s(t) &= i_s(t) + j i_y(t), \\
\hat{\vec{i}}_r(t) &= i_r(t) + j i_y(t),
\end{align*}
\]

in these relations, \( i_s(t), i_y(t), i_r(t) \) and \( i_y(t) \) are direct and indirect components of the stator and rotor currents in the arbitrary reference frame, respectively. Furthermore, \( \hat{\vec{v}}_s(t) \) and \( \hat{\vec{v}}_r(t) \) are space phasors of the rotor and stator voltage in the arbitrary reference frame, respectively. Similar to equation (2), these space phasors are defined by

\[
\begin{align*}
\hat{\vec{v}}_s(t) &= v_s(t) + j v_y(t), \\
\hat{\vec{v}}_r(t) &= v_r(t) + j v_y(t),
\end{align*}
\]

where \( v_s(t), v_y(t), v_r(t) \) and \( v_y(t) \) are direct and indirect components of the stator and rotor voltages in the arbitrary reference frame, respectively. Also, \( \hat{\vec{v}}_s(t) \) and \( \hat{\vec{v}}_r(t) \) are space phasors of the stator flux and rotor flux which are determined as

\[
\begin{align*}
\hat{\vec{\psi}}_s(t) &= L_s i_s(t) + L_m \hat{\vec{v}}_s(t), \\
\hat{\vec{\psi}}_r(t) &= L_m i_r(t) + L_r \hat{\vec{v}}_r(t),
\end{align*}
\]

where \( L_s, L_r \) and \( L_m \) are self-inductance of the stator, rotor and mutual inductance of the windings. In an induction machine with per unit system, \( L_s \) and \( L_r \) are

\[
\begin{align*}
L_s &= L_{sa} + L_{sm}, \\
L_r &= L_{ra} + L_{rm},
\end{align*}
\]

in which \( L_{sa} \) and \( L_{ra} \) show leakage inductance of stator and rotor circuits, respectively. Also, a dynamical equation for the machine speed can be obtained by torque balance relation as

\[
\frac{d}{dt} \omega_r(t) = \frac{1}{J}(T_e(t) - T_1(t) - B \omega_r(t)),
\]

in this relation, \( J, B \) is moment of inertia and friction coefficient of the system and \( T_e, T_1 \) are air gap and load torque. Air gap torque can be determined by

\[
T_e = \frac{3}{4} P L_m (i_{sy}(t) i_{sx}(t) - i_{sx}(t) i_{sy}(t)),
\]

where \( P \) is the number of machine poles. Moreover, observer performance in finding the position and amplitude of the rotor flux will be investigated. The observer solves stator and rotor voltage equations in a reference frame which rotates with the angular speed of the rotor flux \( \omega_m \). As the speed of the reference frame alters, restatement of the mentioned equation (the induction machine model) in the rotor flux frame can be produced by substituting the speed of rotor flux \( \omega_m \) for \( \omega_r \). The angular speed of the rotor flux can be obtained by

\[
\omega_m(t) = \frac{d}{dt} \hat{\psi}_r(t).
\]

The exciting voltage of the rotor windings in a squirrel cage induction machine is zero \( (\hat{\vec{v}}_m(t) = 0) \), and aligned with the real axis in the rotor flux reference frame, the rotor flux is definitely real. Introducing equations (1) and (8) to the rotor flux reference frame and combining them, the real and imaginary parts of the rotor voltage equations are

\[
\begin{align*}
\frac{d}{dt} \hat{\psi}_m &= -\frac{1}{T_r} \hat{\psi}_m(t) + \frac{L_m}{T_r} i_{sx}(t), \\
\frac{d}{dt} \hat{\psi}_r &= \frac{1}{T_r} \hat{\psi}_m(t) i_{sy}(t) + \omega_r(t),
\end{align*}
\]

where \( \alpha \) and \( \beta \) respectively denote real and imaginary variable components in the rotor flux reference frame. Also, \( T_r \) is time constant of the rotor circuit \( (T_r = L_r/R_r) \). In equations (9) and (10), phasor components of the stator current and the motor speed are known as input variables, and by having \( R_s \), the value of \( T_r \) can be obtained. Thus, the amplitude of the rotor flux \( \hat{\psi}_m(t) \) can be determined by using equation (9). Finally, by substituting \( \hat{\psi}_m(t) \) into equation (10), the rotor position \( \rho_r(t) \) is found. Fig. 1 shows RFOC, based on closed-loop estimation process to obtain the magnitude and position of the rotor flux out of the estimated rotor resistance. In this system, flux and speed controllers are given for reg-
ulations in the closed-loop response by the systems. By comparing the reference and output currents via comparators with hysteresis, the voltage source inverter (VSI) is controlled in current mode. The switching control of the VSI produces high amplitude of harmonics in voltage and current. Also, a PWM VSI in current mode control is used to adjust the stator current of the induction machines. In this figure, the obtained values for the amplitude and position of the rotor flux and the estimated value for \( R_r \) are mentioned as \( \hat{\psi}_{\text{rmax}}(t) \), \( \hat{\rho}(t) \) and \( \hat{R}_r \). Furthermore, the process of estimating \( R_r \) by Walsh function will be described in details.

Fig. 1 Closed-loop block diagram of indirect RFOC with rotor resistance estimation.

3 Estimation of rotor resistance

In this section, we will investigate the process of estimator block where the rotor resistance value is estimated. This block works based on the induction machine model in the stator reference frame, where \( \omega_s(t) \) will be set to zero. Restating rotor voltage equation in equation (1) in the stationary reference frame (SRF) and applying the rotor flux in equation (4) in the same frame; alongside with getting integral out of the space phasors of the rotor voltage, one equation can be written as

\[
\frac{d}{dt} \hat{\psi}_r(t) = \frac{1}{T_r} \hat{\psi}_r(t) + \frac{L_m}{T_r} \hat{i}_s(t) + \alpha t + \omega_r(t)
\]

where \( \psi_{\text{r0}} \) shows the initial value for the rotor flux. In equation (11), as the stator current and the rotor speed are available, finding \( R_r \) is essential to know the rotor flux features in the stator reference frame. Applying flux equation in the SRF, then eliminating \( \vec{i}_r(t) \), we come by

\[
\hat{\psi}_r(t) = \frac{L_m}{L'_s} (\psi_s(t) - L'_s \hat{\psi}_s(t)),
\]

where \( L'_s \) is transient inductance of the induction machine which can be given as

\[
L'_s = L_{ss} + \frac{L_m}{L_r} L_{lr}.
\]
In equation (12), $\bar{\psi}_s(t)$ can be obtained from (1) when it is stated in the stator reference frame as

$$\dot{\bar{\psi}}_s(t) = \int_0^t (\bar{\psi}_s(t) - R_s \bar{\psi}_s(t))dt + \bar{\psi}_{s0},$$

(14)

where $\bar{\psi}_{s0}$ indicates the initial value of the stator flux vector. By placing equation (14) into equation (12), we have

$$\begin{align*}
\dot{\psi}_{sd}(t) &= \frac{L_r}{L_m}(\psi_{sd0} - L_s i_{sd}(t)) + \int_0^t (\dot{\psi}_{sd}(t) - R_s i_{sd}(t))dt, \\
\dot{\psi}_{sq}(t) &= \frac{L_r}{L_m}(\psi_{sq0} - L_s i_{sq}(t)) + \int_0^t (\dot{\psi}_{sq}(t) - R_s i_{sq}(t))dt.
\end{align*}$$

(15)

In on-line estimation, the ultimate value is the initial value for the next interval. Mentioning the real and imaginary components of equation (11), the two following relations can be obtained to estimate the rotor resistance value.

$$R_r = \int_0^t \frac{L_m}{L_r} i_{sd}(t) - \frac{1}{L_r} \dot{\psi}_{rd}(t)dt - \psi_{rd}(t)$$

$$- \int_0^t \omega_r(t) \psi_{rd}(t)dt + \psi_{rd0} = 0,$$

(16)

$$R_r = \int_0^t \frac{L_m}{L_r} i_{sq}(t) - \frac{1}{L_r} \dot{\psi}_{rq}(t)dt - \psi_{rq}(t)$$

$$+ \int_0^t \omega_r(t) \psi_{rd}(t)dt + \psi_{rq0} = 0.$$  

(17)

Mentioning rotor resistance as the unknown parameter, based on equations (16) and (17), the estimated value for the rotor resistance ($R_r$) can be obtained from:

$$\begin{bmatrix} x_d(t) \\ x_q(t) \end{bmatrix} = \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} R_r,$$

(18)

where $x_d(t)$, $x_q(t)$, $u_d(t)$ and $u_q(t)$ in equation (18) can be calculated by

$$\begin{align*}
x_d(t) &= \int_0^t \frac{L_m}{L_r} i_{sd}(t) - \frac{1}{L_r} \dot{\psi}_{rd}(t)dt, \\
x_q(t) &= \int_0^t \frac{L_m}{L_r} i_{sq}(t) - \frac{1}{L_r} \dot{\psi}_{rq}(t)dt, \\
u_d(t) &= \dot{\psi}_{rd}(t) + \int_0^t (\omega_r \psi_{rd}(t))dt - \psi_{rd0}, \\
u_q(t) &= \dot{\psi}_{rq}(t) - \int_0^t (\omega_r \psi_{rd}(t))dt + \psi_{rq0}.
\end{align*}$$

(19)

Equation (18) illustrates that the real and imaginary components of the stator current, rotor flux and speed are the input variables for estimating the rotor resistance. Also, by using equations (9) and (10), the amplitude and position of the rotor flux can be dynamically observed when we have values for the rotor resistance, stator current and rotor speed.

### 4 Walsh functions properties

Walsh functions are one of the most applicable functions of PCBF that can be stated in form of orthogonal set of rectangular functions. It is defined on time interval of $[0, 1]$ and varies according to the form of step function and takes values of $+1$ and $-1$. Some use these functions with step values of $0$ and $+1$ which makes no difference in their properties and the interval can be changed into $[0, t]$. Walsh functions with the order of $k = 1, 2, 4, 8, \ldots$ can be produced by constituting and completing Rademacher functions which is another function of PCBF and can be set in different orders [19]. Fig. 2 shows the set of the first eight Walsh functions $(m = 0, \ldots, 7)$ which are arranged in Dyadic or Paley order which is one method to arrange Walsh functions.

---

**Fig. 2** The Walsh functions with $k = 8.

Walsh functions form an orthogonal set on time interval of $t \in [0, 1]$, which means

$$\int_0^1 w_m(t)w_n(t)dt = \delta_{mn}(t),$$

(21)

where $\delta_{mn}(t)$ is Dirac delta-function. A smooth signal like $x(t)$ which is square-integrable on interval $[0, 1]$, can be mentioned by a finite Walsh series as

$$x(t) = \sum_{m=0}^{k-1} c^*_m w_m(t),$$

(22)
where \( c^*_{m} \) is the \( m \)th component of Walsh functions for \( x(t) \), which can be obtained as

\[
c^*_{m} = \int_0^1 x(t)w_m(t)dt, \quad m = 0, 1, 2, \ldots, k - 1.
\]  

(23)

Walsh series can be stated in vector form, hence equation (22) can be restated as

\[
x(t) = w(t)c^T_x,
\]

(24)

where \( c_x \) is the Walsh coefficient vector of \( x(t) \) and its dimension is \( 1 \times k \) and \( w(t) \) is called the Walsh functions vector with the same dimension.

\[
w(t) = (w_0(t) \; w_1(t) \; w_2(t) \cdots w_{k-1}(t)),
\]

(25)

\[
c_x = (c^*_0 \; c^*_1 \; c^*_2 \cdots c^*_k),
\]

(26)

Getting integral out of equation (24) on the same interval, we have

\[
\int_0^1 x(\xi)d\xi = \int_0^1 w(\xi)d\xi c^T_x = w(t)(c,E)^T,
\]

(27)

where \( \xi \) is a dummy variable in the integral operation. In this relation, the operational matrix of the Walsh functions \( E \) with the order \( k \) can be determined as [19]:

\[
E = \begin{bmatrix}
\frac{1}{4} & -\frac{1}{4} & 0 & 0 & \cdots & 0 & \frac{1}{2}I_0 \\
\frac{1}{4} & 0 & -\frac{1}{4} & 0 & \cdots & 0 & \frac{1}{2}I_1 \\
0 & \frac{1}{4} & 0 & -\frac{1}{4} & \cdots & 0 & \frac{1}{2}I_2 \\
0 & \frac{1}{4} & 0 & 0 & \cdots & 0 & \frac{1}{2}I_3 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \frac{1}{4} & 0 \\
\frac{1}{2}I_k & 0 & 0 & 0 & \cdots & \frac{1}{4} & 0 \\
0 & \frac{1}{2}I_k & 0 & 0 & \cdots & 0 & \frac{1}{4} \\
\end{bmatrix}
\]

(28)

where \( I_k \) are zero and identity diagonal matrices with order \( k/2 \).

5 Estimation of Rotor Resistance Based on Walsh Functions

In this section, Walsh functions will be applied to solve the dynamical equations of the induction machine in order to achieve an on-line estimation of the rotor resistance. Mentioing voltage and current vectors in the SRF as known values; their real and imaginary components can be stated as

\[
\begin{align*}
v_{sd}(t) &= w(t)c^T_{sd}, \\
v_{sq}(t) &= w(t)c^T_{sq},
\end{align*}
\]

(29)

\[
\begin{align*}
\dot{\phi}_{sd0} &= \dot{w}(t)c^T_{sd0}, \\
\dot{\phi}_{sq0} &= \dot{w}(t)c^T_{sq0}, \\
\dot{\phi}_{rd0} &= \dot{w}(t)c^T_{rd0},
\end{align*}
\]

(30)

\[
\begin{align*}
\dot{\phi}_{sd0} &= \dot{w}(t)c^T_{sd}, \\
\dot{\phi}_{sq0} &= \dot{w}(t)c^T_{sq}, \\
\dot{\phi}_{rd0} &= \dot{w}(t)c^T_{rd}
\end{align*}
\]

(33)

The rotor speed is known as the output of an encoder. Also, since the dynamical response of the system in equation (6) varies in a low frequency range, the rotor speed can be assumed as a constant value in short-time duty [24]. Therefore, the rotor speed can be represented by the Walsh functions with the zero order as

\[
\dot{\omega}_{r}(t) = c^*_{0}w_0(t).
\]

(31)

Also, the real and imaginary components of the initial conditions of the stator and rotor fluxes in the SRF can be represented in the Walsh functions as

\[
\begin{align*}
\dot{\phi}_{sd0} &= \dot{w}(t)c^T_{sd0}, \\
\dot{\phi}_{sq0} &= \dot{w}(t)c^T_{sq0}, \\
\dot{\phi}_{rd0} &= \dot{w}(t)c^T_{rd0}
\end{align*}
\]

(33)

where \( c^*_{sd0} \), \( c^*_{sq0} \), and \( c^*_{rd0} \) are the Walsh coefficients of the initial conditions’ real and imaginary components of the stator and rotor fluxes in the SRF, respectively. Since all the initial conditions are constant thus we can illustrate them by the first term of the Walsh coefficients and assume other terms as zero. By substituting equations (29), (30) and (32) into equation (15), the real component of the rotor flux can be written as

\[
\dot{\phi}_{sd}(t) = \dot{w}(t)c^T_{sd} = \frac{L_{m}}{L_{m}}w(t)((c_{sd} - R_{s}c_{isd})E + c_{sd0} - L_{s}c_{isd})^T,
\]

\[
\dot{\phi}_{sq}(t) = \dot{w}(t)c^T_{sq} = \frac{L_{m}}{L_{m}}w(t)((c_{sq} - R_{s}c_{isq})E + c_{sq0} - L_{s}c_{isq})^T,
\]

\[
\dot{\phi}_{rd}(t) = \dot{w}(t)c^T_{rd} = \frac{L_{m}}{L_{m}}w(t)((c_{rd} - R_{r}c_{ird})E + c_{rd0} - L_{r}c_{ird})^T,
\]

(34)

(35)

where \( c^*_{sd} \) is the Walsh coefficient of the real component of the rotor flux in the SRF. Similarly, for the imaginary part, we can write

\[
\dot{\phi}_{sq}(t) = \dot{w}(t)c^T_{sq} = \frac{L_{m}}{L_{m}}w(t)((c_{sq} - R_{s}c_{isq})E + c_{sq0} - L_{s}c_{isq})^T,
\]

(35)

where \( c^*_{sq} \) is the Walsh coefficient of the imaginary part of the rotor flux in the SRF. By placing equations (29) to (35) into equations (19) and (20), we have

\[
\begin{align*}
\dot{x}_{sd}(t) &= \dot{w}(t)c^T_{sd} = \dot{w}(t)\frac{L_{m}}{L_{r}}(c_{isd}E - \frac{1}{L_{m}}c_{sd}E)^T, \\
\dot{x}_{sq}(t) &= \dot{w}(t)c^T_{sq} = \dot{w}(t)\frac{L_{m}}{L_{r}}(c_{isq}E - \frac{1}{L_{m}}c_{sq}E)^T,
\end{align*}
\]

(36)
and also,
\[
\begin{align*}
 u_d(t) &= w(t)c_{\psi_{rd}}^T = w(t)(c_{\psi_{rd}} + c_{\psi_{rd}}^0E - c_{\psi_{rd}}^0)T, \\
 u_q(t) &= w(t)c_{\psi_{rq}}^T = w(t)(c_{\psi_{rq}} - c_{\psi_{rq0}}E - c_{\psi_{rq0}})T. \\
\end{align*}
\tag{37}
\]
Substituting equations (36) to (37) into (18), we can write
\[
\begin{bmatrix}
 w(t) \\
 0_{1 \times m}
\end{bmatrix}
\begin{bmatrix}
 c_{\psi_{rd}}^T \\
 c_{\psi_{rq}}^T
\end{bmatrix} \hat{R}_r = \begin{bmatrix}
 c_{\psi_{rd}}^T \\
 c_{\psi_{rq}}^T
\end{bmatrix} = 0,
\tag{38}
\]
where $0_{1 \times m}$ is a zero vector with $1 \times m$ dimensions. To satisfy equation (38) in $t \in [0,1)$, the coefficient vector of the $w(t)$ has to be set to zero. Thus, we have
\[
\begin{bmatrix}
 c_{\psi_{rd}}^T \\
 c_{\psi_{rq}}^T
\end{bmatrix} \hat{R}_r = \begin{bmatrix}
 c_{\psi_{rd}}^T \\
 c_{\psi_{rq}}^T
\end{bmatrix}.
\tag{39}
\]
In this relation, $\hat{R}_r$ is unknown. Therefore, by applying the pseudo inverse method \cite{25,26}, an accurate estimation of the rotor resistance from equation (39) will be achieved by
\[
\hat{R}_r = \frac{c_{\psi_{rd}}^T c_{\psi_{rd}}^0 + c_{\psi_{rq}}^T c_{\psi_{rq0}}}{c_{\psi_{rd}}^T + c_{\psi_{rq}}^T c_{\psi_{rq0}}^0}.
\tag{40}
\]
This process is repeated after the proposed time interval and each time, the last obtained value is used as the next step initial value. Fig. 3 shows the closed-loop speed controlled for an induction machine where the rotor estimator is implemented from equation (34) to (40) by Walsh functions. In this figure, Walsh coefficients of the estimated values for real and imaginary parts of the rotor flux in SRF are mentioned as $c_{\psi_{rd}}$ and $c_{\psi_{rq}}$.

Fig. 3 Closed-loop block diagram of indirect RFOC with rotor resistance estimation.
6 Simulation results

Table 1 presents the parameters of an induction machine and the parameters of the flux and the speed controllers which are presented in Fig. 3. This machine was used in [18] to demonstrate performance of the rotor resistance observer in a closed-loop RFOC drive using fuzzy logic controller. Here, effects of the rotor resistance variation on the system performance (speed, torque and flux) in tracking their references will be investigated. The speed and flux controllers in Fig. 3 are selected as PI controllers which reject the tracking errors in the steady state conditions. Hence, for evaluating the system in different conditions, the reference speed is selected with a uniform and ramp change alongside the time to have a better study on the system dynamic.

Table 1 Parameters of the closed-loop RFOC [18].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>( p_n )</td>
<td>1.1 kw</td>
</tr>
<tr>
<td>Rated line voltage</td>
<td>( V_{ll} )</td>
<td>415 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>( I_n )</td>
<td>2.77 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>( N_n )</td>
<td>1415 r·min(^{-1})</td>
</tr>
<tr>
<td>Poles</td>
<td>( p )</td>
<td>4</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>( R_s )</td>
<td>6.03 ( \Omega )</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>( L_{ls} )</td>
<td>29.3 mH</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>( L_{lr} )</td>
<td>29.3 mH</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>( L_m )</td>
<td>489.3 mH</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>( J )</td>
<td>0.052 Kg·m(^2)</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>( B )</td>
<td>0</td>
</tr>
<tr>
<td>Speed controller proportional gain</td>
<td>( K_{pw} )</td>
<td>0.3</td>
</tr>
<tr>
<td>Integral gain of speed controller</td>
<td>( K_{iw} )</td>
<td>4</td>
</tr>
<tr>
<td>Flux controller proportional gain</td>
<td>( K_{pf} )</td>
<td>1.7</td>
</tr>
<tr>
<td>Integral gain of flux controller</td>
<td>( K_{if} )</td>
<td>34</td>
</tr>
</tbody>
</table>

6.1 Closed-loop performance without estimator

In this section, we shall investigate how the difference between rotor resistance value which is used in observer \( \hat{R}_r \) and its real value will have its influence. The error in the rotor resistance estimated value affects the estimation of the rotor frame angle, where the effect is more sensible when the rotor is running in low speed. This simulation allows the drive to run at 200 r·min in both directions. Figs. 4 and 5 describe the dynamic performances of the system with a prefect current source inverter (CSI) in the two situations.

To evaluate the proposed method in the closed-loop RFOC drive, we consider a variation pattern for \( R_r \) which is shown in Fig. 6. In the first case, the rotor resistance value in the relations of the flux observer (equations (9) and (10)) is assumed equal to the real value \( (\hat{R}_r = R_r) \). In this case, torque and speed of the load had the correct acceleration profile and followed the reference values which is shown in Fig. 4. In the second case, we consider \( \hat{R}_r = 6.08 \Omega \), however the real value of the rotor resistance varies similar to Fig. 6. The dynamic performances of the drive in this case are presented in Fig. 4 \( (\hat{R}_r \neq R_r) \). These results show that without true knowledge of rotor resistance value \( (R_r) \), the output speed of the system is disturbed and cannot track its reference perfectly. Fig. 7 shows that, the rotor flux takes its reference value independent of the estimation value of the rotor resistance.

![Fig. 4 Torque-speed response of the system with CSI.](image)

![Fig. 5 Stator currents (scale: 5A/div) response of the system with CSI.](image)
According to Fig. 4, 1.1 s after abrupt change of speed reference with true knowledge of the rotor resistance value, the rotor speed oscillates and settles down. At $t = 3s$, the speed reference starts to reduce with ramp form which causes the electromagnetic torque to track a lower level torque reference than its real load torque. In $t = 4s$, the rotor rotation begins to change its direction, the reference torque changes its polarity. As $t = 4s$ to $t = 5s$, the electromagnetic torque tracks a reference torque with a higher level than the real load torque where acceleration of motor stands for the reason.

### 6.2 Walsh function estimator and comparison

The problem is to obtain the best estimated value for the rotor resistance in Walsh domain; where the real and imaginary components of the stator voltage and current are known as the input of the estimation block. By using equation (36), $c^T_{xd}$ and $c^T_{sq}$ in Walsh domain with $k = 2$ and an arbitrary $\delta t$, they can be written as

$$
c^T_{xd} = \delta t \begin{bmatrix} -2.7c_{0}^{isl} - 1.4c_{1}^{isl} + 5.1c_{0}^{iap} + 2.6c_{1}^{iap} \\ 1.4c_{0}^{isl} - 2.6c_{1}^{iap} \end{bmatrix}, 
$$

$$
c^T_{sq} = \delta t \begin{bmatrix} -2.7c_{0}^{isq} - 1.4c_{1}^{isq} + 5.1c_{0}^{iasp} + 2.6c_{1}^{iasp} \\ 1.4c_{0}^{isq} - 2.6c_{1}^{iasp} \end{bmatrix}.
$$

Similarly, $c_{ud}$ and $c_{uq}$ from equation (37) can be obtained as

$$
c_{ud} = \delta t(c^1_{ud} c^2_{ud}), \quad c_{uq} = \delta t(c^1_{uq} c^2_{uq}),
$$

in which

$$c^1_{ud} = 76.3c_{0}^{isd} + 8.0c_{1}^{isd} - 2.7c_{0}^{isd} - 1.3c_{1}^{isd} - 1059.9c_{0}^{isp} + c_{0}^{isp} + (0.2c_{0}^{isq} + 0.1c_{1}^{isq} - 2.7c_{0}^{isp} - 1.3c_{1}^{isp})c_{0}^{iwp},$$

$$c^2_{ud} = -8.0c_{0}^{isd} + 60.4c_{1}^{isd} + 1.3c_{0}^{isd} - 0.1c_{1}^{isq} + 1.3c_{0}^{isp}c_{0}^{iwp} - 1059.9c_{1}^{isp} + c_{1}^{isp},$$

$$c^1_{uq} = 76.3c_{0}^{isq} + 8.0c_{1}^{isq} - 2.7c_{0}^{isq} - 1.3c_{1}^{isq} - 1059.9c_{0}^{isp} + c_{0}^{isp} - (0.2c_{0}^{isd} + 0.1c_{1}^{isd} - 2.7c_{0}^{isp} - 1.3c_{1}^{isp})c_{1}^{iwp},$$

$$c^2_{uq} = -8c_{0}^{isp} + 60.4c_{1}^{isp} + 1.3c_{0}^{isp} + 0.1c_{1}^{isd} - 2.7c_{0}^{isp} - 1.3c_{1}^{isp}c_{0}^{iwp} - 1059.9c_{1}^{isp} + c_{1}^{isp}.$$

To check accuracy of the Walsh domain estimator with another technique, a condition which was given in [18] for estimating $R_r$ by neural network algorithm, is used to estimate the rotor resistance in the Walsh functions domain. Speed and output load of the drive are $1000 \cdot \text{min}^{-1}$ and $7.4N \cdot \text{m}$. When the induction motor is in the steady state condition, a step change with $+40\%$ is applied to the rotor resistance. The estimated $R_r$ by neural network algorithm converges within 50 ms. Table 2 shows the numerical estimated value of the rotor resistance with different orders of Walsh functions and system sources. These results illustrate that the accuracy of the estimated resistance by Walsh functions is acceptable for the CSI and the PWM VSI in current mode control. The results in Table 2 show that switching voltage in a PWM VSI which creates voltage harmonics in the line voltages, are not caused as improper effects on estimating the rotor resistance in Walsh functions domain.

<table>
<thead>
<tr>
<th>$R_r$ (Ω)</th>
<th>Estimated rotor resistance $\hat{R}_r$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NNT$^1$ [18]</td>
</tr>
<tr>
<td></td>
<td>CSI</td>
</tr>
<tr>
<td>8.519</td>
<td>6.469</td>
</tr>
<tr>
<td>8.519</td>
<td>7.618</td>
</tr>
<tr>
<td>8.519</td>
<td>7.854</td>
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<tr>
<td>8.519</td>
<td>8.393</td>
</tr>
<tr>
<td>8.519</td>
<td>8.502</td>
</tr>
</tbody>
</table>

1 Neural network technique; 2 Walsh functions method.
7 Closed-loop performance of estimator

Other simulation results are presented to elaborate the effectiveness of proposed estimator in real switching inverters. The value of $\hat{R}_r$ is determined from equation (40) by Walsh functions with $k = 4$. It is clear that a higher order Walsh series will give the answer with more accuracy but it takes more time getting to the answer. In this system, the algorithm is repeated over every $\delta t = 1$ ms. Figs. 8–10 show the performance of the RFOC without and with estimator when the $R_r$ is assumed to be increased with a pattern which has been mentioned in Fig. 6. It can be seen in Fig. 9, when the proposed estimator is applied with consideration of the switching voltage by a VSI, the $R_r$ is exactly tracked and in result, the disturbance of speed and torque due to $R_r$ variation is damped perfectly.

Fig. 8 Transient dynamic response of the system with the VSI in current mode controlled without estimator.

Fig. 9 Transient dynamic response of the system with the VSI in current mode controlled with estimator.

Fig. 10 Transient of the stator currents (scale: 16A/div) with the VSI in current mode controlled.

8 Conclusions

Correct and on-line estimation of the rotor resistance is one of the most important problems in indirect RFOC that has been investigated in this paper. This new on-line method applied the Walsh functions domain of signals on each time interval to solve induction machine equations to estimate the rotor resistance value. First, motor equations were stated in integral form; then, operational matrix of integration was applied to solve equations in the Walsh domain. Finally, finding the estimated value of the rotor resistance was formulized by the use of pseudo-inverse method which brought about a more accurate result. Simulation results showed that the proposed method reduced the noise effects which proved the validity of the applied method.

References

Jalal NAZARZADEH was born in Tehran, Iran, in August 1964. He received the B.S., M.S., and Ph.D. degrees from the Amirkabir University of Technology, Tehran, Iran, in 1990, 1992, and 1998, respectively. He is an associate professor in the Faculty of Engineering, Shahed University. His current research interests include modeling, analysis, and control of wind turbines. His major research interests include analysis and control of electrical machines and drivers, power systems and power electronics based systems. E-mail: hamidreza-shirazi67@gmail.com.

Hamidreza SHIRAZI was born in Yazd, Iran, in 1988. He received the B.S. and M.S. degrees in Electrical Engineering from University of Yazd, Iran in 2010, and Shahed University, Tehran, Iran in 2013, respectively. He is currently doing research on analysis and control of wind turbines. His major research interests include modeling, analysis, and control of electrical machines and drivers, spectral theory in nonlinear optimal control systems, and nonlinear phenomena in power electronics systems. He has published 34 research papers in journals and has presented 40 conference papers. He is the vice-president of Academic Affairs and Graduate Studies in Shahed University. E-mail: nazarzadeh@shahed.ac.ir.