A bi-objective optimization model for designing closed-loop supply chain network by considering the disruption in production centers

Farhad Yazdani1*, Reza Tavakkoli-Moghaddam2, Mahdi Bashiri3

1Department of Industrial Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran (f.yazdani@sin.iaun.ac.ir)
2Department of Industrial Engineering, University of Tehran, Tehran, Iran (tavakoli@ut.ac.ir)
3Department of Industrial Engineering, Shahed University, Tehran, Iran, Iran (bashiri@shahed.ac.ir)

In recent years, due to governmental regulations, environmental issues and social responsibilities, reverse logistics has received more attention by researchers. The present paper deals with the design of a multi-product closed-loop logistics network in which some production centers experience minor and general disruptions. The proposed network has three levels in the direct direction (suppliers, plants and distributors) and three levels in the reverse direction (collection centers, redistribution and disposal). Initially, plants prepare their needed raw materials from the suppliers; after production in plants, the final products are delivered to customers through distribution centers, percentage of these products are collected from customers by collecting centers. In the collection centers, the percentage of products are returned to manufacturing factories for recycling, percentage of them are sent to redistribution centers for sale to secondary customers, and the percentage that do not possess any capability of the cited items will be sent to disposal centers. The mathematical model presented in this study is a two-objective mixed integer nonlinear model that while maximizing the profit, the delivery time to customers will be minimized. For validation of the model, a numerical example is provided by Pareto curve in which the profit is versus the time.

1. Introduction

In today's business world, a company needs its own supply chain and logistics management to be able to effectively compete with other companies. In other words, supply chain management and
logistics tasks can be outlined as such: maximizing the added value and reducing total cost during the business processes with a focus on speed and response to market needs. Nowadays, supply chain management and logistics have become a requirement, especially for manufacturing industries that wish to supply their products at competitive costs and higher quality than their competitors. Reverse logistics models can be divided into two categories: issues that focus only on the recursive network and issues that a recursive network has been integrated with the forward network. Regarding the issues of the first category, the point which is of great importance is that these issues may be affected by sub-optimization, therefore, in recent years the researchers’ focus has been higher on the issues of the second category. Considering the mentioned items, in this study, a closed-loop model has been considered that in addition to maximizing the profit reduces the delivery time to customers. This model is studied in the multi-product and single-period mode in which all the demands of the customers are met. In continuation, initially, an overview of works done in this field will be examined and in the following sections network modeling is provided and by using the proposed solution, the model is solved. Finally, the results obtained from the solving of the model and the conclusions are presented.

2. Literature Review

Much of the literature in the field of logistic network design consists of different facility location models on the basis of the mixed integer linear programming. These models consist of a variety of simple models such as facility location models with unlimited capacity up to more complex models such as multilevel models with limited capacity or multi-commodity models. Also strong algorithms, on the basis of the theory of combinatorial optimization, have been presented for solving these models. In the following, the models presented for supply chain network design is discussed. Salema et al (2010) in their paper have provided a multi-product-multi-period model for supply chain network design and planning by reverse flows and for modeling their network they have used Graph approach on the basis of the traditional notions of nodes and arcs. They performed the strategic design of supply chain along with a practical plan which covered supply, production, storage and distribution sections. El-Sayed et al (2010), in their article developed a multi-period and multi-level reverse-direct logistics network design model. Their model possess 3-level in the direct direction (suppliers, facilities and distribution centers), and two levels in reverse direction (montage and collection centers). In this paper, the customers are divided into two categories: primary consumers and secondary consumers. Primary customers’ demands have been considered as probable demands and secondary customers’ demands have been considered as definitive demands. The model presented by them is a mixed integer linear programming model which maximizes the expected profit. Pishvaee et al (2011) have designed a single-period and single-product, multi-objective and multi-stage deterministic model of MILP for a reverse-direct integrated supply chain network by considering the multiple capacity levels of network centers. The above model is a two-objective model that in the first objective, the author minimizes the costs of the network and the second objective is to maximize the network responsiveness. Stages of network in this model include production, recycling, distribution, collection, inspection and disposal centers. The purpose of this paper to determine the location, number and capacity of production, recycling, distribution, collection, inspection and disposal centers and also to determine the optimal product flow between the facilities. They have used multi-objective Memetic algorithm to obtain a set of non-dominated solutions. Also in another article, Pishvaee et al (2011) have provided a single-product, single-period and multi-level MILP model with limited capacity which includes primary and secondary customers of the market, collection, inspection, recycling, and redistribution and disposal centers for a supply chain network CLSC. The purpose of this paper to determine the location, number and capacity of production, recycling, distribution, collection,
inspection and disposal centers and the optimal product flow between the facilities in a way that the total fixed costs and transportation costs are minimized. In this model, uncertainty has been considered that includes parameters of returned products, the demand for recycled products and transportation costs. For modeling of the uncertainty, they have used the randomized programming. Amin and Zhang (2012) in their study examined a closed-loop supply chain network that includes manufacturing, montage, repair and renovation, and disposal places. Their work is divided into two phases; in the first phase a framework is provided to identify supplier selection criteria. Then a fuzzy approach is designed for the selection of suppliers in order to determine the weight of suppliers. In the second phase a multi-objective mixed integer linear programming model is presented. In this model, the profit and the value of the total are maximized and the number of purchased defective items is minimized. Tabrizi and Razmi (2013) in an article entitled as presenting a mixed integer nonlinear fuzzy model for risk management in the supply chain network design provided a mixed integer nonlinear mathematical model. To consider the uncertainty conditions they used fuzzy set theory and in order to solve their model they applied Benders Decomposition method. By sensitivity analysis and providing numerical examples they evaluated their proposed model. Amin and Zhang (2012) in their study entitled as a multi-objective location model for closed-loop supply chain networks under the conditions of the uncertainty of demand and return provided a mixed integer linear programming model that minimizes the total cost. They also examined the impact of the uncertainty of demands and returns on the form of the network by contingency planning. Ramezani et al (2013) in their article presented a multi-objective probabilistic model under conditions of uncertainty, which has three levels in the direct direction (suppliers, manufacturing and distribution centers), and two levels in reverse direction (collection and disposal). Among the used uncertain parameters we can refer to price of products, production and operating costs, demand and return rates, which are defined by a number of scenarios. In their proposed model, the objective is to maximize profit, customer responsiveness and quality of purchased raw material from the supplier. Also financial risk has been calculated for relative valuation of the objectives. KeyvanShokooh et al (2013), in addition to designing a reverse-direct supply chain network, have applied dynamic pricing for returned products. Their model is a multi-level, multi-period and multi-product model in which the returned products are classified according to their quality and for each category a different price is provided. The sole purpose of the model is to minimize the costs which include fixed costs, operating and maintenance and purchase cost of returned products. For solving their model, they have presented a mixed integer linear programming model. The results indicate that the use of dynamic pricing instead of static pricing and linear pricing approach for these models provided an acceptable answer. Arabzad et al. (2014) proposed a multi-objective optimization algorithm for solving a new multi-objective location-inventory problem in a distribution center (DC) network with the presence of different transportation modes and third party logistics (3PL) providers. In this model 3PL is responsible to manage inventory in DCs and deliver products to customers. Furthermore, they proposed a non-dominated sorting genetic algorithm (NSGA-II) to perform high-quality search using two-parallel neighborhood search procedures for creating initial solutions. The potential of this algorithm has been evaluated by its application to the numerical example. Then, the obtained results have been analyzed and compared with multi-objective simulated annealing (MOSA). Demirel et al (2014) presented a mixed multi-period and multi-piece linear programming model for a closed-loop supply chain network. In their model they followed two general policies which consist of secondary market pricing and incentive policies. In the first policy, customers by orders received goods from distribution centers, but in the next period, they could buy and sell the goods which are returned to the secondary market among themselves. The policy encourages the customer to return the product and the correct amount of collecting at different prices. To solve the model in real size, an improved genetic algorithm is proposed. In this model, the demand is deterministic and shortages are not allowed. Also the capacity
of the entire facility is limited. Vahdani et al (2014) in their article, by presenting a systematic approach designed a reliable network of facilities in a closed-loop supply chain under conditions of uncertainty. Their model consistently designed the intended network by employing an efficient and reliable approach. The network provided by them is a multi-level, multi-facility, multi-product and multi-provider network. To solve the proposed model, a new interactive hybrid approach has been developed by combining a number of effective approaches in the past. This method is a fuzzy two-objective mixed integer linear programming model. To get closer to the real world results a case study on the iron factory has been considered. Finally, the results obtained from the solving of the model are provided to demonstrate the applicability and accuracy of the proposed model. Ghorbani et al (2014) considered a fuzzy goal programming approach for solving a multi-objective model of reverse supply chain design. This model involves of three objectives that they minimize recycling cost, rate of waste generated by recyclers and material recovery time in such a way that the best set of recyclers to allocate products is determined. The main contribution of of the proposed model is that it considers cost, time and efficiency rate to design responsive and efficient reverse supply chain. A numerical example is conducted to validate the model.

Hatefi et al (2014) presented a model for designing an integrated supply chain network. In their model, they used the concepts of reliability to evaluate the damaged facilities. In the proposed model, the facilities are divided into two categories of reliable and unreliable facilities and unreliable facilities lose part of their capacity in the event of disruption. Therefore, in order to meet customer demand, part of the demand is supplied by the residual capacity and part of it is supplied by sharing strategy, in such a manner that goods are carried from reliable facilities to unreliable facilities. This model is a single-product and multi-level model and production and recycling and collection and distribution centers have been considered as a combination. The objective function consists of minimizing fixed costs, transportation, operating and disruption costs in facilities. Parameters of demand and return rates of product, transportation costs, fixed costs, operational costs and the capacity of facilities are considered in a fuzzy form. To deal with the uncertainty, reputation-based imposed math program has been employed. Finally, to validate the model, numerical examples of solutions and the sensitivity analyzes were performed. Pishvaee et al (2014), by using a multi-objective location programming model designed the supply chain network of the pharmaceutical industry. Their proposed model is a stable model which consists of the triple economic, environmental (green) and social objectives. Economic objective includes minimizing fixed costs, operating and transportation costs and environmental objective includes minimizing the environmental impact of transportation and operational activities and the social purpose includes maximizing created jobs and local development and to minimize the risk of consumers and workers injuries. To solve the model, the accelerated Benders Decomposition algorithm was used. Ramezani et al (2014) presented a multi-product and multi-period model for designing a closed-loop supply chain network under fuzzy environment. In the proposed network, the movement of goods between facilities of two levels can be performed by different transport models. Their proposed model consists of four layers in the direct direction (supplier, manufacturer, distribution centers and customers) and three layers in reverse order (customers, collection and disposal centers). Their model includes three objectives: The first objective is to maximize profit that this objective becomes possible by maximizing difference between revenues and costs. The second purpose is to maximize the level of servicing. This goal is achieved by minimizing the transportation time in direct and reverse direction. The third objective maximizes sigma quality level that this goal is performed by minimizing the number of defective raw materials produced by the supplier. In this way the quality of the parts produced in manufacturing centers will increase.
3. Model formulation

The proposed network in this study has three levels in the direct direction (suppliers, producers and distributors) and three levels in reverse direction (collection, redistribution and disposal centers). Initially, the manufacturing factories provide their needed raw materials from the suppliers. Final products manufactured in production centers are sent to distribution centers. Then the products are delivered to customers upon their request. In this network, each customer is allowed only to receive a product from one distribution center. In order to return a percentage of these products are collected from customers by collecting centers. In the collection centers, initially the returned products are inspected, and then those products that are reproducible are again transferred to the manufacturing factories. Some of the products that are no longer usable are sent to disposal centers. Other products that have lost their initial quality but are still reusable are sent to distribution centers to be sent to the secondary customers. Facilities in the supply chain can be disrupted due to variety of reasons, such as natural, humanitarian disasters … and could not be able to work with their intended capacity. These facilities either completely lose their capacity or only a fraction of this capacity is disrupted and the intended facility can continue to operate with the remaining capacity. In this research production centers are divided into two reliable and unreliable categories. Reliable production centers, typically manufacture with the available capacity and the required product is sent to distribution centers, but unreliable centers, may be disrupted with certain probability. If an unreliable center becomes disrupted, it will lose some of its capacity and therefore it would not be capable of answering the needs of its distribution centers. In this study, to deal with such problems, sharing strategy has been used. This means that, if an unreliable production facility does not meet the needs of its distribution center, this distribution center would provide the remaining of its requirements from other distributors who receive products from reliable production centers. Figure 1 shows the configuration of proposed network and the flow of products between different facilities. After configuration of network, a mathematical model is proposed. This model is a multi-product and two-objective model. The first objective is to maximize the expected profit of the network. This objective is achieved from the difference between the proceeds from the sale of products to primary and secondary customers and costs, which include variable costs such as purchase cost, transportation costs, manufacturing costs, distribution costs, collection costs, operating costs and fixed costs of constructing facilities. The second objective maximizes the level of servicing to customers that this objective is achieved from weighted sum of the forward and reverse flow.
3.1. Model decisions

Some of the decisions made in the proposed model are as follows:
1) Determining the location of each facility
2) Determining the number of reliable and unreliable production centers
3) Determining the amount of purchased raw materials and products among facilities

3.2. Model assumptions

In the proposed supply chain network design problem, the following assumptions have been considered in the model:
1) The proposed model is a multi-product and single-period model.
2) The location of suppliers and customers are known and fixed.
3) Potential sites for factories, distributors, and collection and disposal centers are known.
4) Each customer can only get product from one distribution center.
5) The number of facilities that can be opened and their related capacity is limited.
6) The flow of products can only be established between facilities of two successive levels of network layers. Also at the level of distributors, the flow of product can occur in the same facilities.
7) Uncertainty has been considered for some parameters of the model.
8) The level of the quality of the products sent to secondary customers is lower than primary products and they are sold at cheaper prices.

In the following sets, parameters and variables of the model are described:
3.3. Sets

S: The set of fixed locations of suppliers (s = 1, 2... S)
P: The set of potential sites of the trusted factories (p = 1, 2... P)
T: The set of potential sites of unreliable factories (t = 1, 2... T)
H: The set of potential locations of distribution centers (these centers receive products from the P production center) (h = 1, 2... H)
J: The set of potential locations of distribution centers (these centers receive products from the T production center) (j = 1, 2... J)
I: The set of potential locations of collection centers (i = 1, 2... I)
D: The number of potential sites of the disposal centers (d = 1, 2... D)
C: The set of fixed locations of the primary customers (c = 1, 2... C)
K: The set of fixed locations of the secondary customers (k = 1, 2... K)
L: The set of manufactured products (l = 1, 2... L)
M: The set of raw materials (m = 1, 2... M)

3.4. Parameters

Demands:

D_{CL}: demand of the primary customer c, for product L

Variable costs:

P_{cL}: the unit price of the sale of L product to customer C
P_{KL}: the unit price of the sale of L product to secondary customer K
P_{SM}: the unit cost of purchasing raw material M from the supplier S
PP_{pt} = PP_{tL}: the unit cost of production of the product L in the factory (p and t)
CP_{H} = CP_{J}: unit cost of distribution and operations at the distribution center (h and j)
CI_{i}: Unit testing and inspection cost collection center i
CD_{d}: Unit cost of disposal at the disposal center d
CB_{p} = CB_{pl}: unit cost of product recovery at the factory (p and t)
CR_{r}: Unit cost of distribution at redistribution center r

Fixed costs:
FCPR\textsubscript{p}: fixed costs of construction of reliable manufacturing factory \(p\)

FCPU\textsubscript{t}: fixed costs of construction of unreliable manufacturing factory (\(FCPU\textsubscript{t} \leq FCPR\textsubscript{p}\))

\(FCH_h = FCJ_j\): fixed cost of construction of distribution center \((h, j)\)

\(FCC_i\): fixed cost of constructing collection center \(i\)

\(FCD_d\): fixed cost of constructing disposal center \(d\)

\(FCR_r\): fixed cost of construction of redistribution center \(r\)

**Transportation costs:**

\(TC_{spm}\): unit cost of transporting raw material \(m\) from supplier \(s\) to factory \(p\)

\(TC_{stm}\): unit cost of transporting raw material \(m\) from supplier \(s\) to factory \(t\) (\(TC_{spm} = TC_{stm}\))

\(TC_{phl}\): unit cost of transporting product \(l\) from the factory \(p\) to the distribution center \(h\)

\(TC_{tjl}\): unit cost of transporting product \(l\) from the factory \(t\) to the distribution center \(j\) (\(TC_{tjl} = TC_{phl}\))

\(TC_{hcl}\): unit cost of transporting product \(l\) from distribution center \(h\) to customer \(c\)

\(TC_{jcl}\): unit cost of transporting product \(l\) from distribution center \(j\) to customer \(c\) (\(TC_{jcl} = TC_{hcl}\))

\(TC_{cai}\): unit cost of transporting returned product \(l\) from customer \(c\) to collection center \(i\)

\(TC_{ipt}\): unit cost of transporting reproducible product \(l\) from collection center \(I\) to factory \(p\)

\(TC_{ait}\): unit cost of transporting reproducible product \(l\) from collection center \(I\) to factory \(t\) (\(TC_{ait} = TC_{ipt}\))

\(TC_{a1l}\): unit cost of transporting product \(l\) from collection center \(I\) to disposal center \(d\)

\(TC_{a1l}\): unit cost of transporting product \(l\) from collection center \(I\) to redistribution center \(r\)

\(TC_{a1l}\): unit cost of transporting product \(l\) from redistribution center \(r\) to secondary customer \(k\)

**Times**

\(TIPH_{phl}\): Transportation time of each unit of product \(l\) from production center \(p\) to the distributor \(h\)

\(TIIT_{tjl}\): Transportation time of each unit of product \(l\) from production center \(t\) to the distributor \(j\) (\(TIIT_{tjl} = TIPH_{phl}\))

\(TIHC_{hcl}\): Transportation time of each unit of product \(l\) from distribution center \(h\) to the customer \(c\)

\(TIJC_{jcl}\): Transportation time of each unit of product \(l\) from distribution center \(j\) to the customer \(c\) (\(TIJC_{jcl} = TIHC_{hcl}\))

\(TICI_{cit}\): Transportation time of each unit of product \(l\) from customer \(c\) to collection center \(i\)

\(TIIP_{ipt}\): Transportation time of each unit of product \(l\) from collection center \(I\) to production center \(p\)

\(TIIT_{ait}\): Transportation time of each unit of product \(l\) from collection center \(I\) to production center \(t\) (\(TIIP_{ipt} = TIIT_{ait}\))
**Capacity of the facilities:**

- $\text{CAS}_{sm}$: The capacity of supplier, $s$ for raw material $m$
- $\text{CAP}_p$: Factory capacity $p$
- $\text{CAP}_t$: Factory capacity $t$
- $\text{CAD}_h$: The capacity of distribution center $h$
- $\text{CAD}_t$: The capacity of distribution center $t$
- $\text{CAC}_i$: The capacity of collection center $i$
- $\text{CAR}_p$: The capacity of factory $p$ for recovery of products
- $\text{CAR}_t$: The capacity of factory $t$ for recovery of products
- $\text{CAM}_d$: The capacity of disposal center $d$
- $\text{CAN}_r$: The capacity of redistribution center $r$

**Rates:**

- $\text{RR}$: Rate of return of products used by customers
- $\text{RC}$: Rate of reproduction
- $\text{RD}$: Rate of Disposal
- $\text{UR}_{ml}$: The rate of using raw material $m$ in the product $l$
- $\alpha$: The weighting factor for the importance of the forward responsiveness
- $\text{Pu}_t$: The percentage of disturbance in unreliable production center $t$ in the forward direction
- $\text{Pu}_t'$: The percentage of disturbance in unreliable production center $t$ in the reverse direction

**Maximum number of facilities:**

- $\text{MP}$: The maximum number of factories $p$ that can be constructed.
- $\text{MT}$: The maximum number of factories $t$ that can be constructed.
- $\text{MD}$: The maximum number of distribution centers $h$ that can be constructed.
- $\text{MJ}$: The maximum number of distribution center $j$ that can be constructed.
- $\text{MC}$: The maximum number of collection centers that can be constructed.
- $\text{MN}$: The maximum number of disposal facilities that can be constructed.
- $\text{MR}$: The maximum number of redistribution centers that can be constructed.
Continuous variables:

- \( QTY_{s,p,m} \): The amount of raw material \( r \) that supplier sends \( s \) to factory \( p \).
- \( QTY_{s,t,m} \): The amount of raw material \( r \) that supplier sends \( s \) to factory \( t \).
- \( QTY_{p,h,l} \): The amount of product \( l \) which is sent from factory \( p \) to distribution center \( h \).
- \( QTY_{t,j,l} \): The amount of product \( l \) which is sent from factory \( t \) to distribution center \( j \).
- \( QTY_{h,c,l} \): The amount of product \( l \) which is sent from distribution center \( h \) to customer \( c \).
- \( QTY_{j,c,l} \): The amount of product \( l \) which is sent from distribution center \( j \) to customer \( c \).
- \( QTY_{c,i,l} \): The amount of returned product \( l \) which is sent from customer \( c \) to collection center \( i \).
- \( QTY_{i,p,l} \): The amount of reproducible product \( l \) which is sent from collection center \( i \) to factory \( p \).
- \( QTY_{i,t,l} \): The amount of reproducible product \( l \) which is sent from collection center \( i \) to factory \( t \).
- \( QTY_{i,d,l} \): The amount of reproducible product \( l \) which is sent from collection center \( i \) to disposal center.
- \( QTY_{i,r,l} \): The amount of reproducible product \( l \) which is sent from collection center \( i \) to redistribution center \( r \).
- \( QTY_{r,k,l} \): The amount of reproducible product \( l \) which is sent from redistribution center \( r \) to the secondary customer \( k \).

Binary variables:

- \( UX_t \): If unreliable production center \( t \) is constructed 1, otherwise 0
- \( RX_p \): If reliable production center \( p \) is constructed 1, otherwise 0
- \( Y_h \): If the distribution center \( h \) is constructed 1, otherwise 0
- \( Y_j \): If the distribution center \( j \) is constructed 1, otherwise 0
- \( Z_i \): If the collection center \( i \) is constructed 1, otherwise 0
- \( U_d \): If the disposal center \( d \) is constructed 1, otherwise 0
- \( CC_{hc} \): If distribution center \( h \) sends the product to the customer \( c \) 1, otherwise 0
- \( CC_{jc} \): If distribution center \( j \) sends the product to the customer \( c \) 1, otherwise 0
- \( CC_{ci} \): If the collection center \( I \) gets the returned product of the customer \( c \) 1, otherwise 0

3.5. Objective functions

The objective function of the problem includes the maximizing of the profit of supply chain network which will be explained below:
The objective function of the problem is to maximize the expected profit of the network which is achieved from the difference between expected income from the sale and expenses related to the chain. Chain costs include the costs of shipping goods between facilities, variable costs of the supply of the suppliers, the cost of production in factories, processing costs at distribution centers, test and inspection costs at the collection centers, the cost of reproduction in factories, the cost of disposal at disposal centers and fixed costs of establishing facilities.

\[ \text{Max } Z = \text{Income} - \text{Cost} \]

\[ \text{Income} = \sum_{i} \sum_{j} \sum_{k} QTY_{ikj} P_{ij} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} P_{ij} \]  \hfill (1)

\[ \text{Cost} = \sum_{i} \sum_{j} \sum_{k} QTY_{ikj} \text{TC}_{ikj} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TC}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} \]

\[ + \sum_{i} \sum_{j} \sum_{k} QTY_{ikj} \text{TC}_{ikj} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TC}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} \]

\[ + \sum_{i} \sum_{j} \sum_{k} QTY_{ikj} \text{TC}_{ikj} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TC}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} \]

\[ + \sum_{i} \sum_{j} \sum_{k} QTY_{ikj} \text{TC}_{ikj} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TC}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} \]

\[ + \sum_{i} \sum_{j} \sum_{k} QTY_{ikj} \text{TC}_{ikj} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TC}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} + \sum_{i} \sum_{j} \sum_{k} QTY_{ij} \text{TC}_{ij} \]

The objective function of the problem is to maximize the expected profit of the network which is achieved from the difference between expected income from the sale and expenses related to the chain. Chain costs include the costs of shipping goods between facilities, variable costs of the supply of the suppliers, the cost of production in factories, processing costs at distribution centers, test and inspection costs at the collection centers, the cost of reproduction in factories, the cost of disposal at disposal centers and fixed costs of establishing facilities.

\[ \text{Min } f_2 = \alpha \left[ \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TIPH}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TITJ}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TIHC}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TIPC}_{ijk} \right] \]

\[ + (1- \alpha) \left[ \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TICL}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TIPH}_{ijk} + \sum_{i} \sum_{j} \sum_{k} QTY_{ijk} \text{TIPM}_{ijk} \right] \]  \hfill (2)

The second objective function is to maximize the level of servicing to customers. Considering the equation 2, the sum of the time at which the products manufactured in factory are obtained by customer through the distribution centers in forward network and the sum of the time at which the products returned from customers through collection centers are sent to factory for recycling is minimized. As a result, servicing to customers is done in less time and the level of servicing will increase.

3.6. Constraints

Balance Constraint
Constraints 3 and 4 state that at each reliable p and unreliable t production center, for each raw material, the outflow from that factory to distribution centers cannot be more than the total inflow to the same factory from the suppliers and collection centers. Constraint 5 ensures that for each product, the total flow which is sent from a reliable production center p to a distribution center h is equal to the total flow which is sent from this distribution center to other distribution centers j, and outflow from that center to its customers. Constraint 6 ensures that for each product the total flow which is sent from an unreliable production center t to a distribution center and the total flow which is sent from other distributors to this center is equal to the outflow sent from that distribution center to the customers. Constraints 7 and 8 show that the sum of the products sent from distribution centers should be equal with the demands of customers. Constraint 9 shows that customers have single source, this means that customers receive the product only from one distribution center. Constraint 10 shows that how the products returned from the customers is transferred to collection centers. Constraint 11 like the Constraint 12 shows that customers have single source in the reverse direction, in other words, the
products returned from each customer is received by one collecting center. Constraint 13 indicates that all products sent from each collection center are equal to the sum of products that enter to this center. Constraints 14 and 15 show that how the products returned from the customers are transferred to collection centers, factories, dismantling and redistribution centers. Constraint 16 ensures that the total flow sent to each redistribution center is equal to outflow sent from this center to secondary customers.

**Capacity constraint**

\[
\sum_{p} QTY_{spm} \leq CAS_{sm}, \quad \forall s,m \quad (17)
\]

\[
\sum_{h} \sum_{l} QTY_{phl} \leq CAP_{hp}RX_{p}, \quad \forall p \quad (18)
\]

\[
\sum_{h} \sum_{l} QTY_{ijl} \leq CAP_{ij}(1 - pu_{i})UX_{j}, \quad \forall t \quad (19)
\]

\[
\sum_{l} \sum_{t} QTY_{hcl} + \sum_{j} \sum_{t} QTY_{bjl} \leq CAD_{h}Y_{h}, \quad \forall h \quad (20)
\]

\[
\sum_{j} \sum_{l} QTY_{jcl} \leq CAD_{j}Y_{j}, \quad \forall j \quad (21)
\]

\[
\sum_{j} \sum_{l} QTY_{hjl} \leq CAD_{j}Y_{j}, \quad \forall j \quad (22)
\]

\[
\sum_{i} \sum_{l} QTY_{cil} \leq CAD_{c}Z_{i}, \quad \forall i \quad (23)
\]

\[
\sum_{i} \sum_{l} QTY_{ipl} \leq CAR_{ip}RX_{p}, \quad \forall p \quad (24)
\]

\[
\sum_{t} \sum_{i} QTY_{iti} \leq CAR_{i}(1 - pu^{'}_{i})UX_{t}, \quad \forall t \quad (25)
\]

\[
\sum_{l} \sum_{d} QTY_{idl} \leq CAM_{d}U_{d}, \quad \forall d \quad (26)
\]

\[
\sum_{r} \sum_{j} QTY_{irt} \leq CAN_{r}V_{r}, \quad \forall r \quad (27)
\]

Constraint 17 states that the sum of the raw material provided by suppliers cannot be greater than the total available capacity of the supplier. Constraints 18 and 19 ensure that the outflow of each reliable and unreliable factory cannot exceed the capacity of the relevant factory. Constraint 20 shows that the outflow from each distribution center h (the center which receives product form reliable production center) to other distribution centers j and the customer is limited to the capacity of related facilities. Constraint 21 shows that the outflow from each distribution center j (the center which receives the product of an unreliable production center t is limited to the capacity of the related facility. Constraint 22 states that flow that enters into a distribution center from other distribution centers h should not exceed the capacity of this center. Constraint 23 ensures that the outflow from each collection center cannot exceed the capacity of the related facility. Constraints 24 and 25 state that the total outflow from each collection center to the reliable and unreliable production centers for reproduction could not be more than the reproduction capacity of the relevant factory. Constraint 26 states that the inflow to disposal centers is limited to the capacity of the relevant disposal center.

**Maximum number of facility**
\[ \sum_{p} X_{p} \leq MP \]  
\[ \sum_{t} X_{t} \leq MT \]  
\[ \sum_{h} Y_{h} \leq MD \]  
\[ \sum_{j} Y_{j} \leq MJ \]  
\[ \sum_{i} Z_{i} \leq MC \]  
\[ \sum_{r} V_{r} \leq MR \]  
\[ \sum_{d} U_{d} \leq MN \]  

Constraint 27 ensures that the amount of inflow to each redistribution center could not exceed the capacity of the relevant facility. Constraints 35 and 36 imply that the sum of the products which are sent from the collection centers to reliable and unreliable production centers for reproduction should not exceed the total amount of raw materials which are sent from suppliers to production centers. Constraints 37 and 38 state that the sum of the products which are sent from the collection centers to reliable and unreliable production centers for reproduction should not be more than the total flow which is sent from production center to distribution centers.

**Other Constraints**

\[ \sum_{l} \sum_{i} QTY_{ipl \; ml} \cdot UR_{ml} \leq \sum_{s} \sum_{m} QTY_{spm} \quad \forall p \]  
\[ \sum_{l} \sum_{i} QTY_{ipl \; ml} \cdot UR_{ml} \leq \sum_{s} \sum_{m} QTY_{stm} \quad \forall t \]  
\[ \sum_{l} \sum_{i} QTY_{ipl \; phl} \leq \sum_{h} \sum_{l} QTY_{phl} \quad \forall p \]  
\[ \sum_{l} \sum_{i} QTY_{ipl \; tjl} \leq \sum_{j} \sum_{l} QTY_{tjl} \quad \forall t \]  

Constraints (39) and (40) respectively indicate zero and one variables and non-negativity of the variables of the product transfer flow between facilities.

\[ X_{p}, X_{t}, Y_{h}, Y_{j}, Z_{i}, U_{d}, CC_{hc}, CC_{hc}, CC_{ci} \in \{0,1\} \quad \forall p, t, h, j, i, d \]  
\[ QTY_{mp}, QTY_{mt}, QTY_{ph}, QTY_{pm}, QTY_{ph}, QTY_{pm}, QTY_{pl}, QTY_{pl}, QTY_{it}, QTY_{it}, QTY_{it}, QTY_{it}, QTY_{it}, QTY_{it}, QTY_{it}, QTY_{it}, QTY_{it} \geq 0 \quad \forall s, p, t, h, j, i, r, d, m, l \]  

**4. Solution approach**

The intended problem is a two- objective logistics network design problem that for its analysis augmented \( \varepsilon \)-constraint approach was used. Suppose that \( i \) number of objective functions are of maximizing type and \( j \) number of objective functions are of minimizing type:
The steps of the above method are as follows:

1) Initially optimal value of each function is achieved separately and without considering other objectives ($Z^p$).

$$\max f_i(x)$$

$$\min f_j(x)$$

$$s.t.
\begin{align*}
x & \in X
\end{align*}$$

(41)

2) At this stage the optimum solution for each function is placed in other functions and the value of the objective function is obtained. ($Z^n$)

3) The value of $r$ is obtained from the following equation:

$$r = Z^p - Z^n$$

(42)

4) An objective function is considered as the main objective function and for other objectives we consider the value of $e$ to be equal with $Z^n$ and we solve the following model for various $e$ in order to obtain a set of Pareto solutions.

$$\max (f_i(x) + \delta(S_2/r_2 + S_3/r_3 + \ldots S_p/r_p)$$

$$s.t.
\begin{align*}
f_i(x) - S_i & = e_i
f_j(x) + S_j & = e_j
x & \in S, Si \in R^+
\end{align*}$$

(43)

In the above model, $i$ is the index of maximum functions and $j$ is the index of minimum functions. A negligible and usually medial amount is chosen for $\delta$ (Mavrotas and Floris, 2013).

5. Computational results

In order to demonstrate the applicability of the proposed model, a numerical example is presented in this section. In this example, the number of suppliers is 6, the number of potential sites for reliable factories is 4 and for unreliable factories it is 5, by taking into account the percent of capacity lost as a uniform distribution in the interval $(0.1, 0.5)$, for distribution centers 4 and 3, for the collection centers 3, for disposal centers 4, for redistribution centers 3, the number of primary customers is 8 and the number of secondary customers is 2. Also the model will be solved in multi-product mode and by considering two products. Transportation costs are defined according to the distance between the facilities in the network. Other model parameters are shown in Table 1.
The problem has been solved with respect to the intended parameters in definitive mode and in multi-objective form by using augmented $\varepsilon$-constraint approach. Coding of the model and all calculations were conducted by using the GAMS 23.6 software and COUENNE solver. This example is solved by two objective functions and given that this approach offers a set of Pareto solution, the Pareto curve is provided in which the time is set against the profit. According to figure 2, with increase in profits, the travel times also increase and as a result, the level of customer service reduces.
6. Sensitivity analysis

To study the behavior of Pareto curve of profit and time towards the changes in percent of capacity lost, this curve is drawn for different percentages. According to figure 3 the values of Pareto curve are reduced with increasing amounts of lost capacity. This reduction is due to increased costs and reduced profitability of the network.

![Figure 3: Profit versus Time per percent of the lost capacity](image)

7. Conclusion

In this study, a reliable multi-objective nonlinear mixed integer programming model was provided for designing reverse-direct logistics network against the risks caused by the disruption of facilities and risks related to uncertainty of parameters. To study disruption in facilities, two types of production centers were considered, reliable production centers which were stable and do not get damaged and unreliable production centers that in the event of disruption, the percentage of their capacity is lost. In this case the distribution centers, which receive product from these centers, get deficient. This deficiency is compensated by other distribution centers. Objective functions of this model seek to maximize the profit and minimize time (maximizing level of service). Among the main feature of this model we can mention the ability to simultaneously adopt strategic and tactical decisions, its being multi-product, single-sourcing of customers and adopting decisions such as determining the location and time of establishing facilities, the rate of supply of raw materials from each supplier, the production rate of each of the units of production and distribution planning for the final product. The major contribution of this paper is to focus on considering reliability. In proposed supply chain network some plants are unreliable and in disruption condition are allowed to be partially disrupted and thus a percentage of their capacities may be lost. To compensate the lost capacity a sharing strategy is considered. Furthermore second customer echelon has been considered as a new echelon. To demonstrate the applicability and accuracy of the proposed model numerical example was presented and by using augmented $\varepsilon$-constraint approach the Pareto curve for time against the profit was established that by using it the decision maker can chose his intended
answer. This method has the ability to create a variety of efficient solutions. Also in the sensitivity analysis, the impact of the lost capacity of unreliable production centers on the objective function of the problem was expressed.

Further research may consider the uncertain nature of the problem using stochastic programming, such as robust optimization or fuzzy programming. It is also possible to incorporate the reliability concepts into the transportation and inventory decisions to design a more reliable supply chain network. In addition, the proposed reliability concepts can be applied for supplier issues to cope with supplier disruptions. Modelling the different types of disruptions (caused by natural, man-made or technological threats) and their impacts on facilities and/or transportation links through a scenario-based approach would be of particular interest.

8. References


