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A new approach on auxiliary vehicle assignment in capacitated location routing problem

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The location routing problem (LRP) considers locating depots and vehicle routing decisions simultaneously. In classic LRP the number of customers in each route depends on the capacity of the vehicle. In this paper a capacitated LRP model with auxiliary vehicle assignment is presented in which the length of each route is not restricted by main vehicle capacity. Two kinds of vehicles are considered: main vehicles with higher capacity and fixed cost and auxiliary vehicles with lower capacity and fixed cost. The auxiliary vehicles can be added to the transportation system as an alternative strategy to cover the capacity limitations and they are just used to transfer goods from depots to vehicles and cannot serve the customers by themselves. To show the applicability of the proposed model, some numerical examples derived from the well-known instances are used. Moreover the model has been solved by some meta-heuristics for large sized instances. The results show the efficiency of the proposed model and the solution approach, considering the classic model and the exact solution approach, respectively.

Keywords: location routing; auxiliary vehicle; capacitated; simulated annealing

1. Introduction and the literature review

Recently, the location routing problem (LRP) has been one of the most applicable problems in industrial environment and logistic systems. For example, it has been used in designing delivery or distribution systems, such as blood bank location, food and drink distribution (Watson-Gandy & Dohrn, 1973), waste collection (Kulcar, 1996), newspaper delivery (Jacobsen & Madsen, 1980; Madsen, 1983), bills (Lin, Chow, & Chen, 2002), drugs and also other applications that are mentioned in a survey paper by Nagy and Salhi (2007). The LRP consists of two sub-problems, facility location problem (FLP) which is responsible for locating the depots and assigning customers to located depots and vehicle routing problem (VRP) that is going to find the sequence of customers to be served by each vehicle. According to Min, Jayaraman, and Srivastava (1998) methods that treat FLP and VRP separately have some limitations and the effect of this consideration is discussed in Salhi and Rand (1989). LRP also is studied in integrated logistics systems where the optimal location of depots and vehicle routing are considered simultaneously (Samaei, Bashiri, & Tavakkoli-Moghadam, 2012). The benefits of considering these two problems simultaneously are mentioned in survey papers presented by Min et al. (1998) and Nagy and Salhi (2007).

In classic LRP the number of customers in each route depends on the capacity of the vehicles. The customers are assigned to each vehicle until total demands of assigned customers reach the maximum capacity of the vehicle. Hence the route is terminated and the vehicle should return to the depot where it starts its trip.

In this paper we propose a new model that the length of each route is not restricted by vehicle’s capacity. Two kinds of vehicles are considered, main vehicles with higher capacity and fixed cost and auxiliary vehicles with lower capacity and cost. The auxiliary vehicles can be added to the transportation system to cover the capacity limitation as an alternative strategy. Customers cannot be served by the auxiliary vehicles, these vehicles are used only for replenishing the main vehicle. This case can happen when serving customers require specific devices which are installed on main vehicles. Suppose that the cost of initialising main vehicle and its entering to and leaving from depots is high. In this case instead of terminating the route and returning the main vehicle to the depot, an auxiliary vehicle with lower cost can be sent to replenish the main vehicle. As some real situations in this area, we can mention to using auxiliary vessels in ship delivery services. These auxiliary ships with less capacity play an important role in the replenishment of the main ships in harbours as customer points. In many fleet ships companies, auxiliary ships besides cargoes replenishment, play the role of transferring, supporting or repairing as backup ships over the main ship travelling tour. Although the auxiliary vehicles have less cost and more speed, because of their capacity limits, they will not apply for cargoes distribution (Michell, 2011).

Another situation for using auxiliary vehicles is in the transport of hazardous materials. For example, in petrolium
distributing, since the long time of carrying is dangerous, to increase the efficiency of transportation system and prevent returning empty vehicle most of the tour length, they use auxiliary vehicles for replenishment of the containers. Loading equipment in the routes offers enhanced services transport capabilities to customers (Army, United States Government, 2013).

In drug shipment and large distribution systems, for long distances and large volumes, the auxiliary helicopters are used to replenish the ship in order to facilitating drug and health care supply distribution. The cost of these vehicles is absolutely less than returning ships to origins (Nickodemus, 1967).

Another application of this model can be explained for special drugs distributions, which need to be transported under special conditions, so the drugs cannot be carried by a vehicle with a high capacity for a long time. By using auxiliary vehicles the total transportation time for some special drugs can be decreased enough. Also for road drug distribution systems which use the expensive refrigerated trucks, it is efficient to apply auxiliary vehicles for replenishment of the trucks; this helps to manage orders and delivery process and plan for distribution of drugs at the right place, at the right time to better service the customers. So orders can be scheduled and assigned to trucks and auxiliary vehicles for maximum performance transportation planning. Coordinate dispatching between main transport vehicles and auxiliary drug carriers works such a backup for supplying more orders. While if only auxiliary carriers will be implemented, because of their lower capacity the system may not be able to respond to all orders. Especially the farther demand points with large volume of drug demand.

Figure 1 depicts structural differences between the proposed problem and classic LRP.


In distribution problems, recently some works have been done with considering the real situations for vehicle fleets. Ceselli, Righini, and Tresol (2014) studied a drug distribution problem and proposed the combination of multiple distribution modes with heterogeneous vehicle fleet. Mousavi, Niaki, Mehdizadeh, and Tavaroth (2013) presented a mathematical model for multi-facility location allocation problem with probabilistic customer’s locations and demands. Another work in this area which has a specific assumption on vehicles is Shen, Ordóñez, and Dessouky (2009); they proposed a two-stage vehicle routing for solving a distribution problem in emergency situations. Stenger, Schneider, Schwinda, and Vigo (2012) presented a location routing model for small package shipping and they considered the possibility of exchanging the distributor operators. Ho, Chang, and Ku (2013) considered a multi-choice LRP for buying or renting the facilities. Also all above works
consider some assumptions on vehicles and distribution issues; the proposed modelling on using the auxiliary vehicles for replenishment of the main one is an original work in this area.

The rest of the paper is structured as follows: In the next section the problem will be defined. The mathematic formulation of the model is given in Section 3. In Section 4 simulated annealing (SA) approach is proposed for large-scale problems. Some sensitivity analysis is done in Section 5. Section 6 reports some computational results, including the comparison between solutions obtained by the proposed approach and the classic methods. Finally we conclude the paper in Section 7.

2. Problem definition

The simple classic LRP considers a set of potential depots which should serve a set of customers with known demands through a fleet of vehicles by minimum possible costs including fix costs associated with locating depots and routing. The problem solution determines which depots should be opened and which customers should be assigned to each opened depot, also it gives the routing schedule with minimum total cost.

Figure 2 depicts a typical classic LRP solution for this problem. In this solution one depot has been opened, there are 15 customers assigned to the depot and four vehicles which construct four routes.

In this paper we are going to model a new aspect of LRP. To clarify the problem consider a drug distribution company where the drugs must be kept in special conditions such as special temperature; this requires expensive vehicles with high level of technology for transporting the drugs to the demand points. The efficiency of the proposed model will be more tangible in a case with longer travel times and larger number of customers. It means that when the travel time gets longer and the number of customers in a route gets higher the main vehicle should be loaded more when starting the trip, so the cargo spends more time in the vehicle until it reaches its customers. By using auxiliary vehicles this time can be reduced and the main vehicles do not have to load all the customers’ cargo when starting a trip. In another situation when the total demands of customers are more than the capacities of the main vehicle, in classic LRP all the customers cannot be served in one route. If the cost of initialising each route is high using auxiliary vehicles might be economically justifiable.

In the mentioned example, high cost of main vehicles makes it desirable to reduce the number of routes and consequently the number of main vehicles. An appropriate approach could be using auxiliary vehicles which need less cost than the main ones and have a short trip. In this approach each depot has an auxiliary vehicle, in the point that the main vehicle runs out of stocks, the auxiliary vehicle travels from the depot to that point to replenish the main vehicle; this is more beneficial especially when the travel cost of main vehicle to depot is considerably high. By using auxiliary vehicles the tour will not be terminated because of vehicle capacity. Accordingly, the whole vehicle’s set-up costs and the number of routes will be decreased. Figure 3
3. Model formulation

In this section we focus on formulating the problem and presenting a model by considering problem assumptions. Prins, Prodhon, Ruiz, Soriano, and Wolffer Calvo (2007) have given the basic mathematical model for capacitated LRP, here is the mathematical model for our approach that considers the possibility of using auxiliary vehicles. By comparing costs, the model will decide whether to use auxiliary vehicles or not, if using auxiliary vehicle was economical it determines where they should be used. We call the proposed mathematical model Aux-LRP model.

SETS

\( J \) Set of customers
\( I \) Set of potential depots
\( V \) Merge set of customers and potential depots, i.e. \( I \cup J \)
\( K \) Set of vehicles

PARAMETERS AND NOTATION

\( O_i \) Opening cost of depot \( i \)
\( W_i \) Capacity of depot \( i \)
\( d_j \) Demand of customer \( j \)
\( c_{ij} \) Travelling cost between node \( i \) and node \( j \)
\( F_i \) Fixed cost of main vehicles that start their trip from depot \( i \)
\( Q \) Main vehicles capacity
\( F_i^v \) Fixed cost of using auxiliary vehicle at depot \( i \) and the cost of unloading the auxiliary vehicle and replenishing the main one
\( c_i^v \) Capacity of auxiliary vehicles that start their trip from depot \( i \)
\( B \) Total number of customers

VARIABLES

\[ x_{ijk} = \begin{cases} 1 & \text{if node } i \text{ precedes node } j \text{ in route of vehicle } k \\ 0 & \text{otherwise} \end{cases} \quad (\forall i, j \in V, k \in K) \]
\[ y_i = \begin{cases} 1 & \text{if depot } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \quad (\forall i \in I) \]
\[ x_{ij}^v = \begin{cases} 1 & \text{if an auxiliary vehicle is used on the route of vehicle } k \text{ to go from depot } i \text{ to customer } j \\ 0 & \text{otherwise} \end{cases} \quad (\forall j \in J, \forall k \in K, \forall i \in I) \]
\[ f_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is allocated to depot } i \\ 0 & \text{otherwise} \end{cases} \quad (\forall j \in J, \forall i \in I) \]

\( M_{ik} \) variable defined for customer \( j \) for sub-tour elimination in route of vehicle \( k \). (\( \forall j \in J, \forall k \in K \))
\( r_{uv} \) amount of vehicle \( k \)'s load when entering node \( u \). (\( \forall u \in V, \forall k \in K \))

\[
\begin{aligned}
\text{Min} & \sum_{i \in I} O_i y_i + \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ijk} + \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} F_i x_{ijk} \\
&+ \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} F_i^v x_{ij}^v + 2 \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_i^v x_{ij}^v 
\end{aligned}
\] (1)

vehicle is going from depot to the unsatisfied customer location, replenishing the main vehicle and returning to the depot. Comparing Figures 2 and 3 clarifies that the number of main vehicles, main routes and their associated costs in Figure 3 are less than the other one.
subject to

\[ \sum_{k \in K} \sum_{i \in V} x_{ijk} = 1 \quad \forall j \in J \]  
(2)

\[ \sum_{j \in J} d_j x_{ijk} \leq Q + \sum_{i \in I} \sum_{j \in J} x_{ijk} c_{ij} \quad \forall k \in K \]  
(3)

\[ \sum_{j \in J} f_{ij} \leq W_i y_i \quad \forall i \in I \]  
(4)

\[ \sum_{j \in J} x_{ijk} - \sum_{v \in V} x_{ijk} = 0 \quad \forall i \in V, \forall k \in K \]  
(5)

\[ \sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \]  
(6)

\[ M_{jk} - M_{ik} + (B \times x_{ijk}) \leq B - 1 \quad \forall k \in K, \forall v \in V, \forall j \in J \]  
(7)

\[ \sum_{k \in K} x_{ijk} \leq f_{ij} \quad \forall i \in I, \forall j \in J \]  
(8)

\[ x_{ijk} \leq \sum_{v \in V} x_{ijk} \quad \forall i \in I, \forall j \in J, \forall k \in K \]  
(9)

\[ \sum_{i \in I} f_{ij} = 1 \quad \forall j \in J \]  
(10)

\[ \sum_{v \in V} x_{ijk} - f_{ij} \leq 1 \quad \forall i \in I, \forall j \in J, \forall k \in K \]  
(11)

\[ r_{jk} - M (1 - x_{ijk}) - d_j + \sum_{i \in I} x_{ijk} c_{ij} \leq r_{vk} \]  
\[ \forall v \in V, \forall j \in J, \forall k \in K, j \neq v \]  
(12)

\[ r_{jk} + M (1 - x_{ijk}) - d_j + \sum_{i \in I} x_{ijk} c_{ij} \geq r_{vk} \]  
\[ \forall v \in V, \forall j \in J, \forall k \in K, j \neq v \]  
(13)

\[ r_{ik} - M (1 - x_{ijk}) \leq r_{jk} \quad \forall i \in I, \forall j \in J, \forall k \in K, j \neq i \]  
(14)

\[ r_{jk} \leq Q \quad \forall j \in J, k \in K \]  
(15)

\[ x_{ijk} \in \{0, 1\} \quad \forall i \in I, \forall j \in V, \forall k \in K \]  
(16)

\[ y_i \in \{0, 1\} \quad \forall i \in I \]  
(17)

\[ f_{ij} \in \{0, 1\} \quad \forall i \in I, \forall j \in V \]  
(18)

\[ M_{jk} \geq 0 \quad \forall j \in J, \forall k \in K \]  
(19)

\[ r_{vk} \geq 0 \quad \forall v \in V, \forall k \in K \]  
(20)

4. Proposed heuristic solution approach for the Aux-LRP model

4.1. Solving method

Because of complexity of introduced problem, applying meta-heuristics is an efficient method for finding acceptable solutions. We used SA algorithm for solving the discussed problem. SA works based on local search to find optimal solution and it has been successful in a variety of complex problems. One of the effective properties of SA is allowing the search process to continue over all areas even where the objective function goes worse. We chose SA algorithm because it has this capability to search all over the solution area. In SA algorithm, some worse solutions are accepted over the searching process with predefined probability to have a chance in finding better solutions. This ability is our main reason for applying SA as a meta-heuristic in this paper. Moreover our study, recent works in this area, such as Mousavi and Tavakkoli-Moghadam (2013), Şahin, Çavuşlar, Onçan, Şahin, and Akkuş (2013) and Lima and Yob (2012) confirm that SA has good performance in the LRP. This feature makes SA an effective and useful method.

4.2. Solution representation

Our solution representation consists of two number strings. Let ‘n’ and ‘m’ indicate the number of
customers and depots, respectively, first string contains \(\{1, 2, \ldots, n\}\), which are related to \(n\) customers and \(\{n + 1, n + 2, \ldots, n + m\}\) which considers \(m\) potential depots. Zeros indicate the auxiliary vehicles. Second string illustrates the main vehicles and route for each depot. The auxiliary vehicle is sent from depot to the customer that precedes zero in the solution string in order to replenish the main vehicle. Each depot serves the customers who are between two depots number. Tables 2 and 3 give a small instance which is obtained from Perl (1983). Table 2 gives the location and demand of 12 customers which are indicated with numbers 1–12, and Table 3 lists the location, capacity and opening costs of two potential depots which are indicated with numbers 13 and 14. Also in this instance the vehicle capacity is 140.

A solution representation which is used in SA algorithm is shown in Figure 4. A visual illustration of this representation is shown by Figure 5.

In this representation depot 14 is not opened because there is not any customer between this depot and depot 13, in depot 13 a main vehicle starts to serve customers 9, 8, 6, 1, 2, 3, 4, respectively, then because there is zero in representation, an auxiliary vehicle must be sent to replenish the main vehicle at the location of customer 4 and return to depot 13. Then main vehicle continues to serve customers 5, 11, 12, 10, 7 and return to depot 13. Second string shows the number of main vehicles which are used and also determine the routes or depot number 13. The number '1' in second string shows that just one main vehicle is used in this example.

Here are some explanations about how SA representation satisfies these constraints.

Constraint (2) which insures each customer belongs to only one route is satisfied in SA solution representation as each customer is repeated only one time so it will belong to exactly one route. Also constraint (10) which declares that each customer can only be assigned to one depot is satisfied in SA solution representation. Constraints (3) and (4) are related to vehicles and depots capacity, in the proposed representation, for each solution the total demands of assigned customers to vehicles or depots will be compared with related capacity and if it is not satisfied a large penalty will be added to the objective function value. Constraints (5) and (6) ensure that each route remains circular.

<table>
<thead>
<tr>
<th>Customer number</th>
<th>X</th>
<th>Y</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>32</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>33</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>29</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
<td>27</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>31</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
<td>9</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Depot number</th>
<th>X</th>
<th>Y</th>
<th>Opening cost</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>25</td>
<td>19</td>
<td>100</td>
<td>280</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>24</td>
<td>100</td>
<td>280</td>
</tr>
</tbody>
</table>

Figure 5. Visualised solution representation of Figure 4.

<table>
<thead>
<tr>
<th>14</th>
<th>0</th>
<th>13</th>
<th>9</th>
<th>8</th>
<th>6</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>0</th>
<th>5</th>
<th>11</th>
<th>12</th>
<th>10</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0</td>
<td>13</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4. Constraints satisfying strategies.

<table>
<thead>
<tr>
<th>Constraint no.</th>
<th>Satisfying strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 5, 6, 7, 8, 9, 10, 11</td>
<td>Creating the solution representation in feasible mode which remains feasible in the algorithm</td>
</tr>
<tr>
<td>12, 13, 14, 15</td>
<td>Repairing the solution representation to meet the constraint</td>
</tr>
<tr>
<td>3, 4</td>
<td>Adding penalty to the objective function value according to the constraint violation</td>
</tr>
</tbody>
</table>

this issue for the proposed representation is always satisfied as we consider all numbers in representation as consecutive customers so all tours will be circular. Constraint (7) is sub-tour elimination constraints and it is satisfied in the proposed solution representation because all customers’ number and vehicle assignments are repeated one time in each solution. Constraint (8) makes sure that the auxiliary vehicles can only travel from a depot to customers who are assigned to that depot. This constraint is considered in the solution representation as auxiliary vehicles and they are assigned to related main vehicles in each depot tours. Constraint (9) makes sure that the auxiliary vehicle should be sent to a node which belongs to the same depot. This constraint is satisfied in the solution representation as well and in the solution the auxiliary vehicle is assigned to a node within a tour. Constraint (11) links the allocation and routing part and it ensures that a customer can be assigned to a depot if there is a route assigned to that depot, in the proposed solution representation all customers belong to opened depots which serves that customer. Constraints (12)–(14) ensure that the main vehicle can be replenished as much as its empty space by an auxiliary vehicle. In the proposed representation, where auxiliary vehicle are assigned, we calculate the empty space and assign cargoes in the free space of main vehicle, or if the main vehicle is absolutely empty, we calculate the amount of replenishment as much as auxiliary capacity, so this constraint will be satisfied by repairing the solution. Also constraint (15) which declares that maximum amount of empty space of each vehicle is less than its capacity is satisfied through repairing strategy.

The summary of constraints satisfying strategies are presented in Table 4.

Figure 6 gives the visualized solution representation of Figure 4 solved by classic model, as the vehicle capacity is 140 units, there will be more than one route in the solution, but by using auxiliary vehicles there will be just one route for main vehicle. At node 4, the main vehicle does not return to the depot and the route will not be terminated. The parametric results of comparison between Figures 5 and 6 are listed in Table 5. Finally a comparison has been reported. Table 5 verifies the economical benefits of the presented model which leads to reduction in the total costs. As mentioned before, the auxiliary vehicle costs are absolutely less than main vehicles. This difference originates from some cost factors such as fuel equipment, repairing, drivers’ salary, etc. Some obtained data from Barkel and

Figure 6. Visualizing the representation in Figure 4 in usual approaches.

Yalkin (2013) stipulate the ratio of 1/4 for auxiliary and main vehicles costs.

4.3. Initial solution

An initial solution is built according to the proposed solution representation and at least it must be a feasible candidate solution which means all constraints of the proposed model must be satisfied. In this paper some of the initial solutions are built randomly and some of them are obtained after solving the selected instances in classic LRP model. The first kind of solutions which are created randomly, need to be checked and they will be accepted if they are feasible, otherwise we try to repair the infeasible solutions for example by adding vehicles or depots to satisfy capacity constraints. Second kinds of solutions typically are feasible and they are accepted as initial solution for starting the SA algorithm. The algorithm will use aforementioned mechanism for keeping solutions feasible during the searching process.
**Table 5.** Parametric comparison of two mentioned solutions in Figures 5 and 6 (with/without auxiliary vehicle).

<table>
<thead>
<tr>
<th>Costs</th>
<th>Unit cost</th>
<th>Figure 5</th>
<th>Figure 6</th>
<th>Total cost in Figure 5</th>
<th>Total cost in Figure 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of main vehicles</td>
<td>$b$</td>
<td>1</td>
<td>2</td>
<td>$b$</td>
<td>$2b$</td>
</tr>
<tr>
<td>Number of auxiliary vehicles</td>
<td>$b/4$</td>
<td>1</td>
<td>0</td>
<td>$b/4$</td>
<td>0</td>
</tr>
<tr>
<td>Travelled distance by main vehicles</td>
<td>$d$</td>
<td>93.7876</td>
<td>116.7738</td>
<td>93.7876$d$</td>
<td>116.7738$d$</td>
</tr>
<tr>
<td>Travelled distance by auxiliary vehicles</td>
<td>$d/4$</td>
<td>25.6124</td>
<td>0</td>
<td>25.6124$d/4$</td>
<td>0</td>
</tr>
<tr>
<td>Total cost</td>
<td>$5b/4 + 100.1907d$</td>
<td>$2b + 116.7738d$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.4. Neighbourhood

We use a standard SA procedure with a random neighbourhood structure and define some movements in different types for generating neighbourhoods. We work on main elements of the proposed LRP representation including depots, customers, auxiliary vehicles in the first stream and main vehicles in the second one. These movements include two insertion and swap movements for depots in 20% of occasions and for customers in 80% of occasions. The insertion method selects one of the customers randomly and puts it after another randomly selected customer. The swap method selects two customers randomly and changes their positions with each other. Also after swap or insertion movement the vehicles in second stream will be checked by considering customer demands and route capacity and some vehicle allocations in routes may be changed randomly in order to change some routes length. Because of our problem specifications we use another method for generating neighbour solutions; we consider insertion movement for auxiliary vehicles in the first stream. Auxiliary vehicle insertion means to select one auxiliary vehicle randomly and do the insertion method for it so that new routes will be generated. This gives an opportunity to the current solution to improve the objective function with added auxiliary vehicle, it is clear that the auxiliary vehicle may lead to worse solution in some cases but finally after all iterations, the algorithm should decide on using auxiliary vehicles or not and select the best solution. All these methods in all iterations of algorithm will search the solution area more efficiently.

<table>
<thead>
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7-g

Figure 7. Generating neighbourhood solutions of the current solution representation.
Figure 8. The schematic of neighbourhood solutions according to solution representations of Figure 9.

Here is a complete example of six steps in generating neighbourhood solution for a representation and the mechanism of searching neighbour solutions to improve the objective function.

Suppose Figure 7(a) is a solution representation with two depots and 12 customers where the depots are determined by numbers 13 and 14. The current solution representation is shown in Figure 7(a) while the schematic of
this representation is shown in Figure 8(a). Figure 7(b)–(g) is neighbourhood solutions of the current solution which are generated from their previous solution in six steps. For example, representation in Figure 7(b) is generated from representation in Figure 7(a) by inserting customer 3 after customer 1. Figure 7(c) is generated from Figure 7(b) by auxiliary vehicle insertion after customer 6 which results in increasing the tour length. Figure 7(d) is generated from the previous solution representation by inserting customer 10 after customer 2. Also the same insertion occurs for customer 9 in Figure 7(e). Finally Figure 7(f) and 7(g) has swap movements which lead to closing of depot 14. The associated pictures of these representations are depicted in Figure 8. Figure 8(b)–(g) show the steps of changing the proposed representation in six steps of Figure 7(b)–(g) accordingly.
4.5. Parameter setting
Our proposed SA is based on usual SA algorithm and also it contains six parameters as \( N_{iter}, N_{max}, T_0, T_f, K, \alpha \). The \( N_{iter} \) denotes number of iterations in searching process and \( N_{max} \) is the maximum allowable number of accepting worse solutions with a probability. \( T_0 \) and \( T_f \) denote the first and final temperatures during cooling process under cooling ratio of \( \alpha \). Finally \( K \) is Boltzmann constant which determines the probability of accepting worse solutions in Boltzmann function. It is clear that initial parameters would influence the quality of searching process and computational results. So in this paper we applied a central composite design (CCD) experiment to achieve a robust algorithm which has been designed in Minitab 14 software. Objective value and computational time are considered as responses in the experimental design study. Here the tuned parameters are reported:

- \( T_0 = 5,000,000 \);
- \( T_f = 1 \);

Figure 10. The effect of auxiliary vehicle initial cost on the number of main vehicles.

Figure 11. The effect of auxiliary capacity on the number of vehicles.
Figure 12. Comparison between three scenarios of using vehicles.

Table 6. Computational results for comparison of exact and proposed algorithm in solving of the Aux-LRP.

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<th># of depots</th>
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<th>Best solution obtained in 10 runs</th>
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- \( N_{100} = 100; \)
- \( N_{\text{annex}} = 45; \)
- \( \alpha = 0.99; \)
- \( K = 0.001. \)

4.6. Proposed SA pseudo code and flow chart

In this section the pseudo code of proposed SA algorithm is presented and it describes the steps of applied algorithm.

Step 0: Setting parameters obtained by CCD experiment (\( N_{100}; \) \( N_{\text{annex}}, T_0, T_f, K, \alpha \))

Step 1: Generate initial solution (RE) according the proposed solution representation

Step 2: Calculate total amount of these costs for RE objective function value:

- The cost of opened depots
- Travel cost of main vehicles
- Travel cost of auxiliary vehicles
- Fixed costs associated with main vehicles
- Set-up costs associated with auxiliary vehicles which contain the cost and time of loading and unloading goods

Step 3: \( \text{Let } T = T_0; N = 0; N_w = 0; F_{\text{best}} = \text{Obj(RE)}; R_{\text{best}} = \text{RE} \)

Step 4: \( N = N + 1 \)

Step 5: Select one kind of below neighbourhoods and generate a neighbourhood solution (\( X_{\text{RE}} \))

- Insertion movement
- Swap movement
- Auxiliary insertion movement

![Comparison of travelling distance between classic model and proposed model.](image)
Table 7. The comparison between classic and proposed approaches in objective functions results.

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<td>2311</td>
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<td>1609</td>
<td>1423</td>
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<td>1423</td>
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</tr>
</tbody>
</table>

Step 6: Check the main vehicle’s capacity and follow the repair strategies and decide on main vehicle’s assignments in second stream

Step 7: If $\Delta = \text{Obj}(XRE) - \text{Obj}(RE) \leq 0$ (LetRE = XRE;)

Else {Generate $p = \text{random}(0, 1)$;
            If $p < \exp(-\Delta/KT)$ (LetRE = XRE;)

        }

Else {Go to Step3}

Step 8: If $\text{Obj}(RE) < F_{\text{best}}(RE_{\text{best}} = RE; F_{\text{best}} = \text{Obj}(RF); N_w = 0)$;

Step 9: If $N = N_{\text{set}} (T = \alpha T; N = 0; N_w = N_w + 1)$; Perform neighbourhood search on $RE_{\text{best}}$; Check and repair the vehicle assignments in second stream.

} Else {Go to Step 4}

Step 10: If $(T < T_f) \text{Or } N_w = N_{\text{waste}}$. {Terminate the SA algorithm};

Else {Go to Step 4}

Here the flow chart of proposed SA is depicted in Figure 9.

5. Sensitivity analysis

In order to figure out the sensitivity of the model to some important parameters such as the set-up cost and capacity of auxiliary and main vehicles, we consider an example and solve it with different levels of parameters. Figures 10 and 11 illustrate the effect of auxiliary cost and auxiliary capacity on number of used vehicles, respectively.

According to Figure 10 it is clear that using of the auxiliary vehicle will decrease the total cost when its fixed
cost containing its set-up cost and loading/unloading costs is less than the main vehicle. The figure shows that by increasing auxiliary vehicle fixed cost, the model will intend to use the main vehicle instead of the auxiliary one, so the proposed model will be closer to the classic model. Capacity of vehicles is also another important parameter that affects the possibility of using auxiliary vehicles. In this example using auxiliary vehicle is economical only if the auxiliary capacity is more than 25. Figure 11 confirms that by increasing the capacity of the auxiliary vehicle, number of used auxiliary vehicles will be decreased. Finally the auxiliary vehicle will be treated as the main vehicle, in such case again the proposed model will be performed as the classic model.

Moreover, for testing the effect of using auxiliary vehicles, we considered first 10 instances gathered by Tuzun and Burke (1999) and solved them in three scenarios for using vehicles in LRP modelling:

1. Using just auxiliary vehicles to examine the total cost if auxiliary vehicles visit customers in short routes
2. Using just main vehicle which is the classic approach indeed.
3. Using main and auxiliary vehicles which describe our proposed approach.

Figure 12 gives a comparison of above scenarios. As it is clear, our proposed approach has the best performance in all instances.

In the first scenario, while the model uses just auxiliary vehicles, the solution has more routes with short lengths...
which result in using more vehicles and opening more depots, so the total cost will be increased. More computational studies have been done on these instances in the next section.

6. Computational results
In this part of paper computational experiments have been carried out to investigate the desirability of the proposed model, as the problem is in nondeterministic polynomial time (NP) hard classification, appropriate solving method for large-scale instances is meta-heuristic algorithms. SA is applied on benchmarked instances and is coded by MATLAB 7.8.

In order to evaluate the efficiency of proposed method, we applied two categories of data-sets. In the first one, five instances were selected from literature for computational benchmarks. The first five instances have been chosen from Prins (2004) and Perl (1983). The second data-sets are gathered by Tuzun and Burke (1999). The obtained results of solving the instances with presented model and its comparison to classic model are reported in Table 6. We solved the presented model both by GAMS software and SA algorithm. The result indicates the capability of proposed method.

The second data-set contains large size problems which can only be solved by SA algorithm. Tables 7 and 8 contain the results of solving these 36 instances. In these tables, Cl. and Pr. indicate the results obtained from classic and presented models. In this data-set the numbers of customers in first 12 instances are 100, the second 12 instances have 150 customers and the third 12 instances have 200 customers. The number of depots for odd and even instances is set in 10 and 20, respectively. In both classic and auxiliary proposed approach, we run the model 10 times and the obtained results associated with total objective function are reported in Table 7. Table 7 also contains the average of 10 runs and the best and worst solutions objective functions in each method. For each instance, we select the nearest solution to the average among 10 obtained solutions and report the information associated with vehicles and depots in Table 8. In both Tables 7 and 8 the results have been rounded.

Tables 6–8 shows the efficiency of the proposed model especially in problems with large sizes where using auxiliary vehicles could be effective in reducing travelling costs. Following figures depict comparisons between presented and classic models in different factors for 36 instances. Figure 13 compares total travelling distance between classic method which is modelled without using auxiliary and the proposed model with the possibility of using auxiliary vehicles. Total travelling distance in the proposed model is the sum of total travelling distance of main vehicles and total travelling distances of auxiliary vehicles. It shows that by the proposed model total travel will be decreased because some journeys by the main vehicle have been omitted because of using the auxiliary vehicle. Moreover the

![Comparison of total cost between classic model and presented model.](image)

Figure 14. Comparison of total cost between classic model and presented model.

main vehicle tours have become more efficient. Figure 14 shows that in most instances the total cost is reduced when the auxiliary vehicle is added. Figure 15 depicts that in some cases using auxiliary vehicles causes reduction in the number of open depots. The reason is possibility of travelling of the main vehicle to the farthest customers with replenishment by an auxiliary vehicle. Figure 16 illustrates total travelling distance of main vehicles in both classic and

![Comparison of number of depots between classic model and presented model.](image)

Figure 15. Comparison of number of depots between classic model and presented model.

![Comparison of the distance that has been travelled by main vehicles between classic model and proposed model.](image)

Figure 16. Comparison of the distance that has been travelled by main vehicles between classic model and proposed model.
proposed models. In all cases the proposed model has better or equal results. It means that using auxiliary will result in decreasing the total costs of system, the travelled distance by main vehicles which has direct impact on other related costs such as driver’s salary, vehicles maintenance costs and other costs such as taxes and insurance premiums which are dependent on the kind of vehicles.

7. Conclusions
In this paper a new aspect of LRP has been introduced. Two kinds of vehicles are considered, main vehicles with higher capacity and initial cost and auxiliary vehicles with lower capacity and cost. The auxiliary vehicles can be added to the transportation system to cover the capacity limitation as an alternative strategy. The auxiliary vehicles can only transfer goods between depots and main vehicles and servicing is only done by main vehicles. By using auxiliary vehicles the routes are not terminated due to capacity constraints. A mathematical model was presented to formulate this new problem. The model has been solved with GAMS software and the results were inspiring. Some benchmarked instances were taken from the literature and solved by a proposed SA algorithm. Further study in this area could be considering time window constraints together with auxiliary vehicles. Considering time windows lead another constraint to limit the route length, so it might be helpful to use auxiliary vehicles.

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References


