Statistical analysis of total process capability index in two-stage processes with measurement errors

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***Abstract*— The manufacturing processes often involve multistage processes where the process capability of each stage is affected by the process capability of its previous stages. This property is known as cascade property. Therefore, the measured process capability indices in each stage represent the total capability of that stage. In the multistage processes two kinds of process capability index are defined in each stage of the multistage processes as follows: total process capability index and specific process capability index. The process capability indices are proposed under the assumption that there are no measurement errors. However, sometimes in the real application, this assumption is violated. The measurement errors affect the process capability indices significantly. In this paper, the effect of measurement errors on the total process capability index in the second stage of two-stage processes is statistically analyzed.**

***Keywords*— Total process capability index; Two-stage processes; Measurement errors**

I. INTRODUCTION

Process capability indices have been used in industries to measure the performance of the processes. A process is called capable if the product meets customer expectations. There are copious studies on process capability estimations. Kotz and Johnson [1] reviewed the studies on process capability indices during the years 1992-2000 and Yum and Kim [2] provided a bibliography of approximately 530 journal papers and books published during the years 2000-2009. The process capability indices C_p, C_{pk} and C_{pm} are the most commonly used indices.

In the multistage processes, the process in each stage is affected by the processes in the previous stages. Therefore, the capability of the process in each stage is dependent on the capability of the processes in the previous stages and the process capability indices calculate the total capability of the process. Most of the researches on multistage processes are related to control chart and multistage process capability analyses are not well studied. Zhang [3] was the first who designed a control chart referred to as cause-selecting control chart to monitor the multistage processes. Zhang [4-7] developed his work. Wade and Woodall [8] proposed using prediction limits to improve the statistical performance of cause selecting control charts. Yang [9] proposed a new approach to compute the cost model for a two-stage process. Then, he designed economic X chart and cause-selecting control chart to monitor a two-stage process. Yang and Yang [10] proposed an approach to monitor two-stage processes when data are auto-correlated. Yang and Yeh [11] proposed a cause selecting control chart for the two-stage processes with attribute data. Zhang [12] expressed that in each stage of the multistage process, the total process capability index calculate the capability of that stage when affected by the previous stages. The specific process capability index calculates the capability of the process when the effects of previous stages are omitted. He proposed total and specific process capability index for the multistage processes. Linn et al. [13] proposed a method to determine the priority of the process variation reduction in multistage processes to improve the overall capability index. Chen et al. [14] proposed a process capability index for the complex product machining process.

The process capability indices are proposed under the assumption that there is no measurement error. While sometimes in real applications this assumption is violated. Mitag [15] was the first one, who examined the effects of the measurement errors on process capability indices C_p, C_{pk} and C_{pm}. Bardignon and Scagliarini [16] studied the properties of the estimation of process capability indices C_p and C_{pk} in presence of the measurement errors. Bordignon and Scagliarini [17] analyzed the properties of the estimation of process capability index C_{pm} when the observations are affected by measurement errors. Scagliarini [18] examined the properties of the estimation of C_p when observations are autocorrelated and affected by measurement errors. Scagliarini [19] studied the properties of the process capability index C_{pk} when data are autocorrelated in the presence of measurement errors.

In this paper, we analyze the effects of the measurement errors in the total process capability index of the process in the second stage of a two-stage process. Structure of the paper is as follows: The effects of the measurement errors on the regression coefficients are studied in Section 2. The effects of the measurement errors in the total process capability index are examined in Section 3. The statistical analyses of the total process capability index in the presence of the measurement errors are studied in Section 4. Conclusion and future researches are given in the final Section.

II. TWO-STAGE PROCESSES AND MEASUREMENT ERRORS

In the two-stage processes, it is assumed that the parts in stage 1 are fed into stage 2 and the quality characteristic in the first stage is X which follows a normal distribution with mean μ_x and variance of the σ_x^2. The quality characteristic of the second stage is defined as:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \,, \quad (1)$$

where, ε_i follows normal distribution with mean zero and variance of σ_ε^2. The regression coefficients β_0 and β_1 are estimated based on the analyses on historical data by using equations (2) and (3)

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \,, \quad (2)$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i \left(X_i - \bar{X}\right)}{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2} \,, \quad (3)$$

where $\bar{X} = \sum_{i=1}^{n} X_i / n$ and $\bar{Y} = \sum_{i=1}^{n} Y_i / n$ are the mean values of the quality characteristics in the first and second stages, respectively.

The mean and variance of the quality characteristic in the second stage can be expressed as

$$\mu_y = \hat{\beta}_0 + \hat{\beta}_1 \mu_x \,. \quad (4)$$

$$\sigma_y^2 = \hat{\beta}_1^2 \sigma_x^2 + \sigma_\varepsilon^2 \,. \quad (5)$$

The prediction value $\hat{Y}_i$ for the quality characteristic Y_i and can be obtained as follows:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \,. \quad (6)$$

The residuals are obtained by:

$$e_i = Y_i - \hat{Y}_i \,. \quad (7)$$

When the measurement errors are considered in a case, the quality characteristics in the first and second stages are expressed as

$$X_i^e = X_i + V_{1i} \,, \quad (8)$$

$$Y_i^e = Y_i + V_{2i} \,, \quad (9)$$

where V_1 follows normal distribution with mean zero and variance $\sigma_{V_1}^2$. The quality characteristic in the first stage and V_1 are statistically independent. Also, V_2 follows normal distribution with mean zero and variance $\sigma_{V_2}^2$ and it is statistically independent from the quality characteristic in the second stage.

The effect of the measurement errors on the estimation of the regression coefficients of the multistage processes is examined by Ding and Zeng [20]. The relationship between the estimation of the parameter β_1 based on the observed data and the estimation of β_1 based on true data can be expressed as

$$\hat{\beta}_1^e = \frac{\hat{\beta}_1 \sigma_x^2}{\sigma_x^2 + \sigma_{v_1}^2} = \frac{\hat{\beta}_1}{1 + \tau_x^2} \,, \quad (10)$$

where τ_x is defined as follows:

$$\tau_x = \frac{\sigma_{v_1}}{\sigma_x} \,. \quad (11)$$

Therefore, the ratio between the observed and true estimation of the parameter β_1 is decreasing function of τ_x. The true estimation of the parameter β_1 follows normal distribution with mean β_1 and variance of $\sigma_\varepsilon^2 / \sum_{i=1}^{n} (X_i - \bar{X})^2$, when the residuals follow normal distribution with mean zero and variance of σ_ε^2. Therefore, the bias of observed estimation of the parameter β_1 can be obtained as follows:

$$\begin{aligned} B\left(\hat{\beta}_1^e\right) &= E(\hat{\beta}_1^e) - \beta_1 = \frac{\beta_1}{1 + \tau_x^2} - \beta_1 \\ &= \beta_1 \left(\frac{1}{1 + \tau_x^2} - 1\right) . \end{aligned} \quad (12)$$

It is clear from equation (12) that the bias of the observed estimation of the parameter β_1 is increasing function of τ_x and the estimator of β_1 in the presence of measurement errors are smaller than the true value of this parameter. The behavior of the term $(1/1 + \tau_x^2) - 1$ as a function of τ_x is shown in Figure 1.

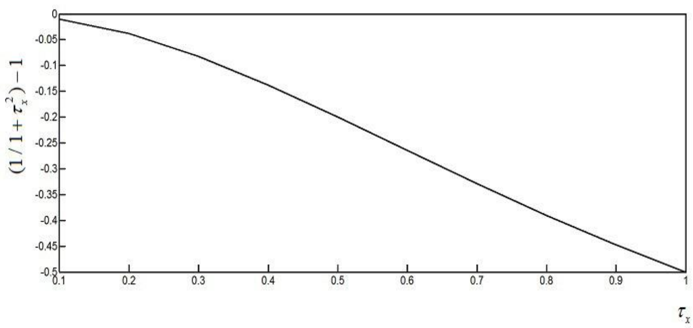


Figure 1: The term $(1/1 + \tau_x^2) - 1$ as a function of τ_x

The prediction value of observed values of the quality characteristic in the second stage can be obtained as follows:

$$\hat{Y}_i^e = \hat{\beta}_0^e + \hat{\beta}_1^e X_i^e$$
$$= \hat{Y}_i - \frac{\beta_1 \tau_x^2}{1+\tau_x^2}(X_i - \bar{X}) + \frac{\beta_1 V_{1i}}{1+\tau_x^2}. \quad (13)$$

It is clear from equation (13) that the prediction values of the second quality characteristic are affected by the measurement errors. The residuals for the observed data are obtained by:

$$e_i^e = Y_i^e - \hat{Y}_i^e = Y_i + V_{2i} - \hat{Y}_i^e$$
$$= \beta_0 + \beta_1 X_i + \varepsilon_i + V_{2i} - \hat{\beta}_0^e - \hat{\beta}_1^e (X_i + V_{1i}). \quad (14)$$

The variance of the residuals in the presence of the measurement error is defined as:

$$(\sigma_\varepsilon^e)^2 = \sigma_\varepsilon^2 + \sigma_{V_2}^2 + \frac{\beta_1^2 \sigma_{V_1}^2}{1+\tau_x^2}. \quad (15)$$

Therefore, in the presence of the measurement error the residuals follow normal distribution with mean zero and variance $\sigma_\varepsilon^2 + \sigma_{V_2}^2 + (\beta_1^2 \sigma_{V_1}^2 / 1+\tau_x^2)$.

III. TOTAL PROCESS CAPABILITY INDEX AND MEASUREMENT ERRORS

Zhang [12] defined two kinds of process capability index in the multistage processes. The first one is total process capability index which calculates the process capability index of total process. The total process capability index for the process in the second stage is defined as:

$$C_{pt} = \frac{USL_y - LSL_y}{6\sigma_y}, \quad (16)$$

where LSL_y and USL_y are the lower and upper specification limits of the quality characteristic in the second stage, respectively. σ_y is the variance of the quality characteristics in the second stage that is affected by the variance of the quality characteristic in the first stage.

Another process capability index is specific process capability index that calculate the specific capability of the process in the second stage when the effects of the quality characteristics in the first stage is omitted. The specific process capability index in the second stage is obtained by:

$$C_{ps} = \frac{USL_y - LSL_y}{6\sigma_\varepsilon}. \quad (17)$$

The ratio between the observed and true value of total process capability index can be expressed as:

$$\frac{C_{pt}^e}{C_{pt}} = \frac{\left(USL_y - LSL_y\right)/6\sqrt{\sigma_y^2 + \sigma_{V_1}^2}}{\left(USL_y - LSL_y\right)/6\sigma_y} = \frac{\sigma_y}{\sqrt{\sigma_y^2 + \sigma_{V_1}^2}}.$$
$$= \frac{1}{\sqrt{1 + \frac{\tau_y^2}{\beta_1^2 \tau_{xy}^2 + 1}}} \quad (18)$$

It is clear from equation (18) that the ratio between the observed and true values of the total process capability is the function of $\tau_y = \sigma_{v_2} / \sigma_\varepsilon$, $\tau_{xy} = \sigma_x / \sigma_\varepsilon$ and the parameter β_1. This ratio is the decreasing function of τ_y and increasing function of τ_{xy} and the parameter β_1.

IV. STATISTICAL ANALYSIS OF TOTAL PROCESS CAPABILITY INDEX

Let the paired observation (X_i, Y_i) i=1, 2,…, n is obtained in ith random sample where X_i is the quality characteristic in the first stage and Y_i is the quality characteristic in the second stage related to X_i. The common estimator of total process capability index is defined as:

$$\hat{C}_{pt} = \frac{USL_y - LSL_y}{6\hat{\sigma}_y}, \quad (19)$$

where $\hat{\sigma}_y$ can be obtained by:

$$\hat{\sigma}_y = \sqrt{\frac{\sum_{i=1}^n \left(Y_i - \bar{Y}\right)^2}{n-1}}. \quad (20)$$

Kotz and Johnson [21] expressed that the expected value and variance of estimator of the process capability index C_p can be obtained by following equations, respectively.

$$E\left(\hat{C}_p\right) = C_p \frac{1}{b_f}, \quad (21)$$

$$\mathrm{Var}\left(\hat{C}_p\right) = \left(\frac{f}{f-2} - b_f^{-2}\right) C_p^{\,2}, \quad (22)$$

where f=n-1 and b_f is defined as:

$$b_f = \left(\frac{2}{f}\right)^{1/2} \frac{\Gamma\left(\frac{1}{2}f\right)}{\Gamma\left(\frac{1}{2}(f-1)\right)}, \quad (23)$$

where Γ(.) is the gamma function. Therefore, the expected value and variance the estimator of the total process capability index is obtained by equations (24) and (25), respectively.

$$E\left(\hat{C}_{pt}\right) = C_{pt} \frac{1}{b_f}. \quad (24)$$

$$\operatorname{Var}\left(\hat{C}_{pt}\right)=\left(\frac{f}{f-2}-b_f^{-2}\right)C_{pt}{}^2 . \qquad (25)$$

Equation (24) shows that the estimator of total the process capability index is a bias estimator. Since, the parameter b_f is smaller than 1, the bias value is positive. The parameter b_f value decreases when the sample number increases. Therefore, the bias of estimator decreases when the sample size increases. When the sample size approaches to infinity, the bias of the estimator is equal to zero.

The estimator of the total process capability index in the presence of measurement errors is defined as:

$$\hat{C}^e_{pt}=\frac{USL_y-LSL_y}{6\hat{\sigma}^e_y}, \qquad (26)$$

where $\hat{\sigma}^e_y$ is obtained by:

$$\hat{\sigma}^e_y=\sqrt{\frac{\sum_{i=1}^{n}\left(Y_i^e-\overline{Y}^e\right)^2}{n-1}}, \qquad (27)$$

where $\overline{Y}^e=\sum_{i=1}^{n}Y_i^e/n$ is the mean value of the quality characteristic in the second stage in the presence of measurement errors. The expected value and variance of the total process capability index when observations contaminated by measurement errors can be expressed as:

$$E\left(\hat{C}^e_{pt}\right)=C^e_{pt}\frac{1}{b_f}=C_{pt}\frac{1}{b_f}\frac{1}{\sqrt{1+\dfrac{\tau_y^2}{\beta_1^2\tau_{xy}+1}}}. \qquad (28)$$

$$\operatorname{Var}\left(\hat{C}^e_{pt}\right)=\left(\frac{f}{f-2}-b_f^{-2}\right)\left(C^e_{pt}\right)^2=$$
$$\left(\frac{f}{f-2}-b_f^{-2}\right)(C_{pt})^2\frac{1}{1+\dfrac{\tau_y^2}{\beta_1^2\tau_{xy}^2+1}}. \qquad (29)$$

The bias of the estimator of the total process capability index in the presence of measurement errors is defined as:

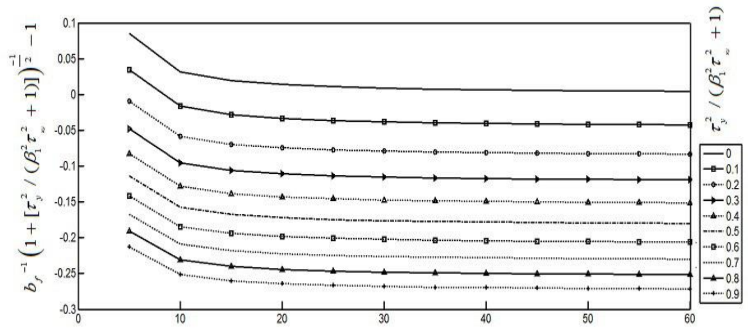
$$B\left(\hat{C}^e_{pt}\right)=C^e_{pt}\frac{1}{b_f}-C_{pt}=$$
$$C_{pt}\left(\frac{1}{b_f}\frac{1}{\sqrt{1+\dfrac{\tau_y^2}{\beta_1^2\tau_{xy}^2+1}}}-1\right). \qquad (30)$$

It is clear from equation (30) that the bias of the estimator of the total process capability index is negative when sample size approaches to infinity. Therefore, the estimator is asymptotically biased. Figure 2 shows the behavior of the term $b_f^{-1}\left(1+[\tau_y^2/(\beta_1^2\tau_{xy}^2+1)]\right)^{\frac{-1}{2}}-1$ as a function of n for different values of the term $\tau_y^2/(\beta_1^2\tau_{xy}^2+1)$. Figure 2 shows that the bias of the estimator of total process capability index is negative when the sample size increases and the bias increases when the term $\tau_y^2/(\beta_1^2\tau_{xy}^2+1)$ decreases. In some values of τ_y, the bias of the estimator of total process capability index is zero. This happens when τ_y is equal to τ_0^t

$$\tau_0^t=\left(\left[\left(\frac{1}{b_f}\right)^2-1\right]\left(\beta_1^2\tau_{xy}^2+1\right)\right)^{1/2}, \qquad (31)$$

while,

$$\begin{aligned} B\left(\hat{C}^e_{pt}\right)>0 \quad for \quad \tau_y<\tau_0^t \\ B\left(\hat{C}^e_{pt}\right)<0 \quad for \quad \tau_y>\tau_0^t \end{aligned}. \qquad (32)$$



Sample size

Figure 2: Term $b_f^{-1}\left(1+[\tau_y^2/(\beta_1^2\tau_{xy}^2+1)]\right)^{\frac{-1}{2}}-1$ as a function of n for different values of the term $\tau_y^2/(\beta_1^2\tau_{xy}^2+1)$

Figure 3 shows the behavior of the term $\left(\left[(b_f)^{-2}-1\right]\left(\beta_1^2\tau_{xy}^2+1\right)\right)^{1/2}$ as a function of the n for different values of the term $\beta_1^2\tau_{xy}^2+1$.

Figure 3 shows that when the sample size increases, threshold value of τ_y decreases. The threshold value is zero, when the sample size approaches to infinity ($n\to\infty$).

Mean square errors of the estimator of the total process capability index when there is no measurement errors can be defined as:

$$\mathrm{MSE}\left(\hat{C}_{pt}\right) = B\left(\hat{C}_{pt}\right)^2 + \mathrm{Var}\left(\hat{C}_{pt}\right) = C_{pt}^{\ 2}\left(\frac{1}{b_f}-1\right)^2 + \left(\frac{f}{f-2} - b_f^{-2}\right)C_{pt}^{\ 2}. \tag{33}$$

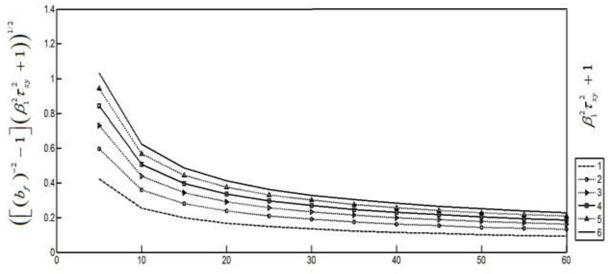


Figure 3: Term $\left(\left[(b_f)^{-2}-1\right]\left(\beta_1^2\tau_{xy}^2+1\right)\right)^{1/2}$ as a function of n and different values of the term $\beta_1^2\tau_{xy}^2+1$

Mean square errors of the estimator of the total process capability index in the presence of the measurement error is defined as:

$$\mathrm{MSE}\left(\hat{C}_{pt}^{e}\right) = B\left(\hat{C}_{pt}^{e}\right)^2 + \mathrm{Var}\left(\hat{C}_{pt}^{e}\right) = C_{pt}^{\ 2}\left(\frac{1}{b_f}\frac{1}{\sqrt{1+\dfrac{\tau_y^2}{\beta_1^2\tau_{xy}^2+1}}}-1\right)^2 + \left(\frac{f}{f-2} - b_f^{-2}\right)C_{pt}^{\ 2}\frac{1}{1+\dfrac{\tau_y^2}{\beta_1^2\tau_{xy}^2+1}}. \tag{34}$$

The equation (35) can be obtained by comparing equations (33) and (34)

$$\begin{cases} \mathrm{MSE}\left(\hat{C}_{pt}\right) > \mathrm{MSE}\left(\hat{C}_{pt}^{e}\right) & \text{for } \tau < \tau_1^T \\ \mathrm{MSE}\left(\hat{C}_{pt}\right) = \mathrm{MSE}\left(\hat{C}_{pt}^{e}\right) & \text{for } \tau = \tau_1^T \\ \mathrm{MSE}\left(\hat{C}_{pt}\right) < \mathrm{MSE}\left(\hat{C}_{pt}^{e}\right) & \text{for } \tau > \tau_1^T \end{cases}, \tag{35}$$

where τ_1^T is obtained by:

$$\tau_1^T = \left(\left[\left(\frac{-f/(f-2)}{[f/f-2]-(2/b_f)}\right)^2 - 1\right](\beta_1^2\tau_{xy}^2+1)\right)^{1/2} \tag{36}$$

Figure 4 shows the ratio $\mathrm{MSE}\left(\hat{C}_{pt}\right)/\mathrm{MSE}\left(\hat{C}_{pt}^{e}\right)$ as a function of n and the term $\tau_y^2/(\beta_1^2\tau_{xy}^2+1)$. Figure 4 shows that for small sample sizes, $\mathrm{MSE}\left(\hat{C}_{pt}\right)$ is larger than $\mathrm{MSE}\left(\hat{C}_{pt}^{e}\right)$. When the sample size is large and the value of term $\tau_y^2/(\beta_1^2\tau_{xy}^2+1)$ is small, this ratio is equal to 1.

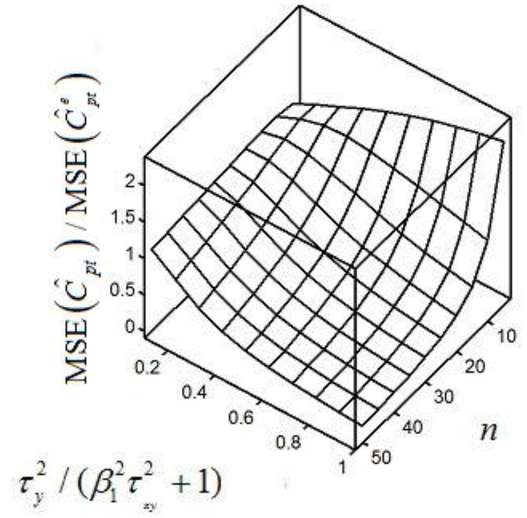


Figure 4: the ratio $\mathrm{MSE}\left(\hat{C}_{pt}\right)/\mathrm{MSE}\left(\hat{C}_{pt}^{e}\right)$ as a function of n and the term $\tau_y^2/(\beta_1^2\tau_{xy}^2+1)$

V. CONCLUSIONS AND FUTURE RESEARCHES

The multistage processes have been used in manufacturing processes. In multistage processes, two kinds of process capability indices are defined in each stage. The process capability indices are proposed under the assumption that there is no measurement error. The measurement errors affect the performance of the process capability indices significantly. In this paper, the effects of measurement errors on the total process capability index of the second stage in the two-stage processes were examined. In addition, the statistical properties of measurement errors on the total process capability index were analyzed in this paper. The bias of the estimator of the total process capability index affect by β_1, τ_{xy} and sample size. In this paper, we show that bias of the total process capability index is positive when there is no measurement error. While, in the presence of the measurement errors, the bias of the total process capability index can be positive or negative. Future studies may include analyzing the properties of the estimation of specific process capability index in the multistage processes.

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