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کواهی می شود که جناب آقای سعید حسینی

در بیست و چهارمین سمینار جبر ایران، که در روزهای ۲۱ و ۲۲ آبان ۹۳ در دانشگاه خوارزمی برگزار شد،

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ضمناً نامبرده مقاله تحت عنوان

" On rational groups whose irreducible characters vanish only on  $p$ -element"

را در این سمینار ارائه نموده است.

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## ON RATIONAL GROUPS WHOSE IRREDUCIBLE CHARACTERS VANISH ONLY ON $p$ -ELEMENTS

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ABSTRACT. A finite group whose irreducible complex characters are rational valued is called a rational group. Our aim in this talk is to study the rational groups whose irreducible characters vanish only on  $p$ -elements for a prime number  $p$ .

This is a joint work with Hesam Sharifi<sup>‡</sup>.

### 1. INTRODUCTION

**Definition 1.1.** Let  $G$  be a finite group, all whose irreducible complex characters are rational valued. Such a group  $G$  is called a **rational group** or a  **$\mathbb{Q}$ -group**.

The importance of studying  $\mathbb{Q}$ -groups can be deduced, looking at the history of character theory. Some examples of  $\mathbb{Q}$ -groups are the symmetric groups  $S_n$ , Dihedral groups of orders 2, 4, 6, 8, 12, elementary abelian 2-groups and the Weyl groups of the complex Lie algebras. One can find some important properties of  $\mathbb{Q}$ -groups in [2].

**Definition 1.2.** Let  $G$  be a finite group and  $\chi$  be a complex character of  $G$ . If  $\chi(g) = 0$  for some  $g \in G$  then  $g$  is called a zero of  $\chi$  and we say  $\chi$  vanishes on  $g$ .

In this context, by a character we always mean a complex character. A well-known theorem of Burnside asserts that every nonlinear irreducible character of a finite group  $H$  vanishes on some element  $h \in H$ . More precisely Malle, Navarro and Olsson in [3], have shown that if  $\chi$  is a nonlinear irreducible character of  $H$  then there exists a  $p$ -element  $h \in H$  such that  $\chi(h) = 0$ , where  $p$  is a prime number dividing the order of  $H$ .

**Definition 1.3.** Let  $H \subseteq F$ , with  $1 < H < F$ . Assume that  $H \cap H^x = 1$  whenever  $x \in F \setminus H$ . Then  $H$  is a **Frobenius complement** in  $F$ . A group which contains a Frobenius complement is called a **Frobenius group**.

Here we are interested on studying rational groups whose irreducible characters vanish only on  $p$ -elements for a fixed prime  $p$ . We claim that if a  $\mathbb{Q}$ -group  $G$  satisfies such property then  $p = 2$  and  $G$  is solvable. Especially we will show, besides the finite elementary abelian 2-groups, every  $\mathbb{Q}$ -group whose irreducible characters vanish only on involutions, satisfies  $G \cong Z(G) \times F$ , where  $F$  is a Frobenius group with an elementary abelian 3-group as Frobenius kernel and  $\mathbb{Z}_2$  as Frobenius complement.

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