"On rational groups whose irreducible characters vanish only on p-element"
ON RATIONAL GROUPS WHOSE IRREDUCIBLE CHARACTERS VANISH ONLY ON p-ELEMENTS

SAEID JAFARI

Abstract. A finite group whose irreducible complex characters are rational valued
is called a rational group. Our aim in this talk is to study the rational groups whose
irreducible characters vanish only on p-elements for a prime number p.

This is a joint work with Hesam Sharifi.

1. Introduction

Definition 1.1. Let G be a finite group, all whose irreducible complex characters are
rational valued. Such a group G is called a rational group or a $\mathbb{Q}$-group.

The importance of studying $\mathbb{Q}$-groups can be deduced, looking at the history of
character theory. Some examples of $\mathbb{Q}$-groups are the symmetric groups $S_n$, Dihedral
groups of orders 2, 4, 6, 8, 12, elementary abelian 2-groups and the Weyl groups of
the complex Lie algebras. One can find some important properties of $\mathbb{Q}$-groups in [2].

Definition 1.2. Let G be a finite group and $\chi$ be a complex character of G. If $\chi(g) = 0$
for some $g \in G$ then g is called a zero of $\chi$ and we say $\chi$ vanishes on g.

In this context, by a character we always mean a complex character. A well-known
theorem of Burnside asserts that every nonlinear irreducible character of a finite group
$H$ vanishes on some element $h \in H$. More precisely Malle, Navarro and Olsson in [3],
have shown that if $\chi$ is a nonlinear irreducible character of H then there exists a p-

Definition 1.3. Let $H \subseteq F$, with $1 < H < F$. Assume that $H \cap H^x = 1$ whenever
$x \in F \setminus H$. Then $H$ is a Frobenius complement in $F$. A group which contains a
Frobenius complement is called a Frobenius group.

Here we are interested on studying rational groups whose irreducible characters vanish only on p-elements for a fixed prime p. We claim that if a $\mathbb{Q}$-group G satisfies
such property then $p = 2$ and G is solvable. Especially we will show, besides the
finite elementary abelian 2-groups, every $\mathbb{Q}$-group whose irreducible characters vanish
only on involutions, satisfies $G \cong Z(G) \times F$, where F is a Frobenius group with an
elementary abelian 3-group as Frobenius kernel and $\mathbb{Z}_2$ as Frobenius complement.

2010 Mathematics Subject Classification. 20C15.

Key words and phrases. Rational group, Zero of character, Frobenius group.

\textsuperscript{1}Faculty of Science, Shahed University.
\textsuperscript{1}Email: sa.jafari@shahed.ac.ir.
\textsuperscript{1}Email: hasbarifi@shahed.ac.ir.