

Green bi-fuel vehicle routing problem with time windows in fuel delivery

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Abstract—In this paper two new Mixed-integer linear programming models for fuel based vehicle routing problem are introduced. By considering the refueling accessibility in customer nodes, these models are formulated as a Fossil Fuel Capacitated Vehicle Routing Problem with Time Windows and the Green Bi-fuel VRPTW. A fuel delivery routing problem is a real world application of these models. Environmental concerns from the pollution emission in routing problem motivated us to investigate one effect of the green fuel. These models can be categorized as extensions of the well-known VRP and the result of them have been compared. In the proposed models not only the routing costs are minimized, but also the pollution costs are minimized as well. Finally, the model validity has been evaluated by numerical examples and sensitivity analysis.

Keywords- Vehicle routing problem; Green routing; Bi-fuel; Time windows; Mathematical model.

I. INTRODUCTION

According to the Intergovernmental Panel on Climate Change (IPCC), freight transport generates ۱۳% of the total amount of anthropogenic greenhouse gases at the global level [۱]. The amount of CO₂ emission of fuel in transportation, such as gasoline is the second largest source of the air pollution. Approximately, ۳۱% of total United State CO₂ emissions and ۲۶% of total green-house gas emissions released from the vehicles [۲]. Therefore, concentration on the goods transportation performance is necessary. It is clear that the amount of vehicles, travelling distances, vehicle weights, types, speeds should be considered [۳].

Vehicle routing and its variants take an important role in freight transport problems. The descriptions of the VRP

and the variants are found in [۴] and [۵]. Environmental effect nowadays is very important for governments and companies. Applying environmental constraints might increase complexity in the routing problem and lead to conflicting interest between economic and ecological requirements. Pollutants such as carbon monoxide, hydrocarbons and particulates are released from the exhaust of a vehicle and affect the quality of the air. So the minimization of vehicle running costs should include not only the operational cost of a vehicle such as time cost, fuel costs, etc. but also environmental costs resulting from the pollution emission. The relative weights of these components in vehicle running costs depend on the types of vehicles, fuel, government regulations, and other issues.

The vehicle routing problem (VRP) was first introduced by Dantzig and Ramser in ۱۹۵۹ [۶]. One of the most applicable VRP is the capacitated vehicle routing problem (CVRP) that assumes a fixed fleet of vehicles with uniform capacity. In the capacitated vehicle routing problem there are a set of customers with predetermined demand, and a set of vehicles with same capacities that stay in a single depot. Each tour consists of a sequence of customers assigned to a vehicle. The objective function minimizes total traveled distances by all the vehicles. In practice, one of the best known variant of vehicle routing problem is VRP with time windows (VRPTW) in which the customers specify a time window to be visited. In the VRPTW problem, each customer has an associated time window defined by the earliest and the latest time to start the customer service. Hard and soft time windows are two kinds of them. In the hard time window, arriving earlier than the predetermined early start time for each customer or after the latest time is not permitted at all. In contrast, the soft time window allows time violations by adding a penalty cost to the objective function [۷].

Several classes of vehicle routing problems have been studied in the literature. In [8] other variants of the VRP is introduced as a rich vehicle routing problem that is not our concern in this article.

Therefore the great interest in the VRP is due to its practical importance, as well as environmental impacts. On the other hand fuel consumption and its pollution nowadays have been concentrated by the researchers.

In article [9] time dependent traveling speed is considered. The pollution routing is introduced in [10] and they assumed fuel costs and speed in their model.

Other works focus on issues that associated with the limited capacity of fuel tanks [11].

There are two fuel strategies related to the refueling and the pollution. One of them is using fossil fuel which has following characteristics: high price, more pollution emission effect, more refueling customer accessibility and less consumption rate. Second strategy is using green fuel with low price, less pollution emission effect, less refueling customer accessibility and more consumption rate. Therefore by using both fuel type benefits we introduce this bi-fuel routing model. This method is known as an alternative refueling strategy in the literature [12]. In [13] the authors solved fuel delivery model but they didn't consider fuel capacity and pollution effect that we consider them in our fuel delivery routing. In [14] authors proposed a model with considering fuel consumption and fuel vehicle capacity.

According to the literature the importance of our research is not only considering more realistic parameters such as fuel capacity, bi-fuel vehicles and time windows, but also concerning on the environmental impact of pollution routing problems.

The structure of this article is as follows. In section 2 the problem definition of our problem is illustrated and compared with the classic model. In Section 3 the proposed mathematical model is formulated. Sensitivity analysis of the model with comparable numerical results is clarified in section 4. Finally, in section 5 the conclusion of our study is explained.

II. PROBLEM DEFINITION

In the classic Vehicle Routing Problem (VRP) there are a set of customers with known demand that a fleet of vehicles must serve them from the specific depot. Minimizing the total distances is the objective function and the problem solution determines the service order and vehicle assignment to each route.

VRP with time windows consider the time accessibility of each customer and in the hard time windows constraints there is no excuse for any delay. The vehicle must consider the early and latest start service time of each customer. To compare our proposed model with the classic VRP with time windows, fuel consumption rate according to vehicle weight, fuel emission pollution rate, fuel price, tank fuel capacity,

using vehicle cost and driver time cost are considered. Therefore these costs should be minimized in the objective function. First we proposed a fuel based capacitated vehicle routing problem with time windows (FCVRPTW) and then the green bi-fuel vehicle routing problem (GBFVRPTW) is introduced as proposed models. In the both models it is assumed that customer nodes have the ability to refuel the vehicles and there is no need to separate refueling stations.

In the real world refueling takes an important role in routing problems and the result of considering refuel able vehicles is closer to the real case. In some cases such as the fuel delivery problem the customers have this feature to supply fuel for the truck vehicles. In the other case when the distances between two customers is much more than the distance between fuel stations and customers, this negligible distance could be omitted.

Here there is a fuel delivery routing problem that a set of truck vehicles must serve \forall customers from the supplier depot node. Driver time cost, vehicle using cost, fuel consumption cost, fuel pollution cost and refueling costs are considered. The objective function is minimization of these costs. Three models are compared in this article to illustrate the effectiveness of the proposed model. Figure 1 depicts the solution result of these three models.

In the GBFVRPTW each vehicle has the ability to use two kinds of fuel. Each vehicle has two fuel tank capacities that both of them are full when the vehicle starts the tour from the depot. Each customer might have the ability to support each kind of fuel. Fossil and green fuels are our assumption kinds of fuel that each of them has its determine price and pollution rate. Fossil and green fuel capacity of each vehicle and goods capacity of them influence on the fuel consumption and fuel pollution emission rate.

According to Figure 1 part (b) or (c) by considering the ability of refueling in customer nodes, when the driver found that the vehicle cannot travel to the next customer because of fuel shortage, he/she decides to refuel in which they have the possibility to refueling and so the length of the tour could be increased and there is no need to depart another vehicle to support the other customers. Fuel constraint is the main difference between VRPTW and FCVRPTW. But the main difference between the GBFVRPTW and FCVRPTW is the ability of vehicle to use two kinds of fuels and customers with the ability to support green fuel. So in GBFVRPTW the pollution emission rate is less than the FCVRPTW. Green features here made us to propose this model and reduce the consumption of fossil fuel and pollution gases.

For more clarifying of the problem here we explain the process of routing in GBFVRPTW. First the truck vehicle starts its trip to serve customer Δ , the driver prefer to use fossil fuel in this part of the route. Then the vehicle goes to the customer Υ by using fossil fuel. According to this fact that the vehicle have full fuel tank when it starts the

trip, there is no need for refueling. The vehicle then depart customer v and by using green fuel it travels to the customer w .

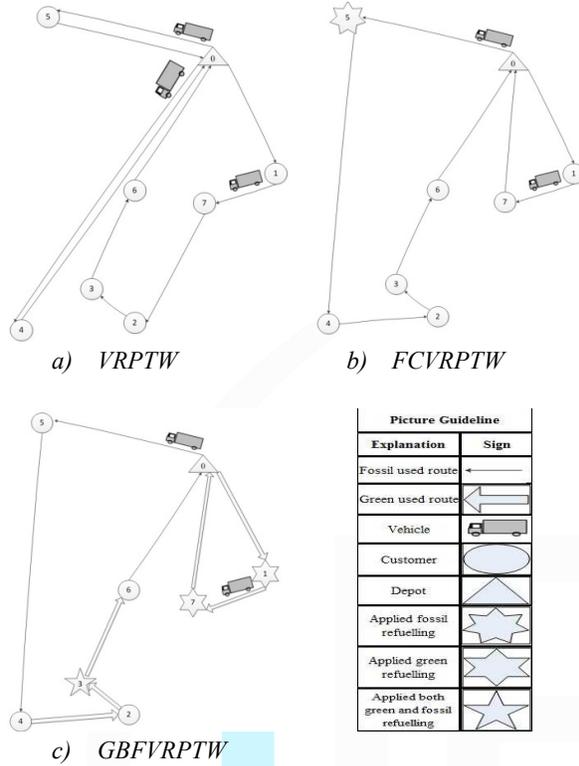


Figure 1. Comparing of the routing solutions for three models.

Then by using green fuel it goes to customer w and because it lost both kinds of fuel in its trip and customer w have this ability to supply both kinds of fuel for the vehicle, the driver decide to refuel green and fossil fuel to continuing the trip. Now the vehicle has a full tank for both fuels. Then it goes to the customer v on using green fuel and at last goes back to the depot by using fossil fuel. So it needs to refuel both kinds of fuel in node w . The other vehicle starts from depot by using green fuel it goes to the customer v and because it needs to refueling green fuel and it can refuel in customer v , it refuels and goes to the customer w by using green fuel and according to loss of green fuel for traveling the last part of the trip, it refuels green fuel in customer w and comes back to the depot. The vehicle here tries to use green fuel because of the environmental effect of green fuel in contrast to fossil fuel.

III. PROBLEM FORMULATION

A. Sets

- I, J, N Set of nodes that consist of the depot 0 and customers from 1 to n
- V Set of customers from 1 to v
- K Set of vehicles from 1 to k
- F Set of fuel kinds, it is equal to 1 for fossil fuel

and 2 for green fuel use

B. Decision variables

- x_{ij}^k Binary variable that is equal to 1, if there is a way between two nodes from i to j by vehicle k that uses fuel kinds off
- y_{v1} Binary variable that shows vehicle k is used for serving customer v
- u_{i1} Positive integer variable that indicates the order of visiting costumers for sub-tour elimination
- $M E_{i1}$ The rest of fossil fuel when departing node i by vehicle k
- $M E_{i2}$ The rest of green fuel when departing node i by vehicle k
- $Z \zeta_{i1}$ Binary variables that indicate refueling fossil fuel in node i by vehicle k
- $Z G_{i1}$ Binary variables that indicate refueling green fuel in node i by vehicle k
- d_k The time cost of using vehicle k by its driver
- $c a_k$ Binary variable that is equal to 1 if vehicle k is used
- $T W_i$ The specific time when the vehicle arrives in customer i
- $T C$ Total costs
- $T L$ Total Traveling distance

C. Parameters

- $Q \zeta$ Vehicle goods capacity
- $n o d$ Number of all nodes
- $F F$ Fossil fuel consumption rate
- $G F$ Green fuel consumption rate
- $F F$ Fossil fuel price per each unite of travel distance
- $G F$ Green fuel price per each unite of travel distance
- $G P$ Green fuel pollution emission rate
- $F P$ Fossil fuel pollution emission rate
- $M F$ Maximum fossil fuel tank capacity
- $M G$ Maximum green fuel tank capacity
- $D 1$ Maximum duration of each tour
- $I T$ Cost of using each vehicle
- $G R_v$ Green refueling cost in customer v
- $F R_v$ Fossil refueling cost in customer v
- q_v Supply or demand of customer v
- s_v Service time of customer v
- $E \zeta_i$ Earliest start time of beginning the service of customer i
- $L \zeta_i$ Latest start time of beginning the service of customer i
- $M A_i$ Maximum time between $E \zeta_i$ and traveling time to node i from the depot
- $R \zeta_v$ Refueling time that is wasted in customer v
- t_{ij} Time between two nodes from i to j
- c_{ij} Distance between two nodes from i to j
- M A very large number

D. Objective Function

The objective function in the classic VRP is sum of tour distances, but here in this model it consist of following Δ parts:

$$TC = P a \tau_1 + P a \tau_2 + P a \tau_3 + P a \tau_4 + P a \tau_5$$

Part τ_1 shows driver time cost that it will grow, if time of travelling increases.

Part τ_2 :

$$\sum_{k \in K} d_k \quad (1)$$

Part τ_3 indicates the both green and fossil fuel costs in all the tours.

Part τ_4 :

$$\sum_{i,j \in N, k \in K} F F . F F . c_i . x_{i j}^1 + \sum_{i,j \in N, k \in K} G F . G F . c_i . x_{i j}^2 \quad (2)$$

Part τ_5 represents refueling cost for both kinds of fuel. It is used to avoid unnecessary refueling and it spends time, energy and made our vehicle stop and start again.

Part τ_6 :

$$\sum_{v \in V, k \in K} F R_v Z \zeta_v + \sum_{v \in V, k \in K} G R_v Z G_v \quad (3)$$

Part τ_7 indicates the rent vehicle cost that is used to avoid renting more vehicles.

Part τ_8 :

$$IT \times \sum_{v \in V, k \in K, f \in F} x_{0v}^f \quad (4)$$

Part τ_9 shows the taxes that government imposes for polluting the air. It differs from green fuel to fossil fuel and depends on fuel rate consumption in vehicles.

Part τ_{10} :

$$\sum_{i,j \in N, k \in K} F P . F F . c_i . x_{i j}^1 + \sum_{i,j \in N, k \in K} G P . G F . c_i . x_{i j}^2 \quad (5)$$

E. Constraints

1) VRP

$$\sum_{f \in F, v \in V} x_{0v}^f \leq 1 \quad \forall k \in K \quad (6)$$

$$\sum_{n \in N, f \in F} x_{n i}^f - \sum_{j \in N, f \in F} x_{i j}^f = 0 \quad \forall i \in N, k \in K \quad (7)$$

$$y_v \leq \sum_{i \in N, f \in F} x_{i v}^f \quad \forall k \in K, v \in V \quad (8)$$

$$\sum_{k \in K} y_v = 1 \quad \forall v \in V \quad (9)$$

$$\sum_{v \in V} q_v y_v \leq Q \zeta \quad \forall k \in K \quad (10)$$

$$u_{0k} = 1 \quad \forall k \in K \quad (11)$$

$$u_{i k} - u_{v k} + n o d . x_{i v}^f \leq n o a - 1 \quad \forall i \in N, v \in V, k \in K, f \in F \quad (12)$$

$$\sum_{i \in N, k \in K, f \in F} x_{i v}^f \leq 1 \quad \forall v \in V \quad (13)$$

$$T L = \sum_{i,j \in N, k \in K, f \in F} c_i . x_{i j}^f \quad (14)$$

2) Green

$$\sum_{f \in F} x_{i j}^f \leq 1 \quad \forall i, j \in N, k \in K \quad (15)$$

$$M E_{10k} \leq M F . Z \zeta_{0k} \quad \forall k \in K \quad (16)$$

$$M E_{20k} \leq M G . Z G_{0k} \quad \forall k \in K \quad (17)$$

$$C a_k = \sum_{v \in V, f \in F} x_{0v}^f \quad \forall k \in K \quad (18)$$

$$M E_{1v} \leq M F . C a_k \quad \forall k \in K, v \in V \quad (19)$$

$$M E_{2v} \leq M G . C a_k \quad \forall k \in K, v \in V \quad (20)$$

$$Z \zeta_{0k} = \sum_{v \in V, f \in F} x_{0v}^f \quad \forall k \in K \quad (21)$$

$$Z G_{0k} = \sum_{v \in V, f \in F} x_{0v}^f \quad \forall k \in K \quad (22)$$

$$M E_{1v} \leq M F - F F . c_{0v} x_{0v}^1 + M \left(1 - \sum_{f \in F} x_{0v}^f + Z \zeta_{0k} \right) \quad \forall v \in V, k \in K \quad (23)$$

$$M E_{1v} \geq M F - F F . c_{0v} x_{0v}^1 - M \left(1 - \sum_{f \in F} x_{0v}^f + Z \zeta_{0k} \right) \quad \forall v \in V, k \in K \quad (24)$$

$$M E_{2v} \leq M G - G F . c_{0v} x_{0v}^2 + M \left(1 - \sum_{f \in F} x_{0v}^f + Z G_{0k} \right) \quad \forall v \in V, k \in K \quad (25)$$

$$M E_{2v} \geq M G - G F . c_{0v} x_{0v}^2 - M \left(1 - \sum_{f \in F} x_{0v}^f + Z G_{0k} \right) \quad \forall v \in V, k \in K \quad (26)$$

$$M E_{1v} \geq M F . Z \zeta_v \quad \forall v \in V, k \in K \quad (27)$$

$$M E_{2v} \geq M G . Z G_v \quad \forall v \in V, k \in K \quad (28)$$

$$M E_{l,l} \leq M E_{v,l} - F F \cdot c_v x_{v,l}^1 \quad (٢٩)$$

$$+ M \left(1 - \sum_{f \in F} x_{v,l,k}^f + Z G_k \right) \quad \forall v, l \in V, k \in K$$

$$M E_{l,l} \geq M E_{v,l} - F F \cdot c_v x_{v,l}^1 \quad (٣٠)$$

$$- M \left(1 - \sum_{f \in F} x_{v,l,k}^f + Z G_k \right) \quad \forall v, l \in V, k \in K$$

$$M E_{l,l} \leq M E_{v,l} - G F \cdot c_v x_{v,l}^2 \quad (٣١)$$

$$+ M \left(1 - \sum_{f \in F} x_{v,l,k}^f + Z G_k \right) \quad \forall v, l \in V, k \in K$$

$$M E_{l,l} \geq M E_{v,l} - G F \cdot c_v x_{v,l}^2 \quad (٣٢)$$

$$- M \left(1 - \sum_{f \in F} x_{v,l,k}^f + Z G_k \right) \quad \forall v, l \in V, k \in K$$

$$x_{i,j}^1 \cdot F F \cdot c_i \leq M F \quad \forall i, j \in N, k \in K \quad (٣٣)$$

$$x_{i,j}^2 \cdot G F \cdot c_i \leq M G \quad \forall i, j \in N, k \in K \quad (٣٤)$$

$$M E_{v,k} \geq F F \cdot c_{v0} x_{v0,k}^1 - M(1 - x_{v0,k}^1 + Z G_{v,i}) \quad \forall v \in V, k \in K \quad (٣٥)$$

$$M E_{v,k} \geq G F \cdot c_{v0} x_{v0,k}^2 - M(1 - x_{v0,k}^2 + Z G_{v,i}) \quad \forall v \in V, k \in K \quad (٣٦)$$

$$M E_{v,k} \geq \sum_{l \in V} F F \cdot c_v x_{v,l}^1 \quad \forall v \in V, k \in K \quad (٣٧)$$

$$M E_{v,k} \geq \sum_{l \in V} G F \cdot c_v x_{v,l}^2 \quad \forall v \in V, k \in K \quad (٣٨)$$

$$\sum_{k \in K} Z G_{v,i} \leq 1 \quad \forall v \in V \quad (٣٩)$$

$$\sum_{k \in K} Z G_{v,i} \leq 1 \quad \forall v \in V \quad (٤٠)$$

$$Z G_{v,i} \leq \sum_{i \in N, f \in F} x_{i,v}^f \quad \forall k \in K, v \in V \quad (٤١)$$

$$Z G_{v,i} \leq \sum_{i \in N, f \in F} x_{i,v}^f \quad \forall k \in K, v \in V \quad (٤٢)$$

٣) Time windows

$$T W_i \leq L_i \quad \forall i \in N \quad (٤٣)$$

$$T W_i \geq E_i \quad \forall i \in N \quad (٤٤)$$

$$T W_0 + s_0 + M A_v - D \cdot \alpha \left(1 - \sum_{f \in F} x_{0v,i}^f \right) \leq T W_i \quad \forall v \in V, k \in K \quad (٤٥)$$

$$T W_0 + s_0 + M A_v + D \cdot \alpha \left(1 - \sum_{f \in F} x_{0v,i}^f \right) \geq T W_i \quad \forall v \in V, k \in K \quad (٤٦)$$

$$T W_k + s_v + t_v - D \cdot \alpha \left(1 - \sum_{f \in F} x_{v,l}^f \right) + R \cdot \beta (Z G_{v,i} + Z G_{v,i}) \leq T W_i \quad \forall v, l \in V, k \in K \quad (٤٧)$$

$$T W_k + \sum_{f \in F} (s_v + t_{v0}) \cdot x_{v0,k}^f + R \cdot \beta (Z G_{v,i} + Z G_{v,i}) \leq D \cdot \alpha \quad (٤٨)$$

$$T W_k + \sum_{f \in F} (s_v + t_{v0}) \cdot x_{v0,k}^f + R \cdot \beta (Z G_{v,i} + Z G_{v,i}) \leq d_k + M \left(1 - \sum_{f \in F} x_{v0,k}^f \right) \quad \forall v \in V, k \in K \quad (٤٩)$$

a) VRP Constraints:

Constraints (٢٩) ensure the maximum times of using each vehicle that starts from the depot. Constraints (٣٠) guarantee that leaving or entering to each customer must be done by same vehicle. Constraints (٣١) represent that only when there is a way to reach customer v by vehicle k then it could be possible to serve customer v by that vehicle. Constraints (٣٢) ensure that each customer must be visited exactly once. Constraints (٣٣) guarantee goods capacity limitation. Constraints (٣٤) and (٣٥) represent the sub-tour elimination constraints. Constraints (٣٦) show that there is at most one way by one vehicle to reach each customer.

b) Green Constraints

Constraints (٣٥) ensure that only one kind of fuel can be used in traveling between two nodes. Constraints (٣٦-٣٧) show the maximum possible level of fuel tank capacity and they represent that both fuel tanks capacity of a vehicle that starts the tour are full. Constraints (٣٨-٣٩) and (٣٩-٣٢) update the level of each fuel tank in the tour. Constraints (٣٧) and (٣٨) declare that when a vehicle refuels in a customer position, its tank fuel capacity becomes full. Constraints (٣٣) and (٣٤) ensure fuel capacity limitation in each route. Constraints (٣٥-٣٨) show that only when a vehicle has enough fuel, it can travel to destination. Constraints (٣٩-٤٢) represent that only when a customer be visited by a vehicle, refueling could be possible on it and it must just one time.

c) Time window Constraint

Constraints (٤٣) and (٤٤) guarantee that each tasks must done at specific time in which the customer allows, according to hard time windows that is used in this model, there is no excuse for violation. Early start shows that the task can't start earlier than that and latest start shows the latest opportunity for starting the task on the customers. The $T W$ decision variables represent the time when serving of customer i start. Constraints (٤٥-٤٧) update the time when a vehicle visit a customer by using refueling time, service time, early start time and the time between two customers. Constraints (٤٨) and (٤٩) consider the maximum duration of each tour that should not be violated and the driver time in each tour.

IV. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

To show the applicability of the proposed model, a simulated problem with V customers is considered and

results have been depicted in Figure ۱. The comparison of results by three models has been reported in Table I.

TABLE I. THREE TRANSPORTATION ROUTING MODELS RESULT

| Model | Number of used vehicle | Total travel distance | Total driver time cost | Total cost |
|----------|------------------------|-----------------------|------------------------|------------|
| VRPTW | ۳ | ۴۷۹.۱۳ | ۶۲ | ۳۰۰۰ |
| FCVRPTW | ۲ | ۳۳۸.۹۷ | ۴۲ | ۲۰۶۰ |
| GBFVRPTW | ۲ | ۳۳۸.۹۷ | ۴۵ | ۱۹۹۱.۸ |

In order to discover the sensitivity and validity of the model to some important parameters such as refueling accessibility, time windows and customer number, we consider examples and solve them with different levels of parameters.

A. Refueling accessibility

Here we compare different models according to accessibility to the customers that have the ability for refueling in the example that we introduced in section ۲. The result is shown in Table II.

When all customers have this ability to refuel vehicles in both fuels the cost of green fuel capacitated vehicle routing problem is better than the others. According to lack of accessibility to green refueling customers, the results become grow up and even the model becomes infeasible because of the loss in green fuel. By comparing the total costs of two proposed models, we figure out that GBFVRPTW have better result. For credibility of our proposed models we compare them with the VRPTW and the same result have been observed while neglecting the green fuel tank weight and having no green fuel in starting a trip. Therefore by increasing the ability to refueling nodes the costs decreased.

TABLE II. REFUELING ACCESSIBILITY ANALYSIS ON THE MODELS PERFORMANCE

| Model | Parameters | Number of green refueling customers | Number of fossil refueling customers | Total green travel cost | Total fossil travel cost | Total cost | |
|--------------|------------------------------|-------------------------------------|--------------------------------------|-------------------------|--------------------------|------------|---------|
| Examples N=۷ | | | | | | | |
| GFCVRPTW | ۱ | ۷ | it is not important | ۳۳۸.۹۷ | - | ۱۷۲۸.۹۴۱ | |
| | ۲ | ۵ | | ۳۳۸.۹۷ | - | ۱۷۲۸.۹۴۱ | |
| | ۳ | ۴ | | infeasible | | | |
| Examples N=۷ | | | | | | | |
| FCVRPTW | ۱ | it is not important | ۷ | - | ۳۳۸.۹۷ | ۲۰۵۹.۹۱ | |
| | ۲ | | ۵ | - | ۳۳۸.۹۷ | ۲۰۶۰.۹۱ | |
| | ۳ | | ۳ | - | ۳۶۶.۸۶ | ۲۱۵۲.۵۸ | |
| | ۴ | | ۲ | - | ۳۹۶.۶۶ | ۲۲۲۲.۹۶۷ | |
| | ۵ | | ۱ | - | ۴۱۶.۸۱ | ۲۳۰۵.۴۲۷ | |
| | ۶ | | ۰ | - | ۴۷۹.۱۳ | ۳۰۰۰ | |
| Examples N=۷ | | | | | | | |
| GBFVRPTW | ۱ | ۷ | ۷ | ۱۵۶.۴۱۷ | ۲۰۹۰.۰۲ | ۱۹۹۱.۸۴ | |
| | ۲ | ۵ | | ۱۴۲.۷۶۷ | ۲۲۳.۳۰۲ | ۲۰۰۴.۴۴ | |
| | ۳ | ۳ | | ۱۱۷.۸۰۹ | ۲۴۹.۴۴۹ | ۲۰۲۹.۹۴۴ | |
| | ۴ | ۲ | | ۸۱.۱۳۴ | ۲۸۷.۸۷ | ۲۰۶۸.۸۷۸ | |
| | ۵ | ۰ | | ۸۱.۱۳۴ | ۲۸۷.۸۷ | ۲۰۶۸.۸۷۸ | |
| | ۶ | no initial green fuel | | - | ۳۷۲.۸۶۸ | ۲۱۶۱.۶۰۳ | |
| | negligible green tank weight | no initial green fuel | | - | - | ۳۳۸.۹۷ | ۲۰۵۹.۹۱ |
| | negligible green tank weight | no initial green fuel | | - | - | ۴۷۹.۱۳ | ۳۰۰۰ |
| Examples N=۷ | | | | | | | |
| VRPTW | ۱ | it is not important | - | - | ۴۷۹.۱۳ | ۳۰۰۰ | |

B. Time windows

By making the hard time windows constraints tighter we observe more costs. According to the Table III when we have to use one separate vehicle for each customer because of the tight time windows that made all nodes critical, same result in our FCVRPTW and VRPTW appears. This observation proves the credibility of our model. Figure ۲ also illustrates the result of costs for these models. It proves that the green one works better.

TABLE III. TIGHT TIME WINDOW EFFECT ON THE SOLUTION PERFORMANCE

| Model | Parameters | vehicle numbers | Latest start service time | Travel distance | Total driver time cost | Total cost |
|--------------|------------|-----------------|----------------------------|-----------------|------------------------|------------|
| Examples N=V | | | | | | |
| VRPTW | 1 | 3 | 100 | 479.13 | 64 | 3000 |
| | 2 | 3 | 60 | 479.13 | 64 | 3000 |
| | 3 | 3 | 20 | 480.28 | 62 | 3002.85 |
| | 4 | 3 | 13 | 537.19 | 55 | 3166.57 |
| | 5 | 3 | 10 | 603.36 | 64 | 3874.09 |
| | 6 | 3 | half of nodes are critical | 680.97 | 60 | 3102.33 |
| | 7 | 5 | 80% of nodes are critical | 689.56 | 71 | 4439.7 |
| | 8 | 7 | all nodes are critical | 858.96 | 81 | 6157.89 |
| Examples N=V | | | | | | |
| FCVRPTW | 1 | 2 | 100 | 338.97 | 42 | 2059.91 |
| | 2 | 2 | 60 | 338.97 | 42 | 2059.91 |
| | 3 | 2 | 20 | 391.72 | 55 | 2231.16 |
| | 4 | 2 | 13 | 535.12 | 47 | 2654.37 |
| | 5 | 3 | 10 | 583.98 | 57 | 3209.93 |
| | 6 | 3 | half of nodes are critical | 647.3 | 66 | 4008.9 |
| | 7 | 5 | 80% of nodes are critical | 689.56 | 71 | 4439.7 |
| | 8 | 7 | all nodes are critical | 858.96 | 81 | 6157.89 |
| Examples N=V | | | | | | |
| GBFVRPTW | 1 | 2 | 100 | 338.97 | 53 | 1999.84 |
| | 2 | 2 | 60 | 338.97 | 53 | 1999.84 |
| | 3 | 2 | 20 | 391.72 | 53 | 2191.1 |
| | 4 | 2 | 13 | 535.12 | 48 | 2599.15 |
| | 5 | 3 | 10 | 596.52 | 58 | 3205.65 |
| | 6 | 3 | half of nodes are critical | 648.85 | 70 | 3921.97 |
| | 7 | 5 | 80% of nodes are critical | 689.56 | 75 | 4554.22 |
| | 8 | 7 | all nodes are critical | 858.96 | 85 | 6177.37 |

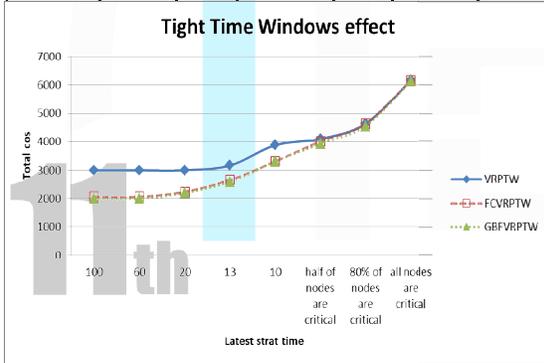


Figure 7. Tight time windows effect on the cost result.

C. Customer number

By increasing the number of customers that have the ability to refueling, the result of proposed models represents better cost. According to the TABLE IV the number of used vehicle becomes more when the customer numbers are increased. Figure 7 illustrated the difference between costs in these models.

TABLE IV. COST RESULT ASSOCIATED WITH CUSTOMER NUMBER

| Model | Parameters | Number of customer | vehicle used number | Travel distance | Total green travel cost | Total fossil travel cost | Total cost |
|----------|------------|--------------------|---------------------|-----------------|-------------------------|--------------------------|------------|
| Examples | | | | | | | |
| VRPTW | 1 | 3 | 1 | 175.17 | 0 | 175.17 | 1047.51 |
| | 2 | 5 | 3 | 479.47 | 0 | 479.47 | 2995.42 |
| | 3 | 7 | 3 | 479.13 | 0 | 479.13 | 3000 |
| | 4 | 9 | 4 | 672.08 | 0 | 672.08 | 4084.26 |
| | 5 | 10 | 4 | 723.67 | 0 | 723.67 | 4238 |
| Examples | | | | | | | |
| FCVRPTW | 1 | 3 | 1 | 175.17 | 0 | 175.17 | 1047.51 |
| | 2 | 5 | 1 | 272.46 | 0 | 272.46 | 1357.39 |
| | 3 | 7 | 2 | 338.97 | 0 | 338.97 | 2059.91 |
| | 4 | 9 | 2 | 390.96 | 0 | 390.96 | 2226.28 |
| | 5 | 10 | 2 | 435.8 | 0 | 435.8 | 2359.42 |
| Examples | | | | | | | |
| GBFVRPTW | 1 | 3 | 1 | 175.17 | 40 | 150.79 | 1053.35 |
| | 2 | 5 | 1 | 272.46 | 78.9 | 217.5 | 1347.96 |
| | 3 | 7 | 2 | 338.97 | 156.4 | 209 | 1991.84 |
| | 4 | 9 | 2 | 390.96 | 227.27 | 205.68 | 2141.57 |
| | 5 | 10 | 2 | 449.62 | 324.92 | 154.19 | 2185.42 |

Figure 7. The effect of customer numbers on the total cost result.

V. CONCLUSIONS

In this paper a new mathematical modeling for an extension of the VRP is introduced that considers fuel capacity and green constraints in classic routing problem with time windows. By considering fuel station in the customer position a fuel delivery system becomes an applicable real problem. A bi-fuel vehicle that decreases the pollution emission effects as well as routing costs is formulated. The difference between our proposed model and classic VRP discussed and it is compared with the fossil fuel capacitated model. The green fuel effects on the pollution of the environment and the important factors in traveling costs have been studied. The model has been solved with GAMS software. The sensitivity analysis confirms validity of the proposed model. Also the numerical results confirm that the proposed model can construct a routing plan with minimum pollution. Further study on this problem could be considering of the possibility of switching fuel during movement between

two nodes. Also Considering other fuel consumption parameters such as traffic time, exact vehicle weight by calculating the goods that carried at each moment time

and the rest of fuel in the fuel tank that might have better cost results are suggested.

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