An Equivalent Lumped Circuit Model for Thin Avalanche Photodiodes With Nonuniform Electric Field Profile

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Abstract—A staircase approximation method is deployed to model nonuniform field in the multiplication region and its surrounding ambient of a thin avalanche photodiode (APD). To the best of our knowledge, this is the first instance of introducing an equivalent circuit model that is taking the effect of the electric field profile in a thin APD’s multiplication region and its surroundings into account. This equivalent circuit model that is developed from the carriers’ rate equations also includes the effect of the tunneling current. The tunneling current that can be induced as a small current injected into the multiplication region results in an enhanced model behavior at high reverse bias voltages near breakdown. The output current obtained from the proposed model reveals excellent model accuracy, in regard to the current levels and prediction of breakdown voltages for both photo and dark currents. Moreover, simulations demonstrate ability of the present model for gain-bandwidth analysis.

Index Terms—Avalanche breakdown, avalanche photodiode (APD), circuit modeling, multiplication, nonuniform electric field.

I. INTRODUCTION

In comparison to photocathode, metal–semiconductor–metal (MSM) photodetector, and PIN photodiode, avalanche photodiode (APD) is by far the most appropriate device for optical networks. This is because, the APD’s internal gain yields higher detection sensitivity and simplifies the design procedure required for an optical receiver preamplifier [1]. The gain originated from the randomness of the carrier’s multiplication processes is accompanied by the excess noise. In order to minimize this noise, the width of the multiplication region should be as short as possible [2]–[4]. In an ultrathin APD two issues have great impact on the device performance: (i) the carriers’ dead spaces [2], [4] and (ii) non-uniformity of the electric field in the multiplication region as well as the adjacent depletion regions in the cladding layers [5]–[7]. A dead space is an average distance over which a carrier gains sufficient energy from an applied electric field to have a non-negligible ionization probability enough to get involved in an impact-ionization process and come into equilibrium with the electric field [4]. There has been a widespread research effort in the modeling of avalanche photodiodes (APDs) with ultrathin multiplication layers [6]–[10]. However, only a few research groups have attempted to introduce circuit models for investigating the APDs characteristics [11]–[18]. Availability of reliable circuit models for APDs facilitates design/simulation of optoelectronic integrated circuits or optical receivers. In fact, the APD can be treated the same as other circuit elements on the chip. Hence, before device fabrication the whole system can be simulated and pre-tested in a single environment. Thus, accuracy of the final product very much depends on the accuracy of the circuit model in predicting the device characteristics [19]–[22].

One of the first circuit models for APDs, suitable for dc, ac and transient analysis, is that of Chen et al. [11]. In their model, however, they have included neither the effects of the carriers’ dead spaces nor that of the nonuniform electric field in high field regions. Hence, their model is not suitable for simulating an ultrathin APD. Then, Jou et al. [12] presented a simple circuit model to account for carriers’ transit time through depletion region. They assumed a uniform field profile across the multiplication region and ignored the effects of carriers’ dead spaces as well as the tunneling current. Later on, Batawy et al. [13]–[15], have introduced efficient circuit models to study the time and the frequency responses of APDs. They have also assumed uniform electric fields within the devices high field regions and ignored the effects of the tunneling current in their models. Meanwhile, Banoushi et al. [16] introduced another circuit model based on carrier’s rate equations developed for layers of separate absorption, grading, charge, and multiplication avalanche photodiodes (SAGCM-APDs). This model also involves a uniform electric field in the high field regions and ignores the tunneling current. Two most recent circuit models have been introduced by Songfeng et al. [17] and Xiang et al. [18]. The former group introduced an effective model to study the single-photon time response of reach-through APDs. The latter group developed an efficient circuit model to predict frequency and bandwidth characteristics of SAGCM-APDs. These two models also involve uniform electric fields in high field regions and both ignore the tunneling current in multiplication regions.

To the best of our knowledge, the present paper is the first instance of reporting an equivalent circuit model that considers a nonuniform electric field profile in an ultrathin multiplication region and its surroundings as well as the tunneling current. In
In order to evaluate $F_m(0)$ and $F_m(W_m)$, one needs to know the value of $x_p$. This can be found, by integrating the field profile over the entire diode and using $F(W_m + x_n) \approx F(-x_p) \approx 0$

$$x_p \approx -W_m(N_n - N_m)/(N_n + N_p) + \left\{N_n(N_n - N_i)(N_p + N_m) W_m^2/N_p(N_n + N_p)^2 + 2\varepsilon_0\varepsilon\varepsilon_0 N_m(V_{th} - V_A)/qN_p(N_n + N_p)\right\}^{1/2}$$

where $V_A$ is the applied bias, and $V_{th}$ is the built-in potential for the APD.

Knowing the field profile, we have implemented two types of staircase approximate models to estimate the exact field profile. Both models are such that the area under each staircase profile equals the area under the real field profile.

$$F(x) = \begin{cases} F_m(0) + qN_p x/\varepsilon_0 \varepsilon \varepsilon_0 & -x_p \leq x \leq 0 \\ F_m(0) - qN_m x/\varepsilon_0 \varepsilon \varepsilon_0 & 0 \leq x \leq W_m \\ F_m(W_m) + qN_m(W_m - x)/\varepsilon_0 \varepsilon_0 & W_m \leq x \leq W_m + x_n \end{cases}$$

Section III, we present a brief description of the rate equations that are employed in our circuit model. Details of the presented circuit model are provided in Section IV. Section V demonstrates the simulation results. Finally, the paper is concluded in Section VI.
A. Model 1

In this model, absolute values of the electric field in the \( i \)th step in \( \eta^+ \), the \( j \)th step in \( \eta^- \), and the \( k \)th step in \( \eta^+ \)-regions are respectively defined as

\[
F_{m_i} = (i-1)F_m(W_m)/(k_n - 1), \quad i = 1, 2, \ldots, k_n
\]
\[
F_{m_j} = F_m(0) - (k_n - j)\eta N_m W_m/(k_n - 1)\approx_\infty_\approx_0, \quad j = 1, 2, \ldots, k_n
\]
\[
F_{p_k} = (k_n - k)F_m(0)/(k_n - 1), \quad k = 1, 2, \ldots, k_n.
\]

where \( k_n \) is the total number of steps in each regions. Note that the steps’ number in each region starts just from the left. In this approximation, no matter what the value of \( k_n \) is, \( F_{m1} = F_{n1} = F_m(W_m), F_{mkn} = F_{p1} = F_m(0) \), and \( F_{n1} = F_{pkn} = 0 \).

B. Model 2

In the second model, value of the field in each step equals the average value of the real field in that section. Absolute values of the electric fields for the \( i \)th step in \( \eta^+ \), the \( j \)th step in \( \eta^- \), and the \( k \)th step in \( \eta^+ \)-regions are respectively defined as

\[
F_{m_i} = (2i-1)F_m(W_m)/2k_n, \quad i = 1, 2, \ldots, k_n
\]
\[
F_{m_j} = F_m(0) - (2k_n - 2j + 1)\eta N_m W_m/2k_n \approx_\infty_\approx_0, \quad j = 1, 2, \ldots, k_n
\]
\[
F_{p_k} = (2k_n - 2k + 1)F_m(0)/2k_n, \quad k = 1, 2, \ldots, k_n.
\]

Contrary to the former model, in the present case, values of field in all steps vary with \( k_n \).

III. RATE EQUATIONS

In order to use both staircase field profiles for developing a circuit model, we need an appropriate set of rate equations. Such equations for \( \eta^+ \), \( \eta^- \), and \( \eta^+ \)-regions of a reversed-biased APD, like that shown in Fig. 1, are respectively

\[
\begin{align*}
\frac{dp_n}{dt} & = \frac{P_{Gn}}{n} - n \tau_{pn} - I_p/q, \\
\frac{dp_m}{dt} & = \frac{P_{Gm}}{m} - \frac{p_m}{\tau_{pm}} + \sigma \beta_p \nu_{pm}, \\
\frac{dn_m}{dt} & = \frac{N_{Gm}}{n} - n \tau_{nm} + \sigma \nu_{nm}, \\
\frac{dn_p}{dt} & = \frac{N_{Gp}}{p} - \frac{p_p}{\tau_{pp}} - I_n/q.
\end{align*}
\]

where \( p_n \) and \( \tau_{pn} \) are the total number of excess minority carriers and their corresponding recombination lifetimes in the \( \eta^+ \)- and \( \eta^+ \)-regions, respectively, and \( p_m, \tau_{pm}, \tau_{mp}, \text{and } \nu_{pm} \) and \( n_m, \tau_{nm}, \tau_{mn}, \text{and } \nu_{nm} \) are the total number of holes and electrons and their corresponding recombination terms life-times and transit time in the m-regions, respectively. Rates of the electron–hole pair photo-generation in (11)–(15) are given by

\[
\begin{align*}
P_{Gn} & = (P_n(1 - R_f)/h\nu)(1 - \exp(-\alpha_n W_m + \alpha_n x_n)), \\
P_{Gm} & = (P_m(1 - R_f)/h\nu)(1 - \exp(-\alpha_p W_m - \alpha_m x_n)), \\
P_{Gj} & = (P_m(1 - R_f)/h\nu)(1 - \exp(-\alpha_m W_m - (k_n - 1)\alpha_n x_n)), \\
P_{Gk} & = (P_m(1 - R_f)/h\nu)(1 - \exp(-\alpha_p W_p + \alpha_k x_p/k_n)).
\end{align*}
\]

where \( P_n \) is the power of incident light, \( R_f \) is the facet reflectivity of the \( \eta^+ \)-region, \( h\nu \) is the photon energy, \( \alpha_p, \alpha_m, \text{and } \alpha_n \) are the absorption coefficients of the \( \eta^+ \), \( \eta^- \), and \( \eta^+ \)-regions of widths \( W_p, W_m, \text{and } W_n \), respectively.

To take the effects of carriers’ dead spaces into account, we have utilized the simple approximation approach presented by [2], [9] that is suitable for our circuit level modeling. Carriers’ dead spaces are defined as [2], [4], [9]

\[
d_{\text{dead}} = E_{\text{dead}}/qF
\]
where $E_{\text{clad},h}$ represents the electron's/hole's ionization threshold energy, indexes "e" and "h" indicate the labels for electrons and holes, respectively.

As pointed out earlier in Section I, in an ultrathin p$^+\cdot$-n$^+$ structure the electric fields in the depletion layers of p$^+$- and n$^+$-regions are non-negligible. Thus, they can energize the carriers before being injected into the multiplication region, from either side. These energetic carriers enter the multiplication region with various initial energies. In order to include the effects of nonuniform electric field in cladding layers in the model, one should include the effects of the carriers’ initial average energy, $E_0$, in the corresponding dead space. The modified carriers’ dead spaces and the corresponding impact ionization coefficients become [6], [7]

$$d_{e,h} = (E_{\text{clad},h} - E_0)/qF$$

$$\alpha(x) = \alpha(F)u(x - d_e)$$

$$\beta(x) = \beta(F)u(x - d_h)$$

where $u(x - d_{e,h})$ is a unit step function, and $E_{\text{clad},h}$ is defined as

$$E_{\text{clad},e} - E_0 = \int_0^{d_e} F(x)dx$$

$$E_{\text{clad},h} - E_0 = \int_{W_m}^{w_{n-1}} F(x)dx.$$  

The electric field dependences of electron and hole ionization coefficients are defined as [2]–[5], [9]:

$$\alpha(F) = \alpha_n \exp(-b_n/F)c^n$$

$$\beta(F) = \alpha_p \exp(-b_p/F)c^p$$

where $F$ (V-cm$^{-1}$) is the electric field, $\alpha_n$ (cm$^{-1}$) and $\alpha_p$ (cm$^{-1}$), $b_n$ (V-cm$^{-1}$) and $b_p$ (V-cm$^{-1}$), and $c_n$ and $c_p$ are all the material constants.

### IV. Circuit Modeling

For circuit modeling, we need to transform our physical parameters to equivalent circuit parameters. For example, the minority carrier charges generated in various space charge regions (i.e., $\eta_p, n_m = p_m$ and $p_h$) are modeled as the equivalent charges on the parallel planes of a nominal capacitor ($C_n$), across which potentials $V_1, V_2,$ and $V_3$ are dropped, such that we can write $q_{\eta_p} = C_n V_1, q_{n_m} = C_n V_{12}, q_{p_m} = C_n V_{23}, q_{p_h} = C_n V_{3}$.

In (20)–(24), $R_{\text{in}}$ is modeled by a voltage variable $V_{\text{in}}$, while $N_{\text{Cov}}, P_{\text{Cov},h}$, and $P_{\text{Cov},e}$ are modeled by the equivalent current passing through the resistances $R_{11}, R_{12}, R_{23},$ and $R_3$ defined as;

$$R_{21} = \frac{h\nu \exp(\alpha_p W_p + (k_m - j)\alpha_n W_m + \alpha_n x_p)}{q(1 - R_f)(1 - \exp(-\alpha_n W_m/k_n))}$$

$$R_{3i} = \frac{h\nu \exp(\alpha_p W_p + (k_m - j)\alpha_n W_m + \alpha_n x_n/k_n)}{q(1 - R_f)(1 - \exp(-\alpha_n x_n/k_n))}$$

Using (32)–(36), the rate equations (11)–(15) reduce to

$$V_{\text{in}}R_{11}^{-1} = C_n dV_1/dt + V_1 R_{12}^{-1} + I_p$$

$$V_{\text{in}}R_{12}^{-1} = C_n dV_2/dt + V_2 R_{12}^{-1} + I_{12}$$

$$V_{\text{in}}R_{33}^{-1} = C_n dV_3/dt + V_3 R_{33}^{-1} + I_n$$

where

$$R_{12} = R_{\text{pr}_n}/C_n$$

$$R_{23} = R_{\text{pr}_p}/C_n$$

$$R_{12} = \frac{R_{12}^{\text{pr}}}{C_n}$$

$$R_{12} = \frac{R_{12}^{\text{pr}}}{C_n}$$

$$I_{12} = C_n dV_2/dt + V_2 R_{12}^{-1} + I_n$$

$$I_{12} = I_{12} + \frac{R_{12}^{\text{pr}}}{C_n}$$

$$I_{12} = I_{12} + \frac{R_{12}^{\text{pr}}}{C_n}$$

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$$I_{12} = I_{12} + \frac{R_{12}^{\text{pr}}}{C_n}$$

Fig. 2 illustrates the equivalent circuit derived from the above equations. The parallel resistors in Fig. 2(a) signify the light absorption in different regions of the device. Currents passing through these resistors are injected into the equivalent circuit models of the corresponding regions by the current sources, $I_1 = V_{\text{in}}/R_{11}, I_{12} = V_{\text{in}}/R_{12}, I_{23} = V_{\text{in}}/R_{33}, I_3 = V_{\text{in}}/R_3$.

Figs. 2(b)–(f) model various regions of the device fed by the photo-generated currents $I_1, I_{12}, I_{23}, I_3,$ and $I_3$. Current components $I_p$ and $I_n$ shown in Figs. 2(b) and (f) can be written in the form of [11]

$$I_p = V_1/R_{pd} + g_{\text{in}} V_{\text{in}} + I_{p0}$$

$$I_n = V_3/R_{nd} + g_{\text{in}} V_{\text{in}} + I_{n0}.$$  

Parameters $R_{pd}, R_{nd}, g_p, g_n, I_{p0}$ and $I_{n0}$ are defined in Appendix I. $I_{\text{tunnel}}$ shown in various parts of Fig. 2(d) represents the current generated by tunneling phenomenon. Since the tunneling current nearly increases in an exponential manner with applied reverse bias voltage, it could originate a breakdown at moderate voltages. The tunneling current for circuit models, is usually presented by a standalone formula, based on a formula given in [23], used to represent

$$I_{\text{tunnel}} = \Theta_1 A V_{\text{in}} F_m \exp(-\Theta_2/F_m)$$

where

$$\Theta_1 = q^2 \sqrt{2m_e^* E_0/4\pi^2 n^2}$$

$$\Theta_2 = \eta m_e^{3/2} E_0^{3/2} / q n.$$

$$\Theta_1 = q^2 \sqrt{2m_e^* E_0/4\pi^2 n^2}$$

$$\Theta_2 = \eta m_e^{3/2} E_0^{3/2} / q n.$$  

$$\Theta_1 = q^2 \sqrt{2m_e^* E_0/4\pi^2 n^2}$$

$$\Theta_2 = \eta m_e^{3/2} E_0^{3/2} / q n.$$
an excessive increase, preventing the model to predict the breakdown voltage accurately. In order to overcome this problem, we have exploited the similarity between the field dependence of (45a) and (30)–(31) to model the tunneling current. In this regard, we have added a small current \( I_{\text{turn}} = k_e \Theta_j A W_j F_{\text{m}j} \) to the circuit model for step \( j \) in the m-region. The multiplier \( k_e \) is a fit parameter that can be obtained by fitting the resulting dark current to the corresponding experimental data.

Fig. 2(g) models the device output. In this figure,

\[
I_{\text{PD}} = I_n + I_{M,k_n} + (C_s + C_{J}) \frac{dV}{dt} + \frac{V_t}{R_{k,n}} \tag{46}
\]

where \( I_{M,k_n} \) that implicitly includes the tunneling is the hole current that follows out of the step \( k_n \) of the depleted layer region in \( p^+ \)-region, and \( R_{k,n} \) and \( R_s \) are the parallel and series parasitic resistances, and \( C_s \) and \( C_J = \varepsilon_0 \varepsilon_r A/(x_n + W_m + x_P) \) are the parasitic and junction capacitances.

V. SIMULATION RESULTS

To verify the model efficiency, a homojunction \( \text{In}_{0.22}\text{Al}_{0.78}\text{As} \) APD has been simulated by Agilent ADS and the results have been compared with the experimental data presented by [3]. Parameters used in the simulation are tabulated in Table I.

Simulations were carried out using models 1 and 2. In both cases, as we increased the total number of steps, \( k_n \), the simulated results become closer to the experimental results. We have realized that there exists a saturation value for \( k_n \): e.g., for \( k_n \geq 20 \) variations in the results obtained by either model are negligible. For \( k_n = 20 \), both models provide almost accurate predictions for the current levels and the breakdown voltages with negligible differences. Nonetheless, for \( k_n < 20 \), there are some differences between the results around the breakdown voltages, predicted by two models. This, in fact, is due to the differences in the approximated values of the electric fields by the two models, within the last couple of steps of the m-region where the high field values control the breakdown voltage. While Model 1 over-estimates the electric field in this section, Model 2 under-estimates it. For this reason Model 1 overestimates the avalanche processes and Model 2 under-estimates them. The results presented henceforth are those obtained by Model 2 for \( k_n = 20 \) that are compared with existing experimental results.

Fig. 3 illustrates the dark current and photocurrent densities for two \( \text{InAlAs-APDs} \) of \( W_m = 100 \) nm and 500 nm, using Model 2 with \( k_n = 20 \) (dashed- and solid-lines for dark- and photo-currents, respectively). These results are compared with those obtained by simplified model in which a uniform electric field is assumed across the multiplication regions (dots for dark- and dotted-dashed for photo-current). A further comparison is made by the existing experimental data presented by [3] ( for dark- and ( for photo-current). This comparison shows that our model resembles the experimental data more accurately. This is more obvious as the applied bias approaches the breakdown voltage. The device breakdown voltage can be calculated by applying some linear relations for multiplication gain and the photocurrent.
Since the background doping concentration of the multiplication region affects the degree of nonuniformity, we have investigated this effect on the device breakdown. Fig. 4 illustrates the simulated results for $W_m$ of 100 and 500 nm (solid- and dashed-lines) that are compared with the experimental data for devices of the same kind ($\bullet$ and $\blacksquare$) [3]. As shown in this figure, the thinner the multiplication region the less important is the effect of its background doping concentration. Furthermore, for the high level of background doping concentration the predicted results approach to each other.

Furthermore, we have simulated 3-dB bandwidth for an InP/InGaAs-APD with separate absorption, graded and charge multiplication (SAGCM), as a function of its mean gain. Fig. 5 illustrates the simulated results (solid line) for a SAGCM-APD with a 200-nm multiplication layer and a 50-nm charge layer that is n-doped with concentration of $1.5 \times 10^{17}$ cm$^{-3}$. The data shown by bullets are those predicted experimentally by [24].

To investigate the credibility of the present circuit model in predicting ultrathin multiplication regions of widths $W_m < 100$ nm, we have predicted mean gain for GaAs-APDs with various $W_m$ ($=49, 150, \text{ and } 480$). Fig. 6 illustrates the predicted gain by the circuit model (solid lines) compared with those predicted by the numerical model of [7] (dashed lines). This numerical model that includes the effects of the carriers’ previous ionization history, are already shown to be in excellent agreement with the experimental data reported by [27].

### Table I

**Parameters Used in Simulations**

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<th>Symbol</th>
<th>Value</th>
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<tr>
<td>$E_{th}$</td>
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</table>

1 For device with $W_m=100nm$
2 For device with $W_m=500nm$

![Fig. 3](image-url) Dark and photo-current densities as a function of reverse bias voltage for APDs with multiplication widths of (a) $W_m = 100$ and (b) $W_m = 500$ nm. Comparisons are made between data obtained by the new circuit model $(k_n = 20)$ with those obtained for uniform E-field $(k_n = 1)$ and also the experimental data of [3].

![Fig. 4](image-url) Breakdown voltage as a function of background doping concentration for APDs of $W_m = 100$ nm and $W_m = 500$ nm $(k_n = 20)$. 
VI. CONCLUSION

A new circuit model for thin APDs with nonuniform electric field has been introduced. Using this circuit model, the photodetector can be treated as a lumped circuit element in circuit simulators. The model is appropriate for ac, dc and transient analysis useful for designing optoelectronic integrated circuits. The main contribution of our work is presenting a suitable approach to model the nonuniform electric field in multiplication region and its surrounding ambient of thin APDs.

To validate our model, we have compared our predicted results with available experimental data for devices of the same kind. Simulations indicate that the new mechanisms for consideration of nonuniform fields and tunneling current in the model have provided excellent agreement with the published experimental data.

This model facilitates circuit simulation for other complicated APD structures such as SACM or SACGM APDs.

Furthermore, we have demonstrated the need for developing a more accurate technique by which the history of the carriers’ previous ionization in ultrathin multiplication regions (\(W_m \sim 50\) nm) can be modeled by some appropriate circuit models.

APPENDIX

Considering \(L_n\) and \(L_p\) as the diffusion lengths of electrons and holes, and \(n_{i0}\) and \(p_{i0}\) as equilibrium electron and hole density in the \(p^+\) and \(n^+\) regions, respectively, and also introducing dimensionless lengths

\[
\ell_{p_n} = W_p / L_{p_n}, \quad \ell_{n_p} = W_n / L_{p_n}, \quad \eta_{p_n} = (W_p - x_p) / L_{p_n}, \quad \eta_{n_p} = (W_n - x_n) / L_{p_n}
\]

we can write

\[
R_{n1} = R_{n_{\text{mp}}} (\cosh \eta_{n_p} - 1) \quad (A2)
\]

\[
R_{p1} = R_{p_{\text{np}}} (\cosh \eta_{n_p} - 1) \quad (A3)
\]

\[
I_{p_n} = \frac{q \eta_{n_p} AL_n}{\tau_{n_{\text{mp}}}} \ell_{n_p}, \quad \cosh \eta_{n_p} + 1
\]

\[
I_{p_n} = \frac{q \eta_{n_p} AL_p}{\tau_{p_{\text{np}}}} \ell_{n_p}, \quad \cosh \eta_{n_p} + 1
\]

\[
g_n = \frac{q(1 - R_f)}{h\nu} \cdot \frac{\alpha_p^2 L_p}{\alpha_p^2 L_p - 1} \cdot \exp (-\alpha_p W_p + \alpha_p x_p)
\]

\[
\times \left\{ \begin{array}{l}
\cosh \eta_{n_p} + 1 \\
\alpha_{n_p} L_{n_p} c \sinh \eta_{n_p}
\end{array} \right. + \frac{1 - \exp (\alpha_{n_p} W_n - \alpha_{n_p} x_n)}{\alpha_{n_p}^2 L_{n_p} (\cosh \eta_{n_p} - 1)} + 1 \right\} \quad (A6)
\]

\[
g_p = \frac{q(1 - R_f)}{h\nu} \cdot \frac{\alpha_n^2 L_n}{\alpha_n^2 L_n - 1} \cdot \exp (-\alpha_n W_p - \alpha_n W_n - \alpha_n x_n)
\]

\[
\times \left\{ \begin{array}{l}
\cosh \eta_{n_p} + 1 \\
\alpha_{n_p} L_{n_p} c \sinh \eta_{n_p}
\end{array} \right. - \frac{1 - \exp (\alpha_n W_n + \alpha_n x_n)}{\alpha_{n_p}^2 L_{n_p} (\cosh \eta_{n_p} - 1)} - 1 \right\}. \quad (A7)
\]
REFERENCES


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