A new fuzzy approach for project time assessment under uncertain conditions

Amir Mohammad Hamzeh*, Seyed Meysam Mousavi

Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran

* Corresponding author: Amir Mohammad Hamzeh (am.hamzeh@shahed.ac.ir)

Abstract

Project control is a part of feedback system intended to find and rectify deviations from desired plan. Earned value management (EVM) and its extensions as project control approaches regard monetary values to assess the project schedule performance. For this reason, they sometimes provide inaccurate duration performance results. In this regard, earned duration management (EDM) which is an effective project time control method, is proposed. EDM is a cost value free technique in evaluating and estimating the project time performance. Since the uncertainty is inherent in projects’ areas, it should be considered in the control and monitoring process. In this paper, a new triangular intuitionistic fuzzy earned duration model is presented to control the project time performance under uncertain conditions. Modified EDM indices and measures are introduced to evaluate the project time performance under uncertainty. Finally, a case study from the recent literature about construction project is given to show the model’s applicability in real-life conditions.

Keywords: Project time control; Earned duration management; Triangular intuitionistic fuzzy numbers; Uncertainty

1. Introduction

Project control and monitoring is one of the main processes of project management with substantial need of growth. It aims at evaluating the state of the project based on a set of time and cost performance metrics during the project execution [1]. Earned value management (EVM) is a cost-based technique established as a project control and forecasting tool. EVM and its derivations include project cost and time performance indicators, but their cost-based time performance indicators provide unreliable results [2].

Earned duration management (EDM) is a new project time control and forecasting tool, which is proposed by Khamooshi and Golafshani [3]. Unlike the EVM, this appropriate method avoids the usage of cost values to evaluate the time performance of projects [4]. In this regard, Vanhoucke et al. [5] illustrated the strengths of the EDM method as an alternative to earned schedule (ES) method.

Project control techniques are applied to real projects with inherent uncertainty. Therefore, considering this uncertainty into the calculations and interpretations helps in considering better performance of a project [6]. In this regard, Naeni et al. [6] proposed a fuzzy earned value model to evaluate the earned value indices, and estimate the project time and cost at completion under uncertainty. In addition, Salari et al. [7] presented a fuzzy earned-value model based on Z-numbers under real-life circumstances. Furthermore, Moradi et al. [8] presented an uncertainty approach using interval type-2 fuzzy sets to improve the applicability of EVM measures in real projects.

However, there is only a little number of studies, which focus on the real-life situations’ uncertainties in the EDM technique. In this sake, Ghanbari et al. [9] integrated the classical fuzzy sets and EDM method to deal with the uncertainty in project time control’ process.

Meanwhile, triangular intuitionistic fuzzy number (TIFN), which is an extension of intuitionistic fuzzy sets (IVFSs) have more capability to deal with project uncertainty than classical fuzzy sets. Moreover, it has better ability to model fuzzy ill-defined information.

In this paper, an earned duration analysis model with triangular intuitionistic fuzzy numbers is proposed to improve the applicability of EDM indices and forecasts. In this regard, the uncertain nature of real project activities is considered. The remaining of this research is organized as follows. In Section 2, the preliminaries of TIFNs are explained. In section 3, the proposed triangular intuitionistic fuzzy EDM model is presented. Then, a case study about the construction project is discussed in section 4 to show the applicability of proposed approach. Finally, the conclusion of the research is represented in section 5.

2. Preliminaries

In this section, some basic concepts about triangular intuitionistic fuzzy numbers (TIFNs), including the definition, operation laws and ranking method are discussed.
**Definition 1.** Let \( \tilde{a} = ((a_1, a_2, \bar{a}_1); w_{\tilde{a}_1}, u_{\tilde{a}_1}) \) be a TIFN, which is a special type of intuitionistic fuzzy set (IFS) on the set \( R \). In this regard, the membership and non-membership functions of TIFN \( \tilde{a} \) are defined as below, respectively [10]:

\[
\mu_{\tilde{a}} (x) = \begin{cases} 
\frac{x - a}{a - \bar{a}}, & \text{if } a \leq x < a \\
w_{\tilde{a}_1}, & \text{if } x = a \\
\frac{\bar{a} - x}{\bar{a} - a}, & \text{if } a < x \leq \bar{a} \\
0, & \text{if } x < a \text{ or } x > \bar{a}
\end{cases}
\]

\[
v_{\tilde{a}} (x) = \begin{cases} 
\frac{a - x + (x - a)u_{\tilde{a}_1}}{r - a}, & \text{if } a \leq x < a \\
u_{\tilde{a}_1}, & \text{if } x = a \\
\frac{x - a + (\bar{a} - x)u_{\tilde{a}_1}}{\bar{a} - a}, & \text{if } a < x \leq \bar{a} \\
1, & \text{if } x < a \text{ or } x > \bar{a}
\end{cases}
\]

where \( 0 \leq w_{\tilde{a}_1} \leq 1, 0 \leq u_{\tilde{a}_1} \leq 1, 0 \leq w_{\tilde{a}_2} + u_{\tilde{a}_2} \leq 1, a, a, \bar{a} \in R \). In this respect, the values of \( w_{\tilde{a}_1} \) and \( u_{\tilde{a}_1} \) show the maximal membership degree and minimal non-membership degree. Furthermore, the membership function and non-membership function of TIFN \( \tilde{a} \) are depicted in Figure 1. In addition, \( \Pi_{\tilde{a}} (x) = 1 - \mu_{\tilde{a}} (x) - v_{\tilde{a}} (x) \) which is called an intuitionistic fuzzy index of \( \tilde{a} \) defines the hesitancy degree of an element \( x \) to \( \tilde{a} \).

![Figure 1. Membership function and non-membership function of TIFN \( \tilde{a} \)](image)

**Definition 2.** Let \( \tilde{a}_1 = ((a_1, a_2, \bar{a}_1); w_{\tilde{a}_1}, u_{\tilde{a}_1}) \) and \( \tilde{a}_2 = ((a_2, a_3, \bar{a}_2); w_{\tilde{a}_2}, u_{\tilde{a}_2}) \) be two TIFNs and \( \gamma \) be a real number. Then, the operation laws for TIFNs are described as follows [10]:

\[
\tilde{a}_1 + \tilde{a}_2 = ((a_1 + a_2, a_1 + a_2, \bar{a}_1 + \bar{a}_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2})
\]

\[
\tilde{a}_1 \tilde{a}_2 = \begin{cases} 
((a_1 \bar{a}_2, a_1 a_2, a_1 \bar{a}_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 > 0 \text{ and } \tilde{a}_2 > 0 \\
((a_1 a_2, a_1 a_2, a_1 a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 < 0 \text{ and } \tilde{a}_2 > 0 \\
((a_1 a_2, a_1 a_2, a_1 a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 < 0 \text{ and } \tilde{a}_2 < 0 \\
((a_1 \bar{a}_2, a_1 a_2, a_1 \bar{a}_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 > 0 \text{ and } \tilde{a}_2 > 0 \\
((a_1 a_2, a_1 a_2, a_1 a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 < 0 \text{ and } \tilde{a}_2 > 0 \\
((a_1 a_2, a_1 a_2, a_1 a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 < 0 \text{ and } \tilde{a}_2 < 0
\end{cases}
\]

\[
\tilde{a}_1 / \tilde{a}_2 = \begin{cases} 
((\bar{a}_1 / a_2, a_1 a_2, \bar{a}_1 / a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 > 0 \text{ and } \tilde{a}_2 > 0 \\
((\bar{a}_1 / a_2, a_1 a_2, \bar{a}_1 / a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 < 0 \text{ and } \tilde{a}_2 > 0 \\
((\bar{a}_1 / a_2, a_1 a_2, \bar{a}_1 / a_2); w_{\tilde{a}_1} \wedge w_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}) & \text{if } \tilde{a}_1 < 0 \text{ and } \tilde{a}_2 < 0
\end{cases}
\]
where “∧” and “∨” shows Min and Max operators. If $a_j \geq 0$ and one of the $a, a_j$ and $a_j$ is not equal to 0, then $\tilde{a} > 0$. Also if $\tilde{a} \geq 0$ and one of the $a, a_j$ and $a_j$ is not equal to 0, then $\tilde{a} < 0$.

**Definition 3.** For TIFNs $\tilde{a}_1$ and $\tilde{a}_2$, the membership function average index and the non-membership function average index are defined as Eqs. (7) and (8), respectively [11].

\[
S(\tilde{a}_j) = \frac{(a_j + 2a_j + \overline{a}_j)}{4} \quad j = 1, 2 \tag{7}
\]

\[
H(\tilde{a}_j) = (a_j + 2a_j + \overline{a}_j)(1 - u_{a_j}) \quad j = 1, 2 \tag{8}
\]

**Definition 4.** Given two TIFNs $\tilde{a}_1$ and $\tilde{a}_2$. The comparison rules are represented as follows [11]:

1. If $S(\tilde{a}_1) > S(\tilde{a}_2)$, or $S(\tilde{a}_1) = S(\tilde{a}_2)$ and $H(\tilde{a}_1) > H(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$.
2. If $S(\tilde{a}_1) = S(\tilde{a}_2)$ and $H(\tilde{a}_1) = H(\tilde{a}_2)$, then $\tilde{a}_1 = \tilde{a}_2$.

**3. Methodology**

The EDM as an effective technique is applied to control the project time. Regarding to the inherent complexity and uncertainty of projects, the EDM measures should be modified to consider the real projects’ conditions. In this regard, some time-based measures are defined to assess the time performance of project in uncertain conditions. Thereby, the earned duration of activity $i$ at any point in time is obtained as follows:

\[
\tilde{ED}_i = BPD_i \times \overline{API}_i = ((\tilde{ED}_i, ED), (\tilde{AP}_i, AP), \{w_{\tilde{ED}_i}, u_{\tilde{ED}_i}\}) = ((\tilde{BPD}_i, \tilde{API}_i, BPD_i \times API_i, \{w_{\tilde{AP}_i}, u_{\tilde{AP}_i}\}) \tag{9}
\]

where $BPD_i$ is the baseline planned duration of activity $i$. Furthermore, $\tilde{AP}_i$ is the triangular intuitionistic fuzzy actual time-based progress of activity $i$ which is presented as Eq. (10).

\[
\tilde{AP}_i = (\tilde{AP}_i, AP, \overline{AP}_i); w_{\tilde{AP}_i}, u_{\tilde{AP}_i}) \tag{10}
\]

The project expert initially determines the $\tilde{API}_i$ for each in-progress activity in terms of linguistic variables. These subjective judgments are converted to triangular intuitionistic fuzzy numbers with regard to Table 1 and then are used to calculate the $\tilde{ED}_i$.

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Triangular intuitionistic fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>$((0,0.05,0.15);0.92,0.06)$</td>
</tr>
<tr>
<td>Low</td>
<td>$((0.1,0.25,0.35);0.89,0.05)$</td>
</tr>
<tr>
<td>Less than half</td>
<td>$((0.3,0.42,0.5);0.91,0.04)$</td>
</tr>
<tr>
<td>Half</td>
<td>$((0.47,0.54,0.6);0.92,0.04)$</td>
</tr>
<tr>
<td>More than half</td>
<td>$((0.58,0.66,0.75);0.9,0.03)$</td>
</tr>
<tr>
<td>High</td>
<td>$((0.72,0.85,0.9);0.93,0.02)$</td>
</tr>
<tr>
<td>Very High</td>
<td>$((0.9,1,1);0.88,0.07)$</td>
</tr>
</tbody>
</table>

Furthermore, the total planned duration of project at time point $t$ is calculated as follows:

\[
TPD_t = \sum_{i=1}^{n} PD_i \tag{11}
\]

where $n$ shows the number of in-progress or accomplished activities up to the time $t$.

Then, the fuzzy total planned duration of project at any point in time is computed as following equation:
\[
\overline{TED} = \sum_{k=1}^{n} \overline{ED}_k = ((TED, TED, TED); w_{TED}, u_{TED}) = ((\sum_{k=1}^{n} ED_k, ED_k, ED_k); \sum_{k=1}^{n} w_{TED}, \sum_{k=1}^{n} u_{TED})
\]  

(12)

where \( n \) shows the number of in-progress or completed activities up to the status date.

Earned duration of project is the corresponding time of total earned duration on total planned duration S-curve that can be obtained as follows:

\[
\overline{ED}(t) = t + \frac{\overline{TED} - TPD}{TPD_{calender\ unit}} \times 1(calender\ unit) = ((ED(t), ED(t), \overline{ED}(t); w_{ED(0)}, u_{ED(0)})
\]

\[
= (t + \frac{TED\_k - TPD\_k}{TPD_{calender\ unit}} - TPD\_k) \times 1(CU), t + \frac{TED\_k - TPD\_k}{TPD_{calender\ unit}} - TPD\_k \times 1(CU); w_{ED(0)}, u_{ED(0)})
\]

(13)

Find \( t \) for \( TED\_k \geq TPD\_k \) and \( TED\_k < TPD\_k \) \((k = 1,2,3)\).

After calculating the aforementioned measures, the time performance of project can be assessed by some indices. In this sake, the fuzzy progress performance index, which expresses the overall time-based progress of project, is calculated as Eq. (14).

\[
\overline{PPI} = \frac{\overline{TED}(t)}{BPD} = ((PPI, PPI, PPI); w_{PPI}, u_{PPI}) = ((\frac{ED(t)}{BPD}, ED(t), \overline{ED}(t); w_{ED(0)}, u_{ED(0)})
\]

(14)

where \( BPD \) is the project baseline planned duration.

In addition, fuzzy duration performance index which shows the project performance in achieving the target completion time is computed as follows:

\[
\overline{DPI} = \frac{\overline{ED}(t)}{AD} = ((DPI, DPI, DPI); w_{DPI}, u_{DPI}) = ((\frac{ED(t)}{AD}, ED(t), \overline{ED}(t); w_{ED(0)}, u_{ED(0)})
\]

(15)

where \( AD \) is the project actual duration.

Fuzzy earned duration index is a time-based measure of total work carried out, in comparison with the work planned until that point in time. It is computed as:

\[
\overline{EDI} = \frac{\overline{TED}}{TPD} = (EDI, EDI, EDI; w_{EDI}, u_{EDI}) = ((\frac{TED}{TPD}, TED, TED; w_{EDI}, u_{EDI})
\]

(16)

Furthermore, the fuzzy duration variance of project from the plan on the critical path is as follows:

\[
\overline{DV} = \overline{TED}(t) - AD = ((DV, DV, DV); w_{DV}, u_{DV}) = ((ED(t) - AD, ED(t) - AD, \overline{ED}(t) - AD; w_{EDI}, u_{EDI})
\]

(17)

In addition, the fuzzy total duration variance of project can be calculated as below:

\[
\overline{TDV} = \overline{TED} - TPD = ((TDV, TDV, TDV); w_{TDV}, u_{TDV}) = ((TED - TPD, TED - TPD, TED - TPD; w_{TDV}, u_{TDV})
\]

(18)

There are various formulations to forecast the completion time of project with different precision. Meanwhile, in the proposed model, the project fuzzy estimated duration at completion is computed as follows:

\[
\overline{EDAC} = AD + \frac{BPD - \overline{ED}(t)}{DPI} = ((EDAC, EDAC, EDAC); w_{EDAC}, u_{EDAC})
\]

\[
= ((AD + \frac{BPD - ED(t)}{DPI}, AD + \frac{BPD - TIED(t)}{DPI}, AD + \frac{BPD - ED(t)}{DPI}; w_{EDI}, u_{EDI})
\]

(19)

The computed time performance indices and estimated project completion time should be interpreted to recognize the project status. In this regard, the values of \( DPI \) and \( EDI \) should be compared to 1 to interpret the project time-based performance. Furthermore, the value of \( EDAC \) should be compared to \( BPD \) to evaluate the project completion time. The interpretations of aforementioned measures are defined in Tables 2, 3 and 4.

### Table 2. Interpretation of \( DPI \)

<table>
<thead>
<tr>
<th>( S(DPI) &lt; 1 )</th>
<th>( VH(DPI) )</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Project is behind the schedule</td>
</tr>
</tbody>
</table>

www.iiiec.ir
Table 3. Interpretation of TIFEDI

<table>
<thead>
<tr>
<th>First comparison</th>
<th>Second comparison</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(\text{TIFEDI}) &lt; 1$</td>
<td>$\forall H(\text{TIFEDI})$</td>
<td>Less amount of work in performed compared to planned work</td>
</tr>
<tr>
<td>$H(\text{TIFEDI}) &lt; 1$</td>
<td></td>
<td>Less amount of work is performed compared to planned work</td>
</tr>
<tr>
<td>$S(\text{TIFEDI}) = 1$</td>
<td>$H(\text{TIFEDI}) = 1$</td>
<td>Same amount of work is performed compared to planned work</td>
</tr>
<tr>
<td>$H(\text{TIFEDI}) &gt; 1$</td>
<td></td>
<td>More amount of work is performed compared to planned work</td>
</tr>
<tr>
<td>$S(\text{TIFEDI}) &gt; 1$</td>
<td>$\forall H(\text{TIFEDI})$</td>
<td>More amount of work is performed compared to planned work</td>
</tr>
</tbody>
</table>

Table 4. Interpreting the comparison among EDAC and BPD

<table>
<thead>
<tr>
<th>First comparison</th>
<th>Second comparison</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(\text{EDAC}) &lt; BPD$</td>
<td>$\forall H(\text{EDAC})$</td>
<td>Project will complete before the BPD</td>
</tr>
<tr>
<td>$H(\text{EDAC}) &lt; BPD$</td>
<td></td>
<td>Project will complete before the BPD</td>
</tr>
<tr>
<td>$S(\text{EDAC}) = BPD$</td>
<td>$H(\text{EDAC}) = BPD$</td>
<td>Project will complete on the BPD</td>
</tr>
<tr>
<td>$H(\text{EDAC}) = BPD$</td>
<td></td>
<td>Project will complete after the BPD</td>
</tr>
<tr>
<td>$S(\text{EDAC}) &gt; BPD$</td>
<td>$\forall H(\text{EDAC})$</td>
<td>Project will complete after the BPD</td>
</tr>
</tbody>
</table>

4. Case Study

In this section, a case study has been represented to show the applicability of the proposed triangular intuitionistic fuzzy EDM model in real-life conditions. The case study is retrieved from a construction project study presented in Agyei [12]. In this regard, the intended house construction project, which has 14 activities, is selected to be analyzed. The project activities information is represented in Table 5 and the project network is displayed in Fig. 2. This project is performed up to the 45th day from the beginning and the time performance of project evaluated at this time.

Table 5. Information of the construction project activities

<table>
<thead>
<tr>
<th>Activity ID</th>
<th>Activity</th>
<th>BPD, (days)</th>
<th>Progress at 45th day</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Site clearing</td>
<td>3</td>
<td>Completed</td>
</tr>
<tr>
<td>B</td>
<td>Foundation</td>
<td>8</td>
<td>Completed</td>
</tr>
<tr>
<td>C</td>
<td>Block Laying</td>
<td>18</td>
<td>Completed</td>
</tr>
<tr>
<td>D</td>
<td>Roofing</td>
<td>10</td>
<td>Completed</td>
</tr>
<tr>
<td>E</td>
<td>Plumbing</td>
<td>5</td>
<td>Completed</td>
</tr>
<tr>
<td>F</td>
<td>Electrical work</td>
<td>10</td>
<td>Less than half</td>
</tr>
<tr>
<td>G</td>
<td>Plastering</td>
<td>11</td>
<td>Very low</td>
</tr>
<tr>
<td>H</td>
<td>Fixing up of doors and windows</td>
<td>17</td>
<td>Not started</td>
</tr>
<tr>
<td>I</td>
<td>Ceiling</td>
<td>9</td>
<td>High</td>
</tr>
<tr>
<td>J</td>
<td>Flooring</td>
<td>9</td>
<td>Not started</td>
</tr>
<tr>
<td>K</td>
<td>Interior Fixtures</td>
<td>4</td>
<td>Not started</td>
</tr>
<tr>
<td>L</td>
<td>Exterior fixtures</td>
<td>7</td>
<td>Not started</td>
</tr>
<tr>
<td>M</td>
<td>Painting</td>
<td>3</td>
<td>Not started</td>
</tr>
<tr>
<td>N</td>
<td>Landscaping</td>
<td>9</td>
<td>Not started</td>
</tr>
</tbody>
</table>
Therefore, the total planned duration of project is calculated at different time points and represented in Table 6.

In addition, the EDM parameters are computed and the results are displayed in Table 7.

Meanwhile, by using computed parameters, the project time performance measures and forecasts are calculated and displayed in Table 8.

Regarding to Table 8 and the interpretation rules which is defined in Tables 2 and 3, the project is behind the schedule. In addition, the estimated completion time shows that the project will be finished after the baseline planned duration.

5. Conclusion

Being able to effectively control and estimate the project schedule performance is necessary to desirable project management. EDM is a new technique to control the project time performance and forecast the project completion time. This methodology, unlike the previous techniques, evaluates project schedule performance utilizing time-based values. In this research, an EDM model was represented with triangular intuitionistic fuzzy numbers to obtain accurate results close to the real world in the project area. In this respect, the formulation of time performance indices and forecasts were designed for uncertain condition. In addition, a case study about the construction project was applied to assess the applicability of proposed model. The results obtained by implementing the proposed approach on the selected real
The project showed that the project was behind the schedule. In addition, the project would be finished after the baseline planned duration.

For the future researches, integrating the EDM method with other extensions of fuzzy sets with regard to different conditions of project environment can be recommended.

References


