ECONOMIC-STATISTICAL DESIGN OF VSSI-MEWMA-DWL CONTROL CHART WITH MULTIPLE ASSESSABLE CAUSES USING GENETIC ALGORITHM

Raziyeh Ghanataiyan*, Amirhossein Amiri

Industrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran.

*M.Sc. student in Department of Industrial Engineering, Faculty of Engineering, Shahed University (Speaker).
rg.ghanataiyan@shahed.ac.ir

ABSTRACT

Desirable properties of the multivariate exponentially weighted moving average (MEWMA) control chart such as the ability to detect small shifts in the process parameters have been caused that the MEWMA has been received significant attention from researchers in recent years. This paper proposes an economic-statistical design (ESD) model of the variable sample size and sampling interval (VSSI) MEWMA control chart by using multiple assignable causes based on Lorenzen and Vance cost function and multivariate Taguchi loss approach. Due to the complexity of the model a genetic algorithm is designed as an optimization technique. A numerical example is provided to illustrate the performance of the model and the solution approach.

Keywords: MEWMA control chart; VSSI; DWL; ESD; Genetic algorithm.

1. INTRODUCTION

In real environments many types of assignable causes may take place. Hence, developing ESD model of the MEWMA control chart that incorporates multi-assessable causes is important. ESD of VSSI-MEWMA-DWL control chart considers minimization of the expected cost per time unit, $E(A)$, as objective function, a lower limit ($AT S_U$) for in-control average time to signal ($AT S_0$) and an upper limit ($AT S_U$) for the out-of-control adjusted average time to signal ($AT S_U$) as statistical constraints. We extend the Lorenzen and Vance cost function considering multiple assignable causes. The multivariate Taguchi loss approach provided by Kapur and Cho [1] is used to obtain the expected external cost per time unit for the in-control and out-of-control process. In addition the Markov chain approach first proposed by Lee [2] is employed to obtain the desired $AT S_0$ and $AT S_U$ of the VSSI MEWMA-DWL control chart.

2. PROPOSED MODEL

The MEWMA vector and the plotted chart statistics are defined as follows respectively:

$$z_t = \gamma(x_t - \mu) + (1 - \gamma)z_{t-1}, \quad (2.1)$$

$$T^2_t = z'_t \Sigma^{-1}_t z_t, \quad (2.2)$$

where $\gamma$ (0 < $\gamma$ ≤ 1) is a diagonal weight matrix and $z_0$ represents a zero vector.

The VSSI-DWL MEWMA is a modification of fixed sample rate MEWMA control chart using two warning lines. The warning line $w_1$ is a guideline to switch between long and short sampling intervals $h_1$ and $h_2$ respectively, whereas the warning line $w_2$ is used to switch between small and large sample sizes $n_1$ and $n_2$ respectively. The next sampling point of the MEWMA-DWL control chart depends on the value of $T^2_t$. In the DWL scheme, there are three possible scenarios including: (i) $w_h < w_1$; (ii) $w_h > w_n$ and (iii) $w_h = w_2$.

The control chart alarms an out-of-control signal when $T^2_t > H$, where $H$ is an upper control limit. The proposed ESD model that is used to determine the VSSI-MEWMA-DWL chart parameters($n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H$) is defined as follows:

$$Min E(A)$$

subject to

$$AT S_0 \geq AT S_U$$

$$AAT S \leq AAT S_U$$

$$0 \leq w_h, w_n \leq H \leq H_{max}$$

$$h_{min} \leq h_2 \leq h_1 \leq h_{max}$$

$$1 \leq n_1 \leq n_2 \leq n_{max} \; (integers)$$

$$0 < \gamma \leq 1$$

3. MODIFIED LORENZEN AND VANCE COST FUNCTION

We make the usual assumption that there are multiple assignable causes ($j = 1, 2, ..., s$) where in each time just one of them can take place and change the mean of the process. Hence, the modified Lorenzen and Vance cost function for computing the expected cost per hour is defined as

$$E(A) = \frac{E(c)}{E(T)} \quad (3.1)$$

$$E(C) = \frac{1}{\lambda} \left( c_0 + \sum_{j=1}^{s} \lambda_j c_1 [AAT S_j + nE + r_1 T_{1j} + r_2 T_{2j}] \right) + a_1 ANF + a_1 ANS + a_2 ANF + \sum_{j=1}^{s} \lambda_j a_{3j} \quad (3.2)$$

$$+ \left( a_1 + a_2 n_1 \right) \sum_{j=1}^{s} \lambda_j (nE + r_1 T_{1j} + r_2 T_{2j}) \quad \lambda$$

$$E(T) = \frac{1}{\lambda} \left( 1 + \sum_{j=1}^{s} \lambda_j (AAT S_j + nE + T_{1j} + T_{2j} + (1 - r_1) T_{0} ANF) \right) \quad (3.3)$$

where $\lambda_j$ is the occurrences rate of $p^{th}$ assignable cause per hour according to Poisson process. Other parameters of the expected cost functions are defined similar to Faraz [3].
4. SOLUTION APPROACH

The proposed ESD model has both continuous and discrete decision variables and can be solved using one of the meta-heuristic methods. In this paper a genetic algorithm is developed to solve the proposed ESD model. Given the process parameters \((p, \lambda_j, T_0, T_1, T_2, \gamma_j, E)\) and the cost parameters \((C_0, C_1, a_1, a_2, a_3, \lambda_j)\) we strive to find the near optimum values for the control chart parameters \((n_1, n_2, h_1, h_2, \gamma, w_h, w_n, H)\). The genetic algorithm parameters involve initial population size \(N_{pop}\), cross over percentage \(P_c\), and mutation percentage \(P_m\) have been set according to Faraz [3]. The pseudo-code for the genetic algorithm of this paper is given in Algorithm 1.

A numerical example is presented to demonstrate the proposed model. Table 1 displays the optimum values of the parameters of the proposed ESD model. The optimum values of the \(ATS_0\), \(AATS\) and \(E(A)\) is illustrated in Table 1.

|  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
|  | \(n_1\) | \(n_2\) | \(h_1\) | \(h_2\) | \(\gamma\) | \(w_h\) | \(w_n\) | \(H\) |
| \(\%\) | 12 | 2.327 | 0.679 | 0.429 | 1.21 | 1.18 | 11.972 |

Table 2: The Optimal values of the \(ATS_0\), \(AATS\), \(E(A)\).

|  |  |  |  |  |
|---|---|---|---|
| \(ATS_0\) | \(AATS\) | \(E(A)\) |  |
| 645 | 0.8123 | 5944 |  |

5. CONCLUSION AND FUTURE RESEARCH

In this study an ESD model of the VSSI-MEWMA-DWL control chart is proposed to consider multiple assignable causes to improve the efficiency of control charts and provide much faster detection of small and moderate process changes. In contrast to previous models we consider multiple assignable causes and more real-life aspects of manufacturing process than existing models in ESD of MEWMA control chart. Considering the problem as a multi-objective model and incorporating the preventive maintenance into the cost model can be a fruitful area for future research.

6. REFERENCES

