ECONOMIC-STATISTICAL DESIGN OF VSSI-MEWMA-DWLCONTROL CHART WITH MULTIPLE ASSIGNABLE CAUSES USING GENETIC ALGORITHM

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ABSTRACT

Desirable properties of the multivariate exponentially weighted moving average (MEWMA) control chart such as the ability to detect small shifts in the process parameters have been caused that the MEWMA has been received significant attention from researchers in recent years. This paper proposes an economic-statistical design (ESD) model of the variable sample size and sampling interval (VSSI) MEWMA control chart by using double warning lines (DWL) by considering multiple assignable causes based on Lorenzen and Vance cost function and multivariate Taguchi loss approach. Due to the complexity of the model a genetic algorithm is designed as an optimization technique. A numerical example is provided to illustrate the performance of the model and the solution approach.
Keywords: MEWMA control chart; VSSI; DWL; ESD; Genetic algorithm.

1. INTRODUCTION

In real environments many types of assignable causes may take place. Hence, developing ESD model of the MEWMA control chart that incorporates multi-assignable causes is important. ESD of VSSI-MEWMA-DWL control chart considers minimization of the expected cost per time unit, E(A), as objective function, a lower limit (ATS_L) for in-control average time to signal (ATS_0) and an upper limit ($AATS_U$) for the out-of-control adjusted average time to signal ($AATS$) as statistical constraints. We extend the Lorenzen and Vance cost function considering multiple assignable causes. The multivariate Taguchi loss approach provided by Kapur and Cho [1] is used to obtain the expected external cost per time unit for the in-control and out-of-control process. In addition the Markov chain approach first proposed by Lee [2] is employed to obtain the desired ATS_0 and $AATS$ of the VSSI MEWMA-DWL control chart.

2. PROPOSED MODEL

The MEWMA vector and the plotted chart statistics are defined as follows respectively:

$$\mathbf{z}_t = \gamma(\mathbf{x}_t - \mu) + (1-\gamma)\mathbf{z}_{t-1}, \tag{2.1}$$

$$T_t^2 = \mathbf{z}_t' \Sigma_z^{-1} \mathbf{z}_t, \tag{2.2}$$

where γ $(0 < \gamma \leq 1)$ is a diagonal weight matrix and $\mathbf{z}_0$ represents a zero vector.

The VSSI-DWL MEWMA is a modification of fixed sample rate MEWMA control chart using two warning lines: The warning line w_h is a guideline to switch between long and short sampling intervals h_1 and h_2 respectively, whereas the warning line w_n is used to switch between small and large sample sizes n_1 and n_2 respectively. The next sampling point of the MEWMA-DWL control chart depends on the value of T_t^2. In the DWL scheme, there are three possible scenarios including: (i) $w_h < w_n$; (ii) $w_h > w_n$ and (iii) $w_h = w_n$. The control chart alarms an out-of-control signal when $T_t^2 > H$, where H is an upper control limit. The proposed ESD model that is used to determine the VSSI-MEWMA-DWL chart parameters$(n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H)$ is defined as follows:

$$\begin{aligned}
&Min\ E(A) \\
&\text{subject to} \\
&ATS_0 \geq ATS_L \\
&\overline{AATS} \leq AATS_U \\
&0 \leq w_n, w_h \leq H \leq H_{\max} \\
&h_{\min} \leq h_2 \leq h_1 \leq h_{\max} \\
&1 \leq n_1 \leq n_2 \leq n_{\max} (\text{integers}) \\
&0 < \gamma \leq 1
\end{aligned} \tag{2.3}$$

3. MODIFIED LORENZEN AND VANCE COST FUNCTION

We make the usual assumption that there are multiple assignable causes $(j = 1, 2, ..., s)$ where in each time just one of them can take place and change the mean of the process. Hence, the modified Lorenzen and Vance cost function for computing the expected cost per hour is defined as

$$E(A) = \frac{E(c)}{E(T)} \tag{3.1}$$

$$\begin{aligned}
E(C) = \frac{1}{\lambda}\{c_0 + \sum_{j=1}^{s} \lambda_j c_{1j}[AATS_j + \bar{n}E + r_{1j}T_{1j} + r_{2j}T_{2j}]\} \\
+ a_3' ANF + a_1 \overline{ANS} + a_2 \overline{ANI} + \frac{\sum_{j=1}^{s} \lambda_j a_{3j}}{\lambda} \\
+ \frac{(a_1 + a_2 n_1)}{h_2} \frac{\sum_{j=1}^{s} \lambda_j (\bar{n}E + r_{1j}T_{1j} + r_{2j}T_{2j})}{\lambda}
\end{aligned} \tag{3.2}$$

$$E(T) = \frac{1}{\lambda}\{1 + \sum_{j=1}^{s} \lambda_j (AATS_j + \bar{n}E + T_{1j} + T_{2j} + (1 - r_{1j})T_0 ANF)\} \tag{3.3}$$

where λ_j is the occurrences rate of j^{th} assignable cause per hour according to Poisson process. Other parameters of the expected cost functions are defined similar to Faraz [3].

Begin

Input: $n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H, AATS_U, ATS_L, H_{\max}, h_{\min}, h_{\max}, n_{\max}, C_0, C_{1j}, a_1, a_2, a_{3j}, a_3', p, \lambda_j, T_0, T_{1j}, T_{2j}, \gamma_{1j}, \gamma_{2j}, E, d, P_c, P_m, P_r, N_{pop}$.

Output: $n_1^*, n_2^*, h_1^*, h_2^*, \gamma^*, w_n^*, w_h^*, H^*, ATS_0{}^*, \overline{AATS}^*, E(A)^*$.

// Initialize generation 0

g=0

k=1

While $k \leq N_{pop}$

Generate initial chromosome P_k consisting of 8 genes $n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H$ randomly.

// Evaluate P_k

Compute the E(A), ATS_0 and $\overline{AATS}$.

If the ATS_0 and $\overline{AATS}$ satisfy the constraints

// Create chromosome $k+1$

end

end while

do

{ **//Create generation** *g+1*

// 1. Cross-over

Select $P_c \times N_{pop}$ members;

pair them up ;

Produce offspring: in each pair three similar genes of chromosomes have been replaced with each other randomly;

Insert the offspring into population of $g+1$;

// 2. Mutation

Select $P_m \times N_{pop}$ members;

In each member, two genes are mutated by size d;

//3. Copy:

Select $P_r \times N_{pop}$ members of current population randomly;

Insert into population of *g+1*;

// Evaluate population

Compute the fitness function: E(A), ATS_0 and $\overline{AATS}$;

// Increment:

g= g+1;

Compute the fitness function: E(A), ATS_0 and $\overline{AATS}$;

}

Sort the best solutions based on E(A) and select the chromosome with minimum E(A) as the best solution.

Algorithm 1: The Pseudo-code of the proposed genetic algorithm.

4. SOLUTION APPROACH

The proposed ESD model has both continuous and discrete decision variables and can be solved using one of the meta-heuristic methods. In this paper a genetic algorithm is developed to solve the proposed ESD model.Given the process parameters$(p, \lambda_j, T_0, T_{1j}, T_{2j}, \gamma_{1j}, \gamma_{2j}, E)$and the cost parameters $(C_0, C_{1j}, a_1, a_2, a_{3j}, a_3')$ we strive to find the near optimum values for the control chart parameters$(n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H)$. The genetic algorithm parameters involve initial population size(N_{pop}), cross over percentage(P_c),and mutation percentage(P_m) have been set according to Faraz [3]. The pseudo-code for the genetic algorithm of this paper is given in Algorithm 1.

A numerical example is presented to demonstrate the proposed model. Table 1 displays the optimum values of the parameters of the proposed ESD model. The optimum values of the ATS_0, $AATS$ and E(A) is illustrated in Table 1.

Table 1: The Optimal Parameters of the ESD of VSSI-MEWMA-DWL Control Chart.

n_1	n_2	h_1	h_2	γ	w_h	w_n	H
7	12	2.327	0.679	0.429	1.21	1.18	11.972

Table 2: The Optimal values of the ATS_0^*, $AATS$, E(A).

ATS_0	$\overline{AATS}$	$E(A)\$$
645	0.8123	5944

5. CONCLUSION AND FUTURE REASEARCH

In this study an ESD model of the VSSI-MEWMA-DWL control chart is proposed to consider multiple assignable causes to improve the efficiency of control charts and provide much faster detection of small and moderate process changes. In contrast to previous models we consider multiple assignable causes and more real-life aspects of manufacturing process than existing models in ESD of MEWMA control chart. Considering the problem as a multi-objective model and incorporating the preventive maintenance into the cost model can be a fruitful area for future research.

6. REFERENCES

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