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Dirac Equation and Ground State of Solvable Potentials: Supersymmetry Method

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Abstract The supersymmetry in non-relativistic quantum mechanics is applied as an algebraic method to obtain the solutions of the Dirac equation with spherical symmetry electromagnetic potentials. We show that some of the superpotentials related to ground state of the solvable potentials in non-relativistic quantum mechanics can be used for studying of the Dirac equation.

Keywords Dirac equation · Solvable potentials · Supersymmetric quantum mechanics

1 Introduction

Solutions of the Dirac equation with physical potential are very useful to investigate the relativistic effects [1]. The Dirac-Coulomb problem is an exactly solvable problem in relativistic quantum mechanics and its solution can be found in text books on quantum mechanics [2, 3]. The Dirac-Coulomb problem has also been studied via supersymmetry quantum mechanics (SUSY QM) [4–11]. In 1989, the relativistic Dirac-Oscillator potential was introduced by adding an off-diagonal linear radial term to the Dirac operator. The relativistic bound states spectrum and its eigen states were also obtained explicitly [12–16]. The Dirac-Mörse problem was solved successfully by Alhaidari [17, 18]. After that, a lot of shape-invariance potentials such as Dirac-Rosen-Mörse, Dirac-Eckart, Dirac-Pöschel-Teller and Dirac-Scarf potentials have been solved exactly [19, 20].

An effective approach for solving the Dirac equation for a charged spinor in spherically symmetric four component electromagnetic potential was introduced by Alhaidari [17–20]. In this method the gauge invariance and spherically symmetry of the electrostatic potential are used to arrive at the radial Dirac equation. In fact, an unitary transformation can be applied to Dirac equation such that the resulting second order differential equation becomes

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Schrödinger-like equation. There are a lot of methods for solving the Schrödinger-like equation, such as SUSY QM [11], shape invariance [21], point canonical transformation (PCT) [22] and etc.

In this work, in Sect. 2, we give a brief introduction for the approach which transform the Dirac equation with spherically symmetric potentials to two Schrödinger equations for upper and lower spinor fields. We factorize these obtained Hamiltonians in terms of two incretion and annihilation differential operators and then try to solve them by SUSY QM method. According to concepts of SUSY in non-relativistic quantum mechanics, the solvable potentials can be stated in terms of superpotentials by a Riccati equation and so we try to relate the well-known superpotentials to our obtained potentials in Dirac equation. It is seen that by inducing one assumption over electrostatic potential and gauge field of Dirac equation, a large class of solvable potentials can be used for solving Dirac equation. We show that the relativistic energy of all of these potentials are constant and the spinor fields of them are obtained by ground state of the solvable potentials. We complete our calculation for two examples in Sect. 3 and we give a table for other potentials. In Sect. 4, the paper ends with a brief conclusion.

2 Dirac Equation and Supersymmetry Method

In atomic units ($m = e = \hbar = 1$) and taking the speed of light $c = \alpha^{-1}$, the Hamiltonian for a Dirac spinor in the four-component electromagnetic potential $(A_0, \vec{A})$ can be written as [2]:

$$H = \begin{pmatrix} 1+\alpha A_0 & -i\alpha\vec{\sigma}.\nabla + \alpha\vec{\sigma}.\vec{A} \\ -i\alpha\vec{\sigma}.\nabla + \alpha\vec{\sigma}.\vec{A} & -1+\alpha A_0 \end{pmatrix}, \tag{2.1}$$

where α is fine structure constant and $\vec{\sigma}$ are the three 2×2 Pauli matrices [17]. Taking into consideration gauge invariance, the form of electromagnetic potential for static charge distribution with spherical symmetry is:

$$(A_0, \vec{A}) = (\alpha\upsilon(r), \hat{r}\omega(r)), \tag{2.2}$$

where $\hat{r}$ is radial unit vector, $\upsilon(r)$ is electrostatic potential and $\omega(r)$ is a gauge field does not contribute to the magnetic field. By substituting the two off-diagonal term $\alpha\vec{\sigma}.\vec{A}$ by $\pm i\alpha\vec{\sigma}.\vec{A}$ in (2.1), the Hamiltonian leads to the following two component radial Dirac equation [2]:

$$\begin{pmatrix} 1+\alpha^2\upsilon(r) & \alpha(\frac{k}{r}+\omega(r)) - \frac{d}{dr} \\ \alpha(\frac{k}{r}+\omega(r)) + \frac{d}{dr} & -1+\alpha^2\upsilon(r) \end{pmatrix}\begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = \varepsilon\begin{pmatrix} G(r) \\ F(r) \end{pmatrix}, \tag{2.3}$$

where ε is the relativistic energy and k is the spin-orbit coupling parameter defined as $k = \pm(j+\frac{1}{2})$ for $l = j \pm \frac{1}{2}$. Equation (2.3) gives two coupled first order differential equations for the radial spinor components. Separating differential term in (2.3); we will have:

$$\begin{pmatrix} \frac{d}{dr} & 0 \\ 0 & \frac{d}{dr} \end{pmatrix}\begin{pmatrix} F(r) \\ G(r) \end{pmatrix} + \begin{pmatrix} -(\frac{k}{r}+\omega(r)) & -\alpha\upsilon(r) \\ \alpha\upsilon(r) & (\frac{k}{r}+\omega(r)) \end{pmatrix}\begin{pmatrix} F(r) \\ G(r) \end{pmatrix} = \begin{pmatrix} 0 & \frac{1-\varepsilon}{\alpha} \\ \frac{1+\varepsilon}{\alpha} & 0 \end{pmatrix}\begin{pmatrix} F(r) \\ G(r) \end{pmatrix}. \tag{2.4}$$

Now, let D be the operator which diagonalizes the matrix that is appeared in the interaction term as [17–20]:

$$D = \begin{pmatrix} \cos\rho(r) & \sin\rho(r) \\ -\sin\rho(r) & \cos\rho(r) \end{pmatrix} \tag{2.5}$$

and multiplying it in the Dirac equation (2.4), with assumption $\rho(r) = constant$, we can obtain:

$$\left[\frac{d}{dr} + \left(\frac{k}{r} + \omega(r)\right)C + \frac{S}{\alpha}\right]\tilde{G}(r) = \left[\left(\frac{\varepsilon}{\alpha} - \alpha\upsilon(r)\right) + \left(\frac{C}{\alpha} - \left(\frac{k}{r} + \omega(r)\right)S\right)\right]\tilde{F}(r), \quad (2.6)$$

$$\left[-\frac{d}{dr} + \left(\frac{k}{r} + \omega(r)\right)C + \frac{S}{\alpha}\right]\tilde{F}(r) = \left[\left(\frac{\varepsilon}{\alpha} - \alpha\upsilon(r)\right) - \left(\frac{C}{\alpha} - \left(\frac{k}{r} + \omega(r)\right)S\right)\right]\tilde{G}(r), \quad (2.7)$$

where $S = \sin(2\rho)$ and $C = \cos(2\rho)$ and:

$$\begin{pmatrix} \tilde{F}(r) \\ \tilde{G}(r) \end{pmatrix} = D \begin{pmatrix} F(r) \\ G(r) \end{pmatrix}. \quad (2.8)$$

By defining the intertwining operators as:

$$A = \frac{d}{dr} + U(r), \quad (2.9)$$

$$A^+ = -\frac{d}{dr} + U(r), \quad (2.10)$$

where

$$U(r) = \left(\frac{k}{r} + \omega(r)\right)C + \frac{S}{\alpha}, \quad (2.11)$$

then we can write (2.6) and (2.7) in the following simple form:

$$A\tilde{G}(r) = (f_1 + f_2)\tilde{F}(r), \quad (2.12)$$

$$A^+\tilde{F}(r) = (f_1 - f_2)\tilde{G}(r), \quad (2.13)$$

where $f_1 = \frac{\varepsilon}{\alpha} - \alpha\upsilon(r)$ and $f_2 = \frac{C}{\alpha} - (\frac{k}{r} + \omega(r))S$. Multiplying A^+ and A in the left of (2.12) and (2.13) respectively, we have:

$$A^+A\tilde{G}(r) = \left[A^+(f_1 + f_2)\right]\tilde{F}(r) + (f_1 + f_2)A^+\tilde{F}(r), \quad (2.14)$$

$$AA^+\tilde{F}(r) = \left[A(f_1 - f_2)\right]\tilde{G}(r) + (f_1 - f_2)A\tilde{G}(r). \quad (2.15)$$

Now by assuming

$$A^+(f_1 + f_2) = 0, \quad (2.16)$$

$$A(f_1 - f_2) = 0, \quad (2.17)$$

then after some calculation, we obtain

$$f_1^2 - f_2^2 = constant \equiv \eta, \quad (2.18)$$

that is, the functions f_1 and f_2 are related to each other by a constant parameter. Hence, as f_1 and f_2 are written in terms of $\upsilon(r)$, $\omega(r)$ and ε, then, we can obtain a relation between the electrostatic potential $\upsilon(r)$, the gauge field $\omega(r)$ and the relativistic energy ε for the radial Dirac equation (2.1).

Also, it is easy to see that, the following eigenvalue equations can be obtained from the above equations

$$A^+A\tilde{G}(r) = \left(f_1^2 - f_2^2\right)\tilde{G}(r) = \eta\tilde{G}(r), \tag{2.19}$$

$$AA^+\tilde{F}(r) = \left(f_1^2 - f_2^2\right)\tilde{F}(r) = \eta\tilde{F}(r), \tag{2.20}$$

that is, by inserting of the constant parameter η, we have obtained two isospectral eigenvalue equations for the upper and lower spinor fields. Also, according to (2.9) and (2.10), we see that the Hamiltonians $H_1 = A^+A$ and $H_2 = AA^+$ have the following potentials

$$V_1(r) = U^2(r) - \frac{dU(r)}{dr}, \tag{2.21}$$

$$V_2(r) = U^2(r) + \frac{dU}{dr}. \tag{2.22}$$

For instance, for upper spinor field, (2.19) leads to following Schrodinger equation

$$-\frac{d^2\widetilde{G}(r)}{dr^2} + \left\{\left[\left(\omega(r) + \frac{k}{r}\right)^2 C^2 + \frac{2CS}{\alpha}\left(\omega(r) + \frac{k}{r}\right) + \frac{S^2}{\alpha^2}\right]\right.$$
$$\left. - \left(\frac{d\omega(r)}{dr} - \frac{k}{r^2}\right)C\right\}\widetilde{G}(r) = \eta\widetilde{G}(r). \tag{2.23}$$

On the other hand, from factorizing of H_1 and H_2 in terms of intertwining operators A and A^+, it is seem that, we can obtain the solution of the radial Dirac equation from the concept of supersymmetry in quantum mechanics. In fact, according to Ref. [11], the Hamiltonians H_1 and H_2 , except for the ground state, are the partner supersymmetry each other and so the spectrum of non-zero energies are the same and except to the ground state, the corresponding eigenfunctions can obtain form each other by acting the ladder or rasing operator on each other. In other word, except for the ground state, the isospectral states of the supersymmetry partner Hamiltonians can be obtained from each other by an algebraic method [11].

Hence, if we could find the electrostatic potential $\upsilon(r)$ and the gauge field $\omega(r)$ for radial Dirac equation (2.1), such that η in (2.18) became a constant, then the upper and lower spinor fields of (2.1) can be solved by supersymmetry method in non-relativistic quantum mechanics. Unfortunately, we could not find such functions and it is might only for $\eta = 0$, that is, only for ground state of the Hamiltonians (2.19) and (2.20). With this condition, these Hamiltonians are not the supersymmetry partner of each other and so, there eigenfunctions are not related to each other in concept of supersymmetric quantum mechanics.

On the other hand, it is easy to see that for $\eta = 0$, we will have

$$\upsilon(r) = \pm\left(\omega(r) + \frac{k}{r}\right)S, \tag{2.24}$$

$$\varepsilon = \pm C, \tag{2.25}$$

that is, the electrostatic potential $\upsilon(r)$ and the gauge field $\omega(r)$ are related to each other by (2.24) and also the relativistic energy ε becomes a constant, the constant C which obtained from the unitary transformation (2.5). Therefore the eigenfunctions of system or the upper and lower spinor fields can obtain from the ground state of (2.19) and (2.20). In other words, they should be solved from (2.19) and (2.20) by inserting $\eta = 0$:

$$-\frac{d^2\tilde{G(r)}}{dr^2} + V_1(r)\tilde{G(r)} = 0, \tag{2.26}$$

$$-\frac{d^2\tilde{F(r)}}{dr^2} + V_2(r)\tilde{F(r)} = 0. \tag{2.27}$$

Since each of the potentials $V_1(r)$ and $V_2(r)$, associated to upper and lower spinor fields, are related to superpotential $U(r)$, by Riccati equation (2.11), then the ground state with zero energy all of the solvable potentials in non-relativistic quantum mechanics can be used in our work. This means that, the Dirac equation for those spherically symmetry electrostatic potentials and gauge fields, which are related to each other by (2.24), have constant relativistic energy and the spinor fields are obtained from the ground state of potentials $V_1(r)$ and $V_2(r)$.

In this work, first we consider (2.26) for upper spinor field and try to compare all of the well-known solvable potentials, given in Ref. [22], to potential $V_1(r)$ which according to (2.23) and (2.24) could be written in terms of $\upsilon(r)$ or $\omega(r)$. Then, for each superpotential given in Ref. [22], we try to obtain $\upsilon(r)$ and $\omega(r)$ for radial Dirac equation (2.1) and so the ground state wave function of solvable potential will be as the upper spinor field of the Dirac equation with relativistic energy $\varepsilon = \pm C$. In other word, the ground spinor field of the radial Dirac equation (2.1) can be obtained from the ground state of the solvable potentials, given in Ref. [22], which the relativistic energy all of them are constant. The lower spinor field is also obtained from the upper by (2.12).

We can also consider the eigenvalue equation (2.27) for lower spinor and do similar formalism for obtaining $\upsilon(r)$ and $\omega(r)$ with constant relativistic energy for radial Dirac equation (2.1). In this case, the upper spinor field is obtained from the lower by (2.13).

In the next section, we calculate the above formalism for two examples, The Dirac-Shifted Oscillator and The Dirac-Coulomb, and end we give a table for a large class of electrostatic potentials and gauge fields which can be obtained from solvable potentials.

3 Examples

3.1 The Dirac-Shifted Oscillator

First, we consider the following superpotential according to Ref. [22]

$$U(r) = \frac{1}{2}\omega r - b, \tag{3.1}$$

where ω and b are the constant value. Corresponding to (3.1) and by using of (2.11) and (2.24), the electrostatic potential $\upsilon(r)$ and the gauge field $\omega(r)$ for radial Dirac equation (2.1) are calculated as:

$$\omega(r) = \pm\left(\frac{\omega}{2\sqrt{1-\alpha^2 b^2}}\right) r - \frac{k}{r}, \tag{3.2}$$

$$\upsilon(r) = \mp\left(\frac{\alpha\omega b}{2\sqrt{1-\alpha^2 b^2}}\right) r, \tag{3.3}$$

where

$$b = -\frac{S}{\alpha}. \tag{3.4}$$

Also, according to (2.25), the relativistic energy for the above electromagnetic potentials is obtained as:

$$\varepsilon = \pm(1 - \alpha^2 b^2)^{\frac{1}{2}}. \tag{3.5}$$

The upper spinor field is also obtained from the ground state of the given superpotential (3.1), which according to Ref. [22], is given as:

$$\widetilde{G}(r) = \exp\left(-\frac{1}{2}g^2\right), \tag{3.6}$$

where

$$g(r) = \left(\frac{\omega}{2}\right)^{\frac{1}{2}}\left(r - \frac{2b}{a}\right). \tag{3.7}$$

By using the upper field, we can calculate the lower spinor field from (2.12) too.

3.2 The Dirac-Coulomb

If we consider the superpotential as [22]:

$$U(r) = \frac{e^2}{2(l+1)} - \frac{(l+1)}{r}, \tag{3.8}$$

then, after some calculation we will have the following gauge field $\omega(r)$ and the electrostatic potential $\upsilon(r)$ for radial Dirac equation from (2.11) and (2.24)

$$\omega(r) = \mp\left(\frac{2(l+1)^2}{[4(l+1)^2 - \alpha^2 e^4]^{\frac{1}{2}}}\right)\frac{1}{r} - \frac{k}{r}, \tag{3.9}$$

$$\upsilon(r) = \pm\left(\frac{e^2}{[4(l+1)^2 - \alpha^2 e^4]^{\frac{1}{2}}}\right)\frac{l+1}{r}, \tag{3.10}$$

where

$$\frac{S}{\alpha} = \frac{e^2}{2(l+1)}. \tag{3.11}$$

We can also obtain the relativistic energy as:

$$\varepsilon = \pm\left(1 - \frac{\alpha^2 e^4}{4(l+1)^2}\right)^{\frac{1}{2}}, \tag{3.12}$$

and for the upper spinor, we have:

$$\widetilde{G}(r) = g^{l+1}(r)\exp\left(-\frac{g(r)}{2}\right), \tag{3.13}$$

Table 1 *Upper spinor* of the ground state and relativistic energy of the radial Dirac equation for electrostatic potential and gauge field, associated with superpotential and internal function. The *lower spinor* can be calculated from (2.12)

Type of Dirac-potentials	Superpotential $U(r)$ / Internal function $g(r)$	Gauge field $\omega(r)$ / Electrostatic potential $\upsilon(r)$	Ground state $\widetilde{G}(r)$ / Relativistic energy ε
Shifted oscillator	$\frac{1}{2}\omega r - b$	$\pm\frac{\omega}{2(1-\alpha^2 b^2)^{\frac{1}{2}}} r - \frac{k}{r}$	$\exp(-\frac{1}{2}g^2(r))$
	$g(r) = (\frac{\omega}{2})^{\frac{1}{2}}(r - \frac{2b}{a})$	$\mp[\frac{b\omega}{2(1-\alpha^2 b^2)^{\frac{1}{2}}}]r$	$\varepsilon = \pm(1-\alpha^2 b^2)^{\frac{1}{2}}$
	$\omega = constant$		$S = -\alpha b$
Three- dimensional oscillator	$\frac{1}{2}\omega r - \frac{l+1}{r}$	$\pm(\frac{\omega}{2})r$ $k = -(l+1)$	$g^{\frac{(l+1)}{2}}(r)\exp(-\frac{g(r)}{2})$
	$g(r) = \frac{1}{2}\omega r^2$	0	$\varepsilon = \pm 1$
	$\omega = constant$		$S = 0$
Coulomb	$\frac{e^2}{2(l+1)} - \frac{l+1}{r}$	$\mp(\frac{2(l+1)^2}{[4(l+1)^2-\alpha^2 e^4]^{\frac{1}{2}}})\frac{1}{r} - \frac{k}{r}$	$g^{l+1}(r)\exp(-\frac{g(r)}{2})$
	$g(r) = (\frac{e^2}{n+l+1})r$	$\pm(\frac{e^2}{[4(l+1)^2-\alpha^2 e^4]^{\frac{1}{2}}})\frac{l+1}{r}$	$\varepsilon = \pm(1 - \frac{\alpha^2 e^4}{4(l+1)^2})^{\frac{1}{2}}$
			$S = \frac{\alpha e^2}{2(l+1)}$
Mörse	$A - B\exp(-ar)$	$\mp\frac{B}{(1-\alpha^2 A^2)^{\frac{1}{2}}}\exp(-ar) - \frac{k}{r}$	$g^{s-n}(r)\exp(-\frac{g(r)}{2})$
	$g(r) = \frac{2B}{a}\exp(-ar)$	$\pm\frac{AB}{(1-\alpha^2 A^2)^{\frac{1}{2}}}\exp(-ar)$	$\varepsilon = \pm(1-\alpha^2 A^2)^{\frac{1}{2}}$
	$a = constant$		$S = \alpha A$
Scarf-II (hyperbolic)	$A\tanh(ar) + \frac{B}{\cosh(ar)}$	$\pm(A\tanh(ar) + \frac{B}{\cosh(ar)}) - \frac{k}{r}$	$(1-g^2(r))^{-\frac{1}{2}} \times \exp(-\lambda\tan^{-1}(-ig(r)))$
	$g(r) = i\sinh(ar)$	0	$\varepsilon = \pm 1$
	$a = constant$		$S = 0$
Rosen-Mörse-II(hyperbolic)	$A\tanh(ar) + \frac{B}{A}$	$\pm\frac{A^2}{(A^2-\alpha^2 B^2)^{\frac{1}{2}}}\tanh(ar) - \frac{k}{r}$	$(1-g(r))^{\frac{(s-n+\overline{a})}{2}} \times (1+g(r))^{\frac{(s-n-\overline{a})}{2}}$
	$B < A^2$		
	$g(r) = \tanh(ar)$	$\pm\frac{AB}{(A^2-\alpha^2 B^2)^{\frac{1}{2}}}\tanh(ar)$	$\varepsilon = \pm(1 - \frac{\alpha^2 B^2}{A^2})^{\frac{1}{2}}$
	$a = constant$		$S = \frac{\alpha B}{A}$
Eckart	$\frac{B}{A} - A\coth(ar)$	$\mp\frac{A^2}{(A^2-\alpha^2 B^2)^{\frac{1}{2}}}\coth(ar) - \frac{k}{r}$	$(g(r)-1)^{\frac{-(s+n-\overline{a})}{2}} \times (g(r)+1)^{\frac{-(s+n+\overline{a})}{2}}$
	$B > A^2$		
	$g(r) = \coth(ar)$	$\pm\frac{AB}{(A^2-\alpha^2 B^2)^{\frac{1}{2}}}\coth(ar)$	$\varepsilon = \pm(1 - \frac{\alpha^2 B^2}{A^2})^{\frac{1}{2}}$
	$a = constant$		$S = \frac{\alpha B}{A}$
Generalized Pöchl-Teller	$A\coth(ar) - \frac{B}{\sinh(ar)}$	$\pm(A\coth(ar) - \frac{B}{\sinh(ar)}) - \frac{k}{r}$	$(g(r)-1)^{\frac{(\lambda-s)}{2}} \times (g(r)+1)^{\frac{-(\lambda+s)}{2}}$
	$A < B$		
	$g(r) = \cosh(ar)$	0	$\varepsilon = \pm 1$
	$a = constant$		$S = 0$
Rosen-Mörse-I (trigonometric)	$A\cot(ar) - \frac{B}{A}$	$\pm\frac{A^2}{(A^2-\alpha^2 B^2)^{\frac{1}{2}}}\cot(ar) - \frac{k}{r}$	$(g^2(r)-1)^{\frac{(s-n)}{2}} \times \exp(\overline{a}ar)$
	$0 \le ar \le \pi$		
	$g(r) = -i\cot(ar)$	$\pm\frac{AB}{(A^2-\alpha^2 B^2)^{\frac{1}{2}}}\cot(ar)$	$\varepsilon = \pm(1 - \frac{\alpha^2 B^2}{A^2})^{\frac{1}{2}}$
	$a = constant$		$S = -\frac{\alpha B}{A}$
Scarf-I (trigonometric)	$-A\cot(ar) + B\csc(ar)$	$\pm(-A\cot(ar) + B\csc(ar)) - \frac{k}{r}$	$(1-g(r))^{\frac{(s-\lambda)}{2}} \times (1+g(r))^{\frac{(s+\lambda)}{2}}$
	$-\frac{\pi}{2} \le ar \le \frac{\pi}{2}$		
	$g(r) = \cos(ar)$	o	$\varepsilon = \pm 1$
	$a = constant$		$S = 0$

where

$$g(r)=\left(\frac{e^2}{n+l+1}\right)r. \tag{3.14}$$

The lower spinor field is calculated by (2.12) too. In Table 1 we have calculated the other cases of the superpotentials in Ref. [22].

4 Conclusion

We have presented an idea for connecting the ground state of the exactly solvable potentials in non-relativistic quantum mechanics to the solution of the radial Dirac equation with spherical symmetry electromagnetic potentials. By using the supersymmetry method in non-relativistic quantum mechanics for each superpotential of the solvable potentials, we have obtained the corresponding electrostatic potential and gauge field for radial Dirac. Then, we have shown that the spinor fields of the obtained electromagnetic potentials can be calculated from the ground state of solvable potentials and the relativistic energy for all of them is constant too.

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