

TPS-RPM Registration in Application of MS Lesions MRI Brain Images

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Abstract— Correspondence problem and transformation function are the main challenges in registration so we try to solve these problems concurrently to achieve reasonable and optimum results. We employ non-rigid registration as a non-rigid point matching problem. We have also used the TPS-RPM algorithm. In addition the binary correspondence improved to fuzzy correspondence through the meaning of soft-assign and deterministic annealing.

TPS-RPM is applied on MS lesion in MRI images which achieved consecutive weeks to follow up the fluctuation. We compare the iterated closest point ICP's result to TPS-RPM in order to demonstrate the power of the TPS-RPM. Our comparison is done by evaluated metrics such as Dice similarity measure and accuracy obtained 0.87 and 0.88 in ICP, respectively. For TPS-RPM the Dice similarity measure and accuracy are 0.9 and 0.89, respectively.

Keywords: *non-rigid matching, Registration, TPS-RPM, MS lesion.*

I. INTRODUCTION

Image registration problems often spread in the medical imaging and computer vision domains [1]. The registration is the process of overlaying two or more images of the same scene taken at different times, from different viewpoints, and by different sensors [2].

In cases of 2D shapes, it can be done manually by an expert. However, in cases of 3D shapes, achieving the corresponding landmarks is a time-consuming. Furthermore, various factors such as the noise, outliers, and deformations can make the problem even more difficult. Therefore, applying accurate correspondences between two shapes is considered as one of the fundamental challenges [3].

The correspondence problem can be solved by three manners [4]. In the first type, two shapes are overlaid and nearest neighbours are considered as corresponding points. In the second type, the structures like lines, curves and surfaces are employed as features to find corresponding points. In the third type the registration and correspondence problems are

solved simultaneously. Registration algorithms are categorized into rigid and non-rigid methods. In rigid methods only global deformations of the shape are considered, so they are applied to register rigid objects such as bones. In contrast, both global and local variations are considered in non-rigid methods [4].

In second type, the features should be selected accurately and also a large number of points are needed to have sufficiently smoothed curves, so deformations of the two shapes should be large.

Finding corresponding landmarks in whole points is very time-consuming and sometimes impossible. In the other hand, always noise and outliers are effected the feature selection step. The third type solves both the correspondence and transformation problems simultaneously. An iterative method is established to find corresponding point in one step and employed the landmarks to find transformation function.

Typically, registration is required in medicine such as combining computer tomography (CT) and NMR data to obtain more complete information about the patient, monitoring tumor growth, treatment verification, comparison of the patient's data with anatomical atlases [1].

The multiple sclerosis (MS) is one of the progressive diseases, that is firstly affected the white matter (WM) of the central nervous system [5]. Quantitative evaluation of the morphological and structural changes in the brain MR images in association with clinical conclusion can provide more accurate appraisal of the disease progress. MRI has superiority to the other imaging techniques for assess the progression of the disease and to evaluate the effect of drug therapy.

The radiological criteria for MS include the number of lesion on the MRI, their locations and their sizes. So these information is employed to check the progression of MS lesions and the effect of drug treatments [6]. To achieve some information about the progress of multiple sclerosis lesions registration of time series MR images can be useful. Human experts can help to identify and estimate properties of them but is extremely time consuming. So, in this paper we proposed a semiautomatic method to register MRI brain images using TPS-RPM method.



II.PREVIOUS WORKS

As before mentioned, first class of corresponding problem uses only the spatial transformation to solve the registration. The method of moments [7] is employed as a classical technique which is applied to solve for the transformation without ever introducing the notion of correspondence. In other word, moment analysis is exerted to determine the center of mass and the principal axis and they are used as criterion for aligning the point sets. The tree searches [8], the Hausdorff distance [9], geometric hashing [10] are another method for solving the corresponding problem. These methods can create reasonable result in rigid transformation cases only.

In second class, registration is done by solving the correspondence problem and includes three methods. Feature-based is the first type which collects the feature points in structures such as lines, curves, or surfaces using object parameterization. Curves and surfaces should be first fitted to extracted features in the images. Feature extraction and Curve fitting are almost difficult for the noisy data or the complex shapes. Also, most of these types often fail in multiple curves or partially occluded curves.

The basic idea in the second type is non-rigidly deforming in shapes and different points at different locations can be assigned different attributes depending on their ways of movement. Distinguish the points and estimate their correspondences through these movement attributes.

In [11] the modal matching method utilizes a mass and stiffness matrix. These obtained by the Gaussian of the distances between point features in either point-set. After computing the mode shape vectors as the eigenvectors, the correspondence is achieved by comparing each point's relative participation in the eigen-modes. Generally outliers can be effected the quality of this method because they can cause the large deformation.

The third type recasts point matching as inexact, weighted graph matching [12]. The spatial relationships between the points in each set are used to constrain the search for the correspondences. Also, there is no direct relationship between the deformable model and the graph used. The graph definition is also a common problem because interrelationships, attributes and link type information can be notoriously brittle and context dependent.

As regards, solving the correspondence and the transformation separately is difficult task. Third class, tries to estimate the non-rigid transformation and the correspondence concurrently. In other words, knowledge of a sensible spatial transformation can be help in the search for correspondence [1]. An iterative method which finds corresponding point in one step and employs the landmarks can be used to estimate transformation function [4].

The well-known Iterative Closest Point (ICP) [13] is a very fast method and guaranteed to converge to a local minimum. ICP uses rigid transformation and iteratively search for binary correspondences between landmarks through nearest-neighbour relationships in each step. It requires that the initial positions of the shapes to be sufficiently close to each other which is not always possible. ICP cannot create a valid result

in the case of non-rigid transformations and also not usually guarantee that the correspondences are one-to-one so it sensitive to outliers [14]. To refine the performance of ICP, fuzzy correspondence is introduced instead binary.

In probabilistic approach [15] point matching is modeled as a probability density estimation problem by a Gaussian mixture model (GMM). In order to solve the matching problem, Expectation Maximization (EM) algorithm is employed. In the Estep, the correspondence is estimated under the given transformation, and in the M-step the transformation is updated based on the present estimate of the correspondence. To control the outliers a uniform distribution in addition to the Gaussian mixture model is considered. Disadvantage of the probabilistic approaches is that they do not enforce one-to-one correspondence. To handle this problem, in [14,16] deterministic annealing and soft-assign, were used to solve the joint optimization problem. One-to-one correspondence is done through the soft-assign used within deterministic annealing.

The non-rigid transformation can be parameterized by the Thin Plate Spline (TPS). It was developed in Wahba [17] as a general purpose spline tool which generates a smooth functional mapping for supervised learning. The smoothness measure is defined as the sum of the squares of the second derivatives of the mapping function over the space. Bookstein [18] firstly use of TPS to generate smooth spatial mappings between two sets of points with known one-to-one correspondences in medical images. The disadvantage of TPS is limitation of known correspondence. Also Robust Point Matching (RPM) is similar to the EM algorithm and easy to implement. RPM contains a dual update process embedded within an annealing scheme

In this paper, we present the TPS-RPM algorithm. Because of using the fuzzy correspondence instead of the binary through the meaning of soft-assign and deterministic annealing, final results are improved. We compare the iterated closest point ICP's result to TPS-RPM in order to demonstrate the power of the TPS-RPM.

III.PROPOSED METHOD

In first step we should remove the skull in MR brain images. In other words brain extraction or skull striping. The brain extraction tool (BET) is employed to remove the skull from an image, leaving only the region occupied by actual brain tissue. It segments these by using the dark space between the skull and brain, is filled by the CSF.

In second step, an initial registration is applied on each pair of our dataset to find geometry transformation between them (Fig.1). Then, images are registered via TPS-RPM in order to measure the size of lesion. The flowchart of our proposed method also is shown in Fig.2.

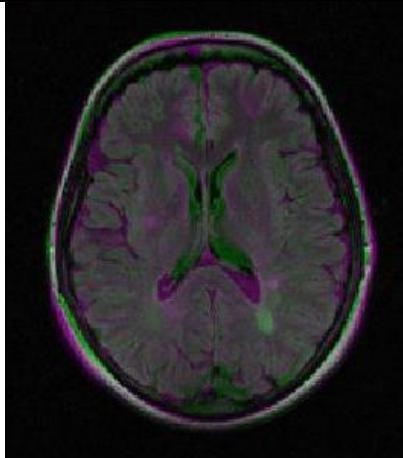


Fig.1. The initial registration (green is the first month as a reference slice and violet is the 7th month later as sensed).

At first binary and fuzzy linear assignments are mentioned as introduction to introduce TPS-RPM method.

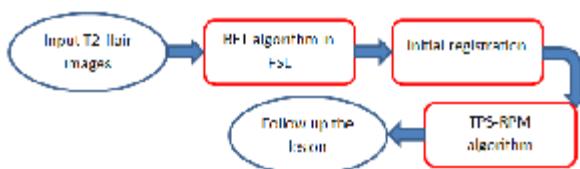


Fig.2.The flowchart of proposed method.

3.1. A binary linear assignment

The non-rigid transformation is represented by general function f in Eq. 1.

$$\min_{Z,f} E(Z,f) = \min_{Z,f} \sum_{i=1}^N \sum_{a=1}^K z_{ai} \|x_i - f(v_a)\|^2 + \lambda \|Lf\|^2 - \xi \sum_{i=1}^N \sum_{a=1}^K z_{ai} \quad (1)$$

Subject to the following constraints [19]

$$\begin{cases} z_{ai} \in \{0,1\}, & \text{for } a = 1,2,\dots,K+1; i = 1,2,\dots,N+1. \\ \sum_{i=1}^{N+1} z_{ai} = 1, & \text{for } i = 1,2,\dots,K. \\ \sum_{a=1}^{K+1} z_{ai} = 1, & \text{for } a = 1,2,\dots,N. \end{cases} \quad (2)$$

Generally, we want to minimize the following binary linear assignment-least squares energy function. In Eq. 1, a point $\{v_a, a = 1,2,\dots,K\}$ is mapped to a new location $u_a = f(v_a)$. An operator L is the smoothness regularization, so smoothness measure $\|Lf\|^2$ is represented by Eq. 3.

$$\|Lf\|^2 = \iint \left[\left(\frac{\partial^2 f}{\partial u^2} \right)^2 + 2 \left(\frac{\partial^2 f}{\partial u \partial v} \right)^2 + \left(\frac{\partial^2 f}{\partial v^2} \right)^2 \right] du dv \quad (3)$$

Regarding to the goal of correspondence problem, the point sets as closely as possible should be matched while rejecting a reasonable fraction of the points as outliers. We define the benefit matrix to specify the outlier and also the correspondence points. The matrix Z or $\{z_{ai}\}$ is the binary correspondence matrix (Fig.1) [14] which involves two parts. The inner $N \times K$ part of Z defines the correspondence. The

correspondence is guaranteed one-to-one because of the row and column summation constraints. If sensed point v_a corresponds to a referenced point $\{x_i, i = 1,2,\dots,N\}$, then $z_{ai} = 1$ and otherwise $z_{ai} = 0$.

The second term is smoothness constraint on the transformation. The third term is the robustness control term preventing rejection of too many points as outliers. Also the parameters λ and ξ are the weight parameters that balance these terms.

z_{ai}	x_1	x_2	x_3	x_4	outlier
v_1	1	0	0	0	0
v_2	0	1	0	0	0
v_3	0	0	0	0	1
outlier	0	0	1	1	

Fig. 3. An example of the binary correspondence matrix

We are face on two main parts in optimization problem of Eq.1, a linear assignment discrete problem on the correspondence and a least-squares continuous problem on the transformation. The non-rigid point matching problem becomes difficult if both of mentioned problems supposed to solve simultaneously. In addition, even if such an approach is employed and binary one-to-one correspondences (and outliers) solved at each step is not meaningful when the transformation is far away from the optimal solution. A method like EM algorithm with dual-step and also two techniques such as soft-assign and deterministic annealing are employed to achieve reasonable correspondence and transformation concurrently.

Soft-assign method is applied to slack the binary correspondence to be a continuous valued matrix. The continuous matrix can save the fuzzy and continues value so the correspondence improves gradually. Also the optimization is done without jumping around in the space of binary matrices (and outliers). This fuzziness directly is controlled by adding an entropy term ($T \sum_{i=1}^{N+1} \sum_{a=1}^{K+1} m_{ai} \log m_{ai}$) in deterministic annealing meaning. The parameter T is called the temperature parameter. By reducing T , the energy function is minimized through a process similar to physical annealing [1].

3.2. A fuzzy linear assignment

The original binary assignment by soft-assign and deterministic annealing is converted to the fuzzy assignment energy function (Eq.2) [14, 16].

$$\begin{aligned} E(M,f) = & \sum_{i=1}^N \sum_{a=1}^K m_{ai} \|x_i - f(v_a)\|^2 + \lambda \|Lf\|^2 + \\ & T \sum_{i=1}^N \sum_{a=1}^K m_{ai} \log m_{ai} - \xi \sum_{i=1}^N \sum_{a=1}^K m_{ai} \end{aligned} \quad (4)$$

Subject to



$$\begin{cases} 0 \leq m_{ai} \leq 1, \text{ for } i = 1, 2, \dots, K+1; a = 1, 2, \dots, N+1 \\ \sum_{i=1}^{K+1} m_{ai} = 1, & \text{for } a = 1, 2, \dots, N. \\ \sum_{a=1}^{N+1} m_{ai} = 1, & \text{for } i = 1, 2, \dots, K. \end{cases} \quad (5)$$

When the temperature T reaches zero, the fuzzy correspondence becomes binary. Also, $\sum_{i=1}^{N+1} m_{ai} = 1$ for $a \in \{1, \dots, K\}$ and $\sum_{a=1}^{K+1} m_{ai} = 1$ for $i \in \{1, \dots, N\}$.

The parameter ξ represents the percentage of outliers in both point-sets. Since there is no special way to compute it, the outlier variable is considered as a cluster center with a very large variance (T_0) and each point that cannot be matched is placed in this cluster. The outlier cluster for each point-set is placed at the center of mass.

The parameter λ is set to control the prior smoothness term. If λ is selected large value, the range of non-rigidity of the transformation is limited and in small value of λ the algorithm becomes unstable. Reducing λ using an annealing schedule can be improved the results [3].

3.3. The TPS-RPM algorithm

The TPS-RPM algorithm solves the optimization problem in (4) and (5) as follow, which is similar to the EM algorithm involving a dual update process within an annealing. The two-step update is briefly summarized.

Step 1: Update the correspondence m by fixing the transformation f . For correspondence points $a=1, 2, \dots, K$ and $i=1, 2, \dots, N$.

$$m_{ai} = \frac{1}{T} \exp\left(-\frac{(x_i - f(v_a))^T(x_i - f(v_a))}{2T}\right) \quad (6)$$

And for outliers possibilities in $a=K+1$ and $i=1, 2, \dots, N$.

$$m_{K+1,i} = \frac{1}{T_0} \exp\left(-\frac{(x_i - v_{K+1})^T(x_i - v_{K+1}))}{2T_0}\right) \quad (7)$$

And for outliers possibilities in Y , $a=1, 2, \dots, K$ and $i=N+1$.

$$m_{a,N+1} = \frac{1}{T_0} \exp\left(-\frac{(x_{N+1} - f(v_a))^T(x_{N+1} - f(v_a))}{2T_0}\right) \quad (8)$$

Where v_{K+1} and x_{N+1} are outlier cluster centers and T_0 is a very large variance. Iterated row and column normalization algorithm is then applied to the m in (6)–(8) to satisfy the constraints (5) until convergence is reached,

$$m_{ai} = \frac{m_{ai}}{\sum_{b=1}^{K+1} m_{bi}}, \quad i = 1, 2, \dots, N \quad (9)$$

$$m_{ai} = \frac{m_{ai}}{\sum_{j=1}^{N+1} m_{aj}}, \quad a = 1, 2, \dots, K \quad (10)$$

Step 2: Update the transformation f by fixing the correspondence m . The TPS-RPM algorithm simplifies this step by minimizing the following standard TPS bending energy function, a simplified form of Eq. (5):

$$\min_f E(f) = \min_f \sum_{a=1}^K \|y_a - f(v_a)\|^2 + \lambda T \|Lf\|^2 \quad (11)$$

Where,

$$y_a = \sum_{i=1}^N m_{ai} x_i \quad (12)$$

Where y_a is considered as estimated positions of the point-set $\{x_i\}$ that corresponds to $\{v_a\}$. Creating smooth mapping between two sets of points often is one of the challenges in spline theory. Because of non-rigidity property, there are an infinite number of ways to map one point-set onto another.

Once non-rigidity is allowed, there are an infinite number of manners to map one point-set onto another. Choosing a specific smoothness measure, can be used as criterion to control the mapping behavior. The space integral of the square of the second order derivatives of the mapping function is employed as well-known smoothness measure. The thin-plate spline is formed based on this measure according to Eq.13.

$$E_{TPS}(f) = \sum_{a=1}^K \|y_a - f(v_a)\|^2 + \lambda \int \int \left[\left(\frac{\partial^2 f}{\partial x^2} \right)^2 + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 + \left(\frac{\partial^2 f}{\partial z^2} \right)^2 \right] dx dy \quad (13)$$

Through minimizing mentioned energy function, the corresponding point-sets $\{y_a\}$ and $\{v_a\}$ is fitted by a mapping function $f(v_a)$. Also y_a can be defined as a vector $(1, y_{ax}, y_{ay})$ and the vector $\varphi(v_a)$ is described the TPS kernel where each entry $\varphi_b(v_a) = \|v_b - v_a\|^2 + \log\|v_b - v_a\|$. A unique minimizer f which includes two matrices d and w is written as Eq.14.

$$f(v_a, d, w) = v_a \cdot d + \varphi(v_a) \cdot w \quad (14)$$

Where d and w represents the affine transformation and warping coefficient matrix, respectively also if the solution for f in Eq.14 is substituted into Eq.13, the TPS energy function becomes

$$E_{TPS}(d, w) = \|Y - Vd - \emptyset w\|^2 + \lambda \text{trace}(w^T \varphi w) \quad (15)$$

The point coordinates y_a and v_a are concatenated in Y and V , respectively. One of the very nice properties of the TPS is decompose the transformation into a global affine and a local non-affine. A QR decomposition [43] is employed to separate the affine and non-affine warping space in Eq.16.

$$V = [Q_1 Q_2] \begin{pmatrix} R & \\ & 0 \end{pmatrix} \quad (16)$$

Where Q_1 and Q_2 are orthonormal matrices and the matrix R is upper triangular. From Eq.16 and also Eq.15,

$$\begin{aligned} E_{TPS}(\gamma, d) &= \|Q_2^T Y - Q_2^T \emptyset Q_2 \gamma\|^2 + \|Q_1^T Y - R d - Q_1^T \emptyset Q_2 \gamma\|^2 + \\ &\quad \lambda \gamma^T Q_2^T \emptyset Q_2 \gamma \end{aligned} \quad (17)$$

$$w = Q_2 \gamma$$

The least-squares energy function in (15) can be first minimized w.r.t γ and then w.r.t. d . The final solution for w and d are:

$$\hat{w} = Q_2 \hat{\gamma} = Q_2 (Q_2^T \emptyset Q_2 + \lambda I_{(K-D-I)})^{-1} Q_2^T Y \quad (18)$$

$$\hat{d} = R^{-1}(Q_1^T V - \emptyset \hat{w}) \quad (19)$$

The minimum value of the TPS energy function is computed in the optimum (\hat{w}, \hat{d}) , so bending energy is defined as Eq.20. The Pseudo-code of the TPS-RPM Algorithm is given in Algorithm 1.

$$E_{bending} = \lambda \text{trace}[Q_2(Q_2^T \emptyset Q_2 + \lambda I_{(K-D-I)})^{-1} Q_2^T Y Y^T] \quad (20)$$

	TPS-RPM	ICP
#01	0.91	0.89
#05	0.88	0.88
#07	0.9	0.89
#10	0.89	0.87
#12	0.9	0.88

Algorithm 1

Initialize parameters T, λ .
Initialize parameters M, d and w .

Apply Deterministic Annealing algorithm.
Update correspondence matrix M using (6)–(8).
Update transformation parameters (d, w) using (11).
Decrease T and λ .
End

IV.RESULTS AND DISSCUSSION

In this paper MRI brain images were used to follow up the MS lesions via non-rigid registration (TPS-RPM). We used the MRI T2-flair which achieved in Kourosh Medical diagnostic imaging center in Tehran, Iran.

We implemented our method in MATLAB environment and ran in a personal computer with AMD-Radeon(TM) HD 6630M, 2.3GHz CPU and 6GB dynamic RAM, running Windows 7.

Also in order to remove the skull from an image, the brain extraction tool (BET) is employed [20]. The TPS-RPM is applied to our database and typical steps are shown in Fig.3. ITK-SNAP [21] and OSIRIS [22] were used to Visualization of the images.

To compare our result in terms of accuracy, ICP results are collected. Dice similarity is measured to compare our method with gold standard.

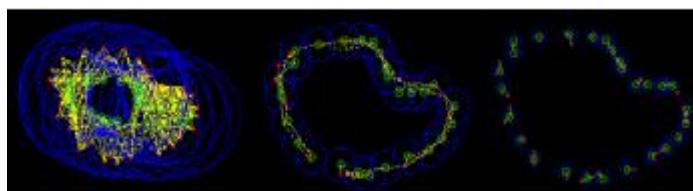


Fig.4. The steps of TPS-RPM procedure from left to right.

Table 1.COMPARISON OF ACCURACY OF TPS-RPM and ICP METHOD.

	Method

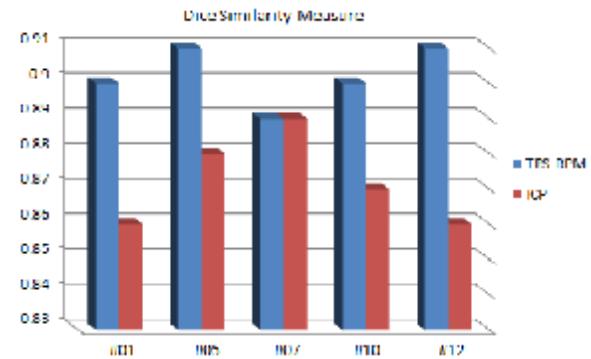


Fig.5. Typical Dice similarity measures for different images.

VI.CONCLUSION AND FUTURE WORKS

In this paper, we proposed a TPS-RPM registration method to follow up the MS lesion in T2-flair MRI images. As seen in Fig.5 and Table 1 our method was superior to the ICP algorithm.

In future we are going to implement our method to the most of different dataset (another type of modalities) and also employ an optimum clustering to classify the types of lesions.

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