Int. J. Electron. Commun. (AEÜ) 89 (2018) 153–159

Contents lists available at ScienceDirect

Int. J. Electron. Commun. (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Regular paper

Separable transmit beampattern design for MIMO radars with planar colocated antennas

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ARTICLE INFO

Keywords:
MIMO radar
Transmit beampattern
Uniform rectangular arrays

ABSTRACT

Multiple-input multiple-output (MIMO) radar transmit beampattern design for one-dimensional arrays has been widely studied in literatures. In this paper, transmit beampattern design is considered for two-dimensional (2D) arrays. As the size of the array is increased, the computational complexity and time for pattern optimization are increased drastically. To overcome these problems, we introduced the conditions upon which the 2D beampattern design for uniform rectangular arrays (URA) can be achieved via the product of two perpendicular transmit beampatterns of uniform linear arrays (ULA). The transmit beampattern design is accomplished under special characteristics such as minimum integrated sidelobe level or special 3 dB beamwidth in azimuth and elevation with much lower computation time.

1. Introduction

MIMO radar is a concept that has attracted many researches to it. In general, a MIMO radar is a radar architecture that employs a collection of antennas as transmitter and receiver and each antenna emits its own waveform apart from others [1,2]. According to the layout of the antennas, two major types of MIMO radars are assumed; MIMO radar with widely separated antennas and MIMO radar with colocated antennas. MIMO radars with widely separated antennas capture the spatial diversity of the target cross-section and improve the ability of detection and estimation of target parameters. Many studies have been conducted to show the improvements in target detection and localization and the Cramer-Rao lower bound (CRLB) of the target parameters estimation [2–8].

The colocated MIMO radars offer the improved angular resolution along with improved detection and parameter identifiability, increased number of detectable targets and extended array aperture by virtual sensors [1,9,10].

Waveform design is another topic in MIMO radar in order to achieve enhanced specification such as optimizing the radar ambiguity function [11–14] or improving the target detection performance or target parameters estimation performance [4–6,15,16].

Beampattern designing is an important topic in most systems that utilize array antennas [17–23]. In MIMO radars with colocated antennas, the transmit beampattern is designed to focus the transmit energy in the desired sector and improve the target detection performance, direction of arrival estimation, interference rejection and so on. In some literature, the transmit beampattern is designed for ULA [24–29] and in a few ones, the transmit beampattern for planar arrays such as URA is studied [30,31]. Some literature focuses on partitioning the planar array and introduces phased-MIMO radars in which coherent transmit gain is achievable and cost reduction for practical radars is some how possible [32–35]. In transmit beam pattern design approaches, some researchers try matching the beampattern to a specific pattern under minimum mean square error. Some others design the pattern maintaining minimum sidelobe level or specific beamwidth. Transmit beampattern is carried out by designing the transmit waveforms' covariance matrix or weights matrix. Transmit beampattern design for large URAs costs much more computation time and memory due to the extensive variables, as the optimization must be performed on a two-dimensional angular grid (azimuth and elevation) and for a large number of antennas, the simulation time will be cumbersome.

In this paper, we focus on designing the transmit beampattern for MIMO radars with symmetric planar arrays under different constraints such as uniform power distribution over antennas and specific 3 dB beamwidth. As it is mentioned before, designing such beampatterns for large planar antennas is time-consuming, so we derive the conditions under which the two-dimensional beampattern can be made via the product of two one-dimensional patterns in azimuth and elevation directions. With this beampattern separation, beampattern design for large planar arrays can be evaluated with much lower computational time.

In this paper, lowercase italic letters are used to denote scalers, uppercase letters in bold denote matrices and lowercase letters in bold

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https://doi.org/10.1016/j.aeue.2018.03.028
Received 30 September 2017; Accepted 21 March 2018

denote vectors. Superscripts signs of T and H represent the transpose and Hermitian operators of vectors and matrices respectively. $vec\{.\}$ is the vectorization operation on matrices, $diag\{.\}$ denote the diagonal of a square matrix, $tr\{.\}$ is the trace of a square matrix, $\|.\|$ is the Euclidean norm of a vector, $\otimes$ is the Kronecker product.

The remainder of this paper is organized as follows. Section 2 introduces the system model for a MIMO radar with planar array transmit antenna. In Section 3, the problem formulation for the transmit-beampattern-design problem is developed. In Section 4, the computational complexity of the proposed method is addressed. Simulation examples are given in Section 5 and the conclusion is made in Section 6.

2. System model

Consider a colocated URA MIMO radar system in which there are N antennas in each row and M antennas in each column (Fig. 1), which are separated by d_x and d_y in x and y directions, respectively. The array is assumed to be in the xy plane. Assume each transmit antenna emits a weighted sum of K independent orthonormal baseband waveforms which are expressed by $\boldsymbol{\phi}(t) = [\phi_1(t),\phi_2(t),...,\phi_K(t)]^T$ and $\int_0^{T_p} \phi_i(t)\phi_j^*(t)dt = \delta(i-j)$, $i,j = 1,2,...,K$ where T_p is the pulse duration of radar. The weights are described by the matrix $\mathbf{W} = [\mathbf{w_1}, \mathbf{w_2},\cdots,\mathbf{w_k}]$ where $\mathbf{W} \in \mathbb{C}^{MN\times K}$. So the signals transmitted by the MN antennas can be described by [24]:

$$\mathbf{s}(t) = \mathbf{W}\boldsymbol{\phi}(t) \tag{1}$$

The covariance matrix of the transmitted signals is:

$$\mathbf{R} \triangleq \int_0^{T_p} \mathbf{s}(t)\mathbf{s}^{\mathbf{H}}(t)dt = \int_0^{T_p} \mathbf{W}\boldsymbol{\phi}(t)\boldsymbol{\phi}^H(t)\mathbf{W}^{\mathbf{H}}dt = \mathbf{W}\mathbf{W}^{\mathbf{H}} \tag{2}$$

and $Rank\{\mathbf{R}\} = K$. In general $K \leqslant MN$ and as shown in [30], K can be chosen according to the number of effective eigenvalues of the following matrix:

$$\mathbf{A}(\theta,\varphi) = \int_\Theta \int_\Phi \mathbf{a}(\theta,\varphi)\mathbf{a}^H(\theta,\varphi)d\varphi d\theta \tag{3}$$

where $\mathbf{a}(\theta,\varphi)$ is the steering vector which will be defined later.

Regarding Fig. 1, the steering vectors in x-direction is defined as:

$$\mathbf{u}(\theta,\varphi) = \left[e^{-j\frac{2\pi}{\lambda}d_x\left(-\frac{M-1}{2}\right)\sin\theta\cos\varphi}, e^{-j\frac{2\pi}{\lambda}d_x\left(-\frac{M-3}{2}\right)\sin\theta\cos\varphi},...,e^{-j\frac{2\pi}{\lambda}d_x\left(\frac{M-1}{2}\right)\sin\theta\cos\varphi}\right]^T \tag{4}$$

and the steering vector in y-direction is defined as:

$$\mathbf{v}(\theta,\varphi) = \left[e^{-j\frac{2\pi}{\lambda}d_y\left(-\frac{N-1}{2}\right)\sin\theta\sin\varphi}, e^{-j\frac{2\pi}{\lambda}d_y\left(-\frac{N-3}{2}\right)\sin\theta\sin\varphi},...,e^{-j\frac{2\pi}{\lambda}d_y\left(\frac{N-1}{2}\right)\sin\theta\sin\varphi}\right]^T \tag{5}$$

The overall steering vector will be [30]:

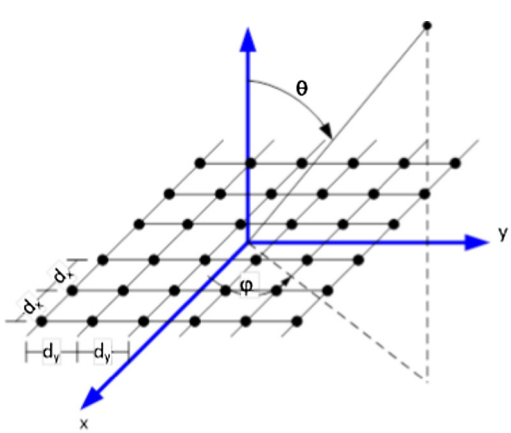

Fig. 1. Array geometry.

$$\mathbf{a}(\theta,\varphi) = vec\{\mathbf{u}(\theta,\varphi)\mathbf{v}^T(\theta,\varphi)\} \tag{6}$$

Introducing $\psi_x = \frac{2\pi}{\lambda}d_x\sin\theta\cos\varphi$ and $\psi_y = \frac{2\pi}{\lambda}d_y\sin\theta\sin\varphi$, Eqs. (4) and (5) can be rewritten as:

$$\mathbf{u}(\psi_x) = \left[e^{-j\psi_x\left(-\frac{M-1}{2}\right)}, e^{-j\psi_x\left(-\frac{M-3}{2}\right)},...,e^{-j\psi_x\left(\frac{M-1}{2}\right)}\right]^T \tag{7}$$

$$\mathbf{v}(\psi_y) = \left[e^{-j\psi_y\left(-\frac{N-1}{2}\right)}, e^{-j\psi_y\left(-\frac{N-3}{2}\right)},...,e^{-j\psi_y\left(\frac{N-1}{2}\right)}\right]^T \tag{8}$$

and the overall steering vector is:

$$\mathbf{a}(\psi_x,\psi_y) = vec\{\mathbf{u}(\psi_x)\mathbf{v}^T(\psi_y)\} \tag{9}$$

Assuming a ULA along the x-axis with M omnidirectional antennas in the array, the related transmit beampattern is defined as [19]:

$$P(\psi_x) = \mathbf{u}^H(\psi_x)\mathbf{R_x}\mathbf{u}(\psi_x) \tag{10}$$

where $\mathbf{u}$ is the steering vector and $\mathbf{R_x}$ is the covariance matrix of the transmitting signals and is defined as follows:

$$\mathbf{R_x} \triangleq \mathbf{W_x}\mathbf{W_x^H} \tag{11}$$

In Eq. (11), $\mathbf{W_x}$ is the corresponding weights matrix of the ULA. With the same approach, the transmit beampattern of a ULA with N omnidirectional antennas along the y-axis is defined as:

$$P(\psi_y) = \mathbf{v}^H(\psi_y)\mathbf{R_y}\mathbf{v}(\psi_y) \tag{12}$$

where $\mathbf{R_y}$ is the covariance matrix of the transmitting signals.

Dealing with planar arrays, the transmit beampattern of a URA with $M \times N$ omnidirectional antennas is:

$$P(\psi_x,\psi_y) = \mathbf{a}^H(\psi_x,\psi_y)\mathbf{R}\mathbf{a}(\psi_x,\psi_y) \tag{13}$$

3. Two dimensional transmit beampattern design

For transmit beampattern design in MIMO radars with planar array antennas, various methods are applicable. Most methods provide an optimization problem under certain conditions such as low integrated sidelobe level (ISL), low peak sidelobe level (PSL) and specific 3 dB bandwidth and the solution of such a problem is the desired $\mathbf{W}$ matrix. Here, especially for a large number of antennas and large angular grid, the calculation of $\mathbf{W}$ via optimization methods is highly time consuming. In some cases, the two-dimensional beampattern has symmetry along x and y-axis. In these cases, the transmit beampattern can be evaluated as the product of two perpendicular one-dimensional beampatterns belonging to two ULAs assumed to be placed along the x and y-axis. So the optimization procedure can be performed on these ULAs and after calculating $\mathbf{R_x}$ and $\mathbf{R_y}$ (for the two ULAs along x and y-axis), the final $\mathbf{R}$ matrix for the planar arrays can be evaluated using the following theorems.

Theorem 1. *If $\mathbf{R} = \mathbf{R_y} \otimes \mathbf{R_x}$, then the beampattern of the URA can be expressed as the product of two perpendicular beampatterns of two ULAs along x and y directions and vice versa.*

Proof.

$$P(\psi_x,\psi_y) = \mathbf{a}^H(\psi_x,\psi_y)\mathbf{R}\mathbf{a}(\psi_x,\psi_y) \tag{14}$$

Using Eq. (9), Eq. (14) can be expressed as:

$$P(\psi_x,\psi_y) = [vec\{\mathbf{u}(\psi_x)\mathbf{v}^T(\psi_y)\}]^H\mathbf{R}[vec\{\mathbf{u}(\psi_x)\mathbf{v}^T(\psi_y)\}] \tag{15}$$

Using $\mathbf{R} = \mathbf{R_y} \otimes \mathbf{R_x}$ and the matrix relations $vec\{\mathbf{a}\mathbf{b}^T\} = \mathbf{b} \otimes \mathbf{a}$ and $vec\{\mathbf{a}\mathbf{b}^T\}^H = \mathbf{b}^H \otimes \mathbf{a}^H$ [36,37], Eq. (15) can be rewritten as:

$$P(\psi_x,\psi_y) = [\mathbf{v}^H(\psi_y) \otimes \mathbf{u}^H(\psi_x)][\mathbf{R_y} \otimes \mathbf{R_x}][\mathbf{v}(\psi_y) \otimes \mathbf{u}(\psi_x)] \tag{16}$$

Using $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$ we have:

$$\begin{aligned} P(\psi_x,\psi_y) &= [(\mathbf{v}^H(\psi_y)\mathbf{R_y}) \otimes (\mathbf{u}^H(\psi_x)\mathbf{R_x})][\mathbf{v}(\psi_y) \otimes \mathbf{u}(\psi_x)] \\ &= [\mathbf{v}^H(\psi_y)\mathbf{R_y}\mathbf{v}(\psi_y)] \otimes [\mathbf{u}^H(\psi_x)\mathbf{R_x}\mathbf{u}(\psi_x)] \\ &= P(\psi_y) \otimes P(\psi_x) \\ &= P(\psi_x)P(\psi_y) \quad \square \end{aligned} \tag{17}$$

The reverse relationship can be easily proved, such that the two-dimensional beam pattern can be expressed as the product of two one-dimensional beampatterns; then, $\mathbf{R} = \mathbf{R_y} \otimes \mathbf{R_x}$.

Theorem 2. *The following relationship also holds:* $\mathbf{W} = \mathbf{W_y} \otimes \mathbf{W_x}$

Proof.

$$\begin{aligned} \mathbf{R_y} \otimes \mathbf{R_x} &= (\mathbf{W_y}\mathbf{W_y}^H) \otimes (\mathbf{W_x}\mathbf{W_x}^H) \\ &= (\mathbf{W_y} \otimes \mathbf{W_x})(\mathbf{W_y}^H \otimes \mathbf{W_x}^H) \\ &= (\mathbf{W_y} \otimes \mathbf{W_x})(\mathbf{W_y} \otimes \mathbf{W_x})^H \end{aligned} \tag{18}$$

As $\mathbf{R} = \mathbf{W}\mathbf{W}^H$ and $\mathbf{R} = \mathbf{R_y} \otimes \mathbf{R_x}$ the above theorem is proved. $\square$

In phased array antennas, such a condition is accomplished when $w_{mn} = w_m w_n$, where w_{mn} is the weight of the mnth array and w_m and w_n are the weights of mth and nth arrays of the two perpendicular ULAs [19].

3.1. Transmit beampattern design via approximation method

A method of beampattern designing is to optimize the matrix $\mathbf{W}$ (or $\mathbf{R}$) such that the resulted beampattern is much alike a specific shape. For this purpose, minimizing the error or mean square error between the desired beampattern and the resulted one is typically considered. Using the method described in [25–27], the 2D beampattern can be evaluated by solving the following optimization problem in the (ψ_x,ψ_y) space.

$$\min_{\mathbf{R}} \max_{\psi_x,\psi_y} |P_d(\psi_x,\psi_y) - \mathbf{a}(\psi_x,\psi_y)^H \mathbf{R}\mathbf{a}(\psi_x,\psi_y)| \tag{19}$$

$$\text{s.t.} \quad diag\{\mathbf{R}\} = \frac{P_T}{MN}\mathbf{1_{MN\times 1}} \tag{20}$$

$$Rank\{\mathbf{R}\} = K \tag{21}$$

In Eq. (19), $P_d(\psi_x,\psi_y)$ is the desired beampattern and $\mathbf{R}$ is the transmit signal covariance matrix. Eq. (20) maintains the equal transmit power across each antenna, which is the desired condition through practical consideration such as designing the similar power supplies and power amplifiers. P_T is the total radiated power and $\mathbf{1_{MN\times 1}}$ is the $MN \times 1$ vector. Eqs. (19)–(21) are non-convex optimization problems due to the rank constraint in Eq. (21). Therefore the semidefinite relaxation technique [38] can be used to recast it as a convex one. Using some matrix relationships, Eqs. (19)–(21) can be rewritten as:

$$\min_{\mathbf{R}} \max_{\psi_x,\psi_y} |P_d(\psi_x,\psi_y) - tr\{\mathbf{a}(\psi_x,\psi_y)\mathbf{a}(\psi_x,\psi_y)^H \mathbf{R}\}| \tag{22}$$

$$\text{s.t.} \quad diag\{\mathbf{R}\} = \frac{P_T}{MN}\mathbf{1_{MN\times 1}} \tag{23}$$

$$Rank\{\mathbf{R}\} \succeq 0 \tag{24}$$

Eqs. (22)–(24) can be solved using optimization technique such as interior point method [39,40]. When the solution of $\mathbf{R}$ is found, then the weight matrix $\mathbf{W}$ must be evaluated.

As mentioned earlier, solving the above optimization problem over large angular grid and for large array size, is computationally time-consuming. By assuming the symmetry of beampattern and using the Theorem 1, the beampattern design problem can be changed to the following problems which are much faster because of the reduced array size and angular grid size. For any of the ULAs along the x-axis, $\mathbf{R_x}$ is calculated by solving Eqs. (25)–(27).

$$\min_{\mathbf{R_x}} \max_{\psi_x} |P_d(\psi_x) - tr\{\mathbf{u}(\psi_x)\mathbf{u}(\psi_x)^H \mathbf{R_x}\}| \tag{25}$$

$$\text{s.t.} \quad diag\{\mathbf{R_x}\} = \frac{\sqrt{P_T}}{M}\mathbf{1_{M\times 1}} \tag{26}$$

$$Rank\{\mathbf{R_x}\} \succeq 0 \tag{27}$$

and for any of the ULAs along the y-axis, $\mathbf{R_y}$ is calculated by solving Eqs. (28)–(30).

$$\min_{\mathbf{R_y}} \max_{\psi_y} |P_d(\psi_y) - tr\{\mathbf{v}(\psi_y)\mathbf{v}(\psi_y)^H \mathbf{R_y}\}| \tag{28}$$

$$\text{s.t.} \quad diag\{\mathbf{R_y}\} = \frac{\sqrt{P_T}}{N}\mathbf{1_{N\times 1}} \tag{29}$$

$$Rank\{\mathbf{R_y}\} \succeq 0 \tag{30}$$

After evaluating $\mathbf{R_x}$ and $\mathbf{R_y}$ and by the help of Theorem 1, $\mathbf{R}$ is calculated.

3.2. Specific 3 dB beamwidth for transmitting beampattern

One of the most desirable cases in transmit-beampattern design is to design the beampattern with a specific 3 dB beamwidth and the lowest possible sidelobe level. The half-power beamwidth for a planar array antenna is a contour in (θ,ϕ) or (ψ_x,ψ_y) space. This contour can be approximated by a circle $(M = N)$ or an ellipse $(M \neq N)$. Usually, two perpendicular planes are chosen to characterize the beamwidth in azimuth and elevation.

If the 2D beampattern can be approximated as the product of two perpendicular patterns, then we can design each pattern with the desired specifications and resolve the final beampattern by the use of Theorem 1. In this section we will study a method for designing a two-dimensional beampattern, suppressing ISL, with specific 3 dB beamwidth in azimuth and elevation. Assuming a beampattern with 3 dB beamwidth in azimuth as β_{3dB} and 3 dB beamwidth in elevation as ε_{3dB} (or the equivalents in (ψ_x,ψ_y) space), suppressing the ISL and following the discussion in [41], we will have to solve the following optimization problems in (ψ_x,ψ_y) space. For each ULA, assume that ψ_0 is the direction of the beam and $\psi_2 - \psi_1$ specify the 3 dB beamwidth. So the optimization problem will be [41]:

$$\min_{\mathbf{R},t} \quad t \tag{31}$$

$$\text{s.t.} \quad 0.5P(\psi_0) \leqslant P(\psi) \leqslant P(\psi_0) \quad \forall\, \psi \in \Psi_m \tag{32}$$

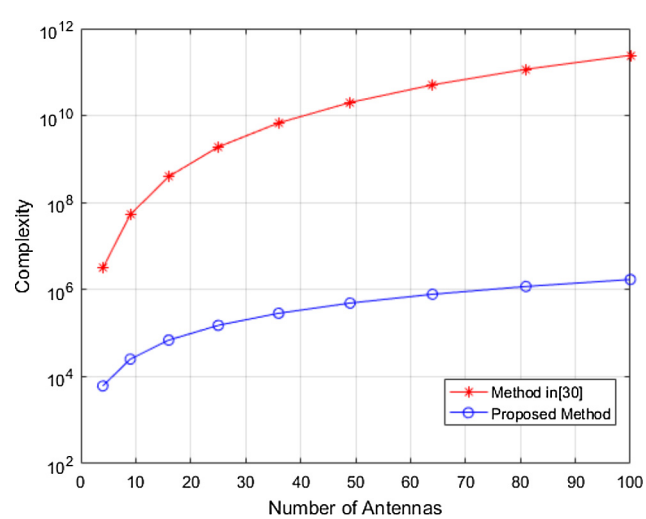

Fig. 2. A comparison between the complexity of the proposed method and the method of [30].

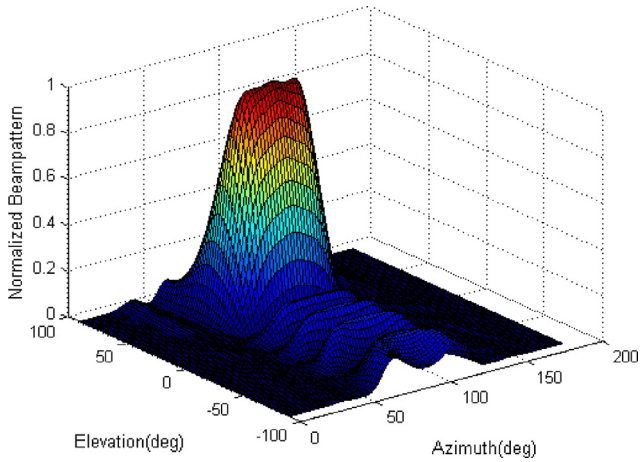

Fig. 3. Normalized transmit beampattern designed via separable beampattern method.

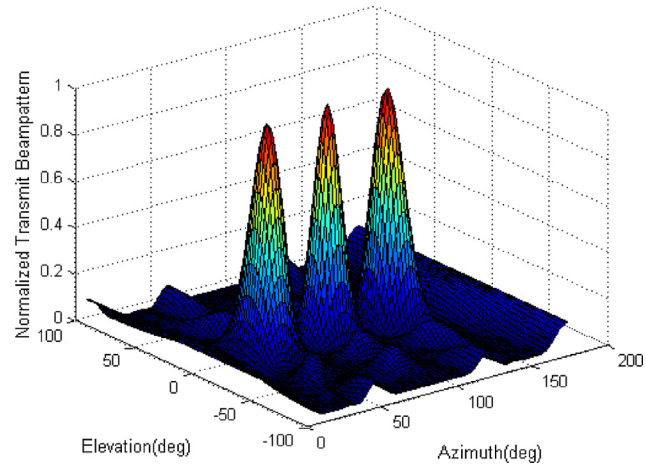

Fig. 5. Normalized transmit beampattern designed via separable beampattern method (3 mainlobes).

$$\int_{\Psi_s} P(\psi)d\psi \leqslant t \tag{33}$$

$$P(\psi_1) = 0.5P(\psi_0) \tag{34}$$

$$P(\psi_2) = 0.5P(\psi_0) \tag{35}$$

$$diag\{\acute{\mathbf{R}}\} = P \tag{36}$$

$$\mathrm{Rank}\{\acute{\mathbf{R}}\} \geq 0 \tag{37}$$

In the above equations, Eq. (32) assure that the radiation intensity inside the 3 dB beamwidth is higher than half power intensity, Eq. (33) means minimizing the integrated sidelobe level, Ψ_m is the region of half-power beamwidth, and Ψ_s is a proper subset of the sidelobe region. Eqs. (34) and (35) assure the 3 dB beamwidth criteria. In Eq. (26), P is the total power radiated by the array. The above optimization problem will be solved for ULAs in x and y directions.

When the above problem is solved for the ULA along the x-direction, Ψ_m,Ψ_s, and ψ are replaced with Ψ_{xm},Ψ_{xs}, and ψ_x respectively and P is equal to $\frac{\sqrt{P_T}}{M}\mathbf{1_{M\times 1}}$ and $\acute{\mathbf{R}}$ denotes $\mathbf{R_x}$. On the other hand, when it is solved for the ULA along the y-direction, Ψ_m,Ψ_s, and ψ are replaced with Ψ_{ym},Ψ_{ys}, and ψ_y respectively and P is equal to $\frac{\sqrt{P_T}}{N}\mathbf{1_{N\times 1}}$ and $\acute{\mathbf{R}}$ denotes $\mathbf{R_y}$. The URA beampattern will be achieved by using the result of Theorem 1.

Once $\mathbf{R}$ is evaluated via solving the above optimization problem, it's time to calculate $\mathbf{W}$ from $\mathbf{R}$ as $\mathbf{R} = \mathbf{W}\mathbf{W}^H$. If $Rank\{\mathbf{R}\}$ is K, then $\mathbf{W}$ can be evaluated simply by the eigenvalue decomposition of $\mathbf{R}$. Assume $\mathbf{R} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H$ where $\mathbf{\Lambda}$ is the diagonal eigenvalue matrix and $\mathbf{Q}$ is the matrix whose columns are the corresponding eigenvectors. So, we have

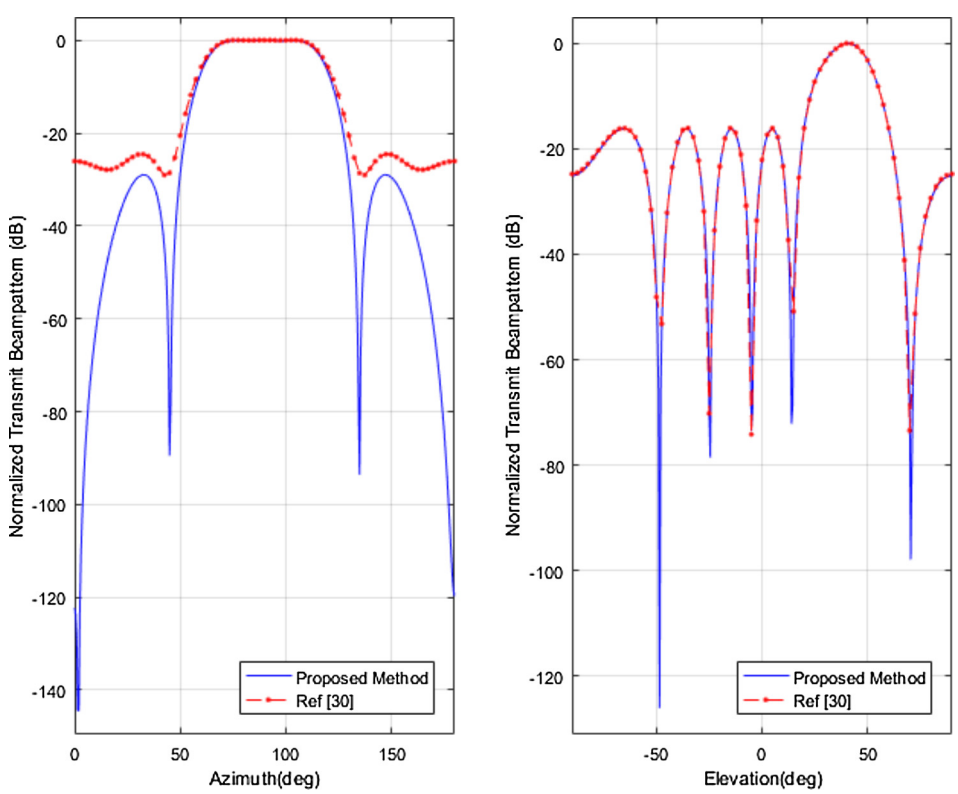

Fig. 4. Azimuth and elevation cross-sections of the beampattern shown in Fig. 3 at elevation = 30° and azimuth = 90° respectively, compared with the same pattern obtained via the method of [30].

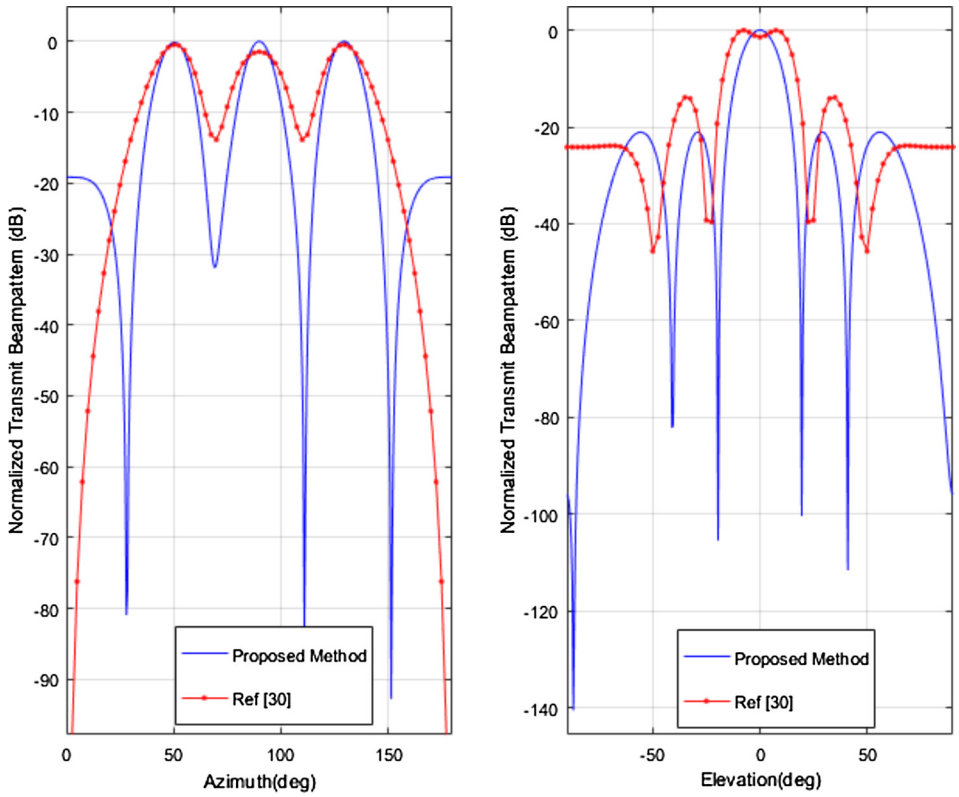

Fig. 6. Azimuth and elevation cross-sections of the beampattern shown in Fig. 5 at elevation = 0° and azimuth = 90° respectively, compared with the same pattern obtained via the method of [30].

$\mathbf{W} = \mathbf{Q}\mathbf{\Lambda}^{1/2}\mathbf{U_0}$ in which $\mathbf{U}$ is an arbitrary unitary matrix. If $Rank\{\mathbf{R}\} > K$, which happens often, then we can select the K effective eigenvalues and eigenvectors. Simulation shows that if K is taken as the number of effective eigenvalues of the semidefinite matrix defined in Eq. (3), then the resulting pattern may differ far from the optimized pattern. Thus, we propose to select K as the number of effective eigenvalues of $\mathbf{R}$ rather than the matrix defined in Eq. (3) and simulations show that the transmit beampattern obtained by the calculated $\mathbf{W}$ from effective eigenvalues, differ slightly from the pattern obtained by $\mathbf{R}$.

4. Computational complexity

One of the major benefits of the proposed method is the computational complexity and computational time. Following the interior point algorithm used by most convex optimization toolboxes which handle the semidefinite programming, the computational complexity is in the worst case $\mathcal{O}(M^{4.5}\log(1/\varepsilon))$ in which M is the array size and $\varepsilon > 0$ is the solution accuracy[38]. Using the proposed algorithm in [40], the computational complexity will reduce to $\mathcal{O}(M^{3.5}\log(1/\varepsilon))$. By considering the grid size of the coverage angle, the computational complexity is $\mathcal{O}(L_{az}L_{el}(MN)^{3.5}\log(1/\varepsilon))$. For the proposed beam separation method, the complexity is $\mathcal{O}(L_{az}M^{3.5}\log(1/\varepsilon)) + \mathcal{O}(L_{el}N^{3.5}\log(1/\varepsilon))$. Assuming the same number of antennas in each row and column (M = N = P), and the same grid size in azimuth and elevation directions ($L_{az} = L_{el} = L$), the computational complexity of the method proposed in [30] is $\mathcal{O}(L^2P^7\log(1/\varepsilon))$ and the computational complexity of the beam separation method is $\mathcal{O}(2LP^{3.5}\log(1/\varepsilon))$. A comparison between the proposed method complexity and the method of [30] for squared planar arrays is shown in Fig. 2. As can be seen, the proposed beam separation method is much computational complexity efficient, especially for a large number of arrays and a large grid size of the angular sector of interest.

5. Simulation results

In the first simulation, a 7 × 7 URA is assumed with $d_x = d_y = \lambda/2$ where λ is the wavelength. A 2D transmit beampattern is designed via the approximation method. The main lobe region is [70°,110°] in azimuth and [30°,50°] in elevation with a transition of 20° in azimuth and 10° in elevation on each side. The transmit beampattern can be assumed as the product of two beams in azimuth and elevation. In Fig. 3, the resulted beampattern via the proposed method is shown. Fig. 4 illustrates the azimuth and elevation cross-section of beampattern and compares them with the same patterns obtained by the approach expressed in [30]. As can be seen, both approaches result nearly the same patterns but the proposed method is much faster because of its lower computational complexity. In the next simulation, the above array is

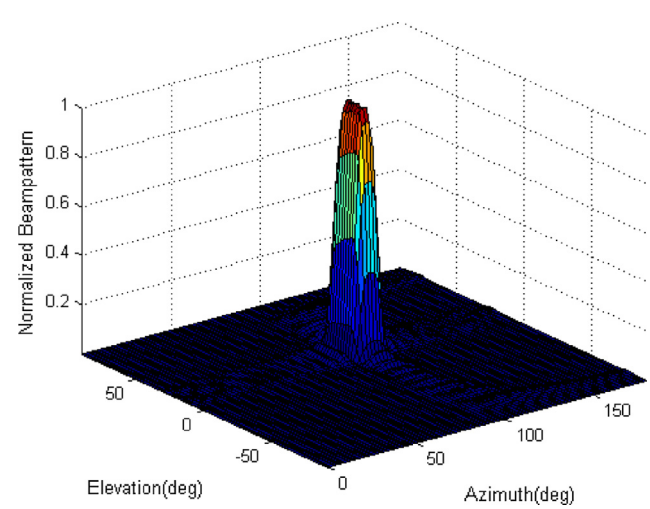

Fig. 7. Normalized transmit beampattern for the 20 × 15 URA.

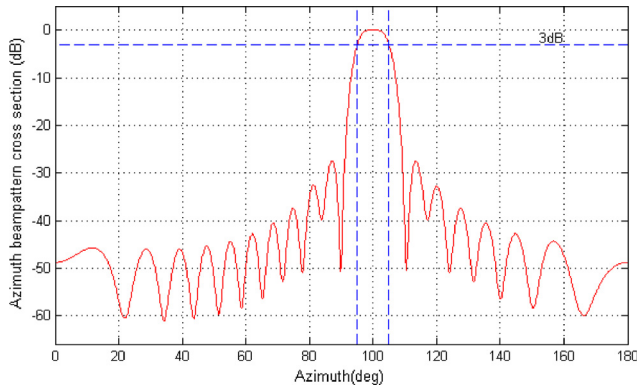

Fig. 8. Azimuth cross-section of the transmit beampattern.

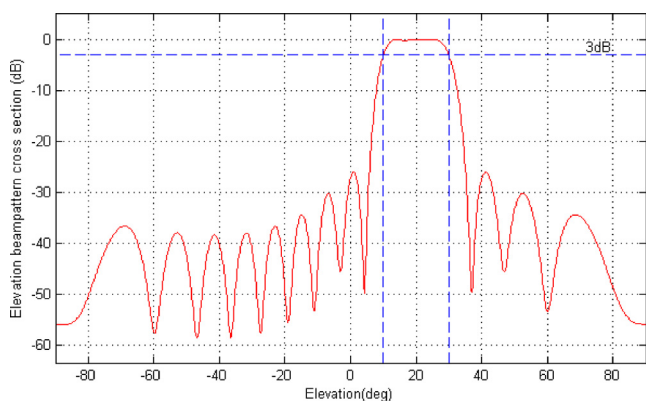

Fig. 9. Elevation cross-section of the transmit beampattern.

optimized to have three identical beams located at [45°,55°],[85°,95°] and [105°,115°] in azimuth with a transition of 10° on each side and [−5°,5°] in elevation with the same transitions as the azimuth. The transmit beampattern is evaluated via the proposed method and the method of [30]. In Fig. 5, the normalized 3D beampattern is shown and in Fig. 6, the cross-sections of the beampattern in azimuth and elevation are shown and compared with the result obtained by the method of [30]. As shown in Fig. 6, the proposed method yields lower sidelobe level and deeper nulls. In the next simulation, the problem of designing URA beampattern with minimum possible ISL and specific 3 dB beamwidth in azimuth and elevation is simulated. Here, a larger planar

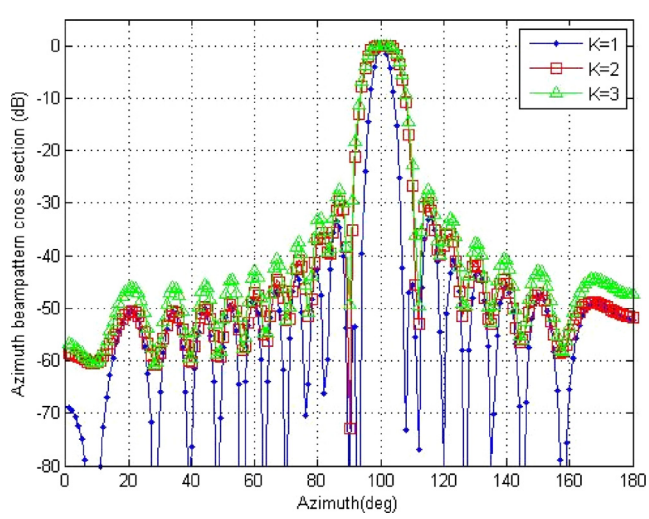

Fig. 10. Effect of K on the beampattern, azimuth cross-section.

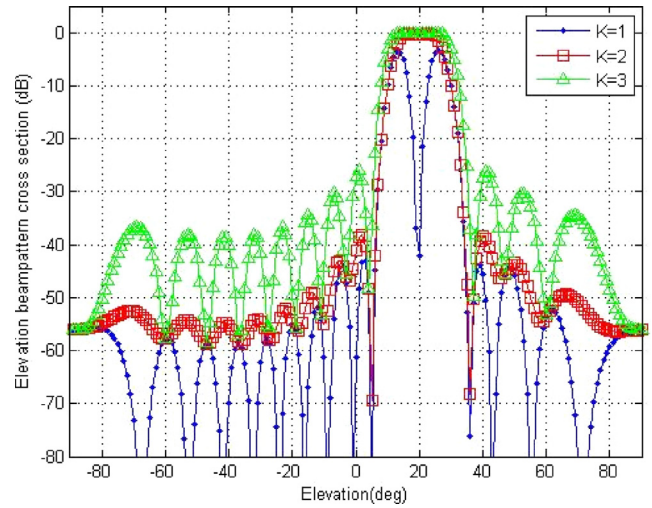

Fig. 11. Effect of K on the beampattern, elevation cross-section.

array, a 20×15 URA is assumed. The antennas are separated by $d_x = d_y = \lambda/2$. The main lobe in azimuth is at 100° with a 3 dB beamwidth of 10° and in elevation is at 20° with a 3 dB beamwidth of 20°. Fig. 7 presents the beampattern of the planar array. Figs. 8 and 9 show the cross-sections in azimuth plane and elevation plane, respectively. The straight lines show the 3 dB region. Also, Figs. 10 and 11, show the effect of choosing K effective eigenvalues of **R** for evaluating **W** on the beampattern. As presented, by increasing the value of K, the resulted pattern is much alike the patterns shown in Figs. 8 and 9. For this special case, K = 3 and for values of $K \geqslant 3$, the beampattern does not change noticeably.

6. Conclusion

In the present study, the problem of designing transmit beampattern of MIMO radars with URA is addressed. The transmit beampattern is evaluated via approximation method under the constraints such as uniform transmit power across antennas. Moreover, the 2D transmit beampattern with minimum ISL and specific 3 dB beamwidth in azimuth and elevation is designed. As the array size grows and for large angular resolution, meaning large grid size, the computational load for solving the optimization problem is very high. To overcome this problem, beampattern separation method is studied and the conditions and relationships between ULAs and URA beampatterns, are evaluated and the computational complexity of the optimization problem is drawn. Our results show that for large arrays and large angular grid size, the proposed method is much faster and computationally efficient. Also, assessing the effect of choosing K on the resulting beampattern revealed that if K is smaller than the number of effective eigenvalues of **R**, then the resulting beampattern differ far from the desired beampattern.

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