Robust stability for affine T-S fuzzy impulsive control systems subject to parametric uncertainties

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Abstract—**The main objective of this paper is to investigate the robust stability problem of affine fuzzy impulsive control systems in which the system is presented by affine Takagi-Sugeno fuzzy model. By using extended Lyapunov stability theory, some matrix inequalities conditions are derived to guarantee the stability of affine Takagi-Sugeno fuzzy impulsive control systems subject to parametric uncertainties. A numerical example is given to confirm the analytical results and illustrate the effectiveness of the proposed strategy.**

***Keywords-** Affine Takagi-Sugeno Fuzzy system; Impulsive control strategy; Parametric uncertainties*

I. Introduction

Motivated from impulsive differential equations, the effects of impulses can be used as a strategy in control theory which is named as ‘‘impulsive control strategy’’. The main idea of impulsive control strategy is to change the states of continuous dynamic systems via discontinuous control inputs at certain time moments. Impulsive control strategy has attracted many literatures [1, 2] and have been widely used for stabilization and synchronization of complex delayed dynamical networks [1-4], switched systems [5-7], optimal control for switched systems [8], large-scale systems [9], control of biological models [10], control of chaotic systems [11] and etc. Among of these systems, which have used impulsive control strategy, fuzzy systems have attracted many interests [12-17]. Many theories and relevant method were developed for fuzzy systems in previous decade and applied with more or less success depending on the specific problem [18-23]. Extended and innovative methods and relevant conditions for stability analysis of fuzzy impulsive control systems were found. Some methods and conditions for stability in continuous and discrete fuzzy impulsive control systems are presented in [24-27]. To the best of our knowledge, stability analysis of affine T-S fuzzy impulsive control system has not yet been fully investigated and this will be the goal of this paper. Another important subject that should be attended is stability analysis in the presence of uncertainties, which is considered in this paper.

In this correspondence, we consider two classes of uncertain affine T-S fuzzy impulsive control systems. By using impulsive control strategy, we obtain some sufficient conditions to analyze (uniformly, asymptotically) stability of uncertain affine T-S fuzzy impulsive control systems in the presence of parametric uncertainties will be given.

The contributions of this paper are organized as follows. After an introduction, we introduce the concepts of impulsive differential equations and then affine T-S fuzzy impulsive control system is explained in section three. In the section four, stability analysis of two classes of uncertain affine T-S fuzzy impulsive control systems are considered. Base on extended Lyapunov theory, sufficient conditions for stability analysis in the presence of parametric uncertainties are extracted in terms of matrix inequalities. Finally, results are argued in conclusion section.

II. Impulsive Control Strategy

In this section, we present the concepts and definitions of impulsive control strategy that we need to introduce affine T-S fuzzy impulsive control systems. First, we address the issues which concern the necessity of impulsive control strategy from [10] and mentioned previously in [28-30] by the authors of the paper. This part is given originally from [10]. Consider the following impulsive functional equation

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & t \geq t_0, \\ x(t_k) = I(t_k, x(t_k^-)), & k \in \mathbb{Z}^+, \end{cases} \tag{1a}$$

where $f: [t_0, \infty) \times PC \to \mathbb{R}^n$ and $I: [t_0, \infty) \times s(\rho) \to \mathbb{R}^n$. Here, $s(\rho) = \{x(t) \in \mathbb{R}^n : \|x(t)\| < \rho\}$, PC denotes the space of piecewise right-continuous function. $\|\cdot\|$ is a norm in $\mathbb{R}^n$, $0 < t_k < t_{k+1}$ with $t_k \to \infty$ as $k \to \infty$. $\dot{x}(t)$ denotes the right-hand derivative of $x(t)$ and $\mathbb{Z}^+$ denotes the set of all positive integers. For stability analysis, we assume that $f(t, 0) = 0$ and $I(t_k, 0)=0$ so that $x(t) = 0$ is a solution of (1a). Motivated from the nonlinear impulsive equation (1a), the following impulsive control system (strategy) can be considered

$$\begin{aligned} &\dot{x}(t) = f(t, x(t)), \\ &y(t) = \varphi(x(t)), \end{aligned} \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $y(t) \in \mathbb{R}^m$ is the output variable, $f(t, x(t))$ and $\varphi(x(t))$ are continuous function in their respective domains of definition. An impulsive control law of (1b) is given by a sequence $\{\tau_k, I_k(y(\tau_k))\}$, where $0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots$, $\tau_k \to \infty \; as \; k \to \infty$, and $I_k(y(t))$ is a continuous function which maps $\mathbb{R}^m$ to $\mathbb{R}^n$ for all $k = 1,2, \cdots$ which is named as ‘‘impulsive function’’. Clearly, if the solution of (1b) exists, we can rewrite (1b) as follows:

$$\begin{aligned} &\dot{x}(t) = f(t, x(t)), \quad t \neq \tau_k, \\ &y(t) = \varphi(x(t)), \quad t \neq \tau_k, \end{aligned}$$

$$\Delta x(t) = I_k(y(t)), \quad t = \tau_k,$$
$$x(t_0) = x_0, \quad k = 1,2,\cdots. \qquad (2)$$

where $\Delta x(\tau_k) = x(\tau_k{}^+) - x(\tau_k{}^-), x(\tau_k{}^+) = \lim_{t\to\tau_k{}^+} x(t)$. We call (2) as ''impulsive control system''. Without loss of generality, we assume $f(t,0) \equiv 0$ and $\varphi(0) = 0$, so that system (2) admits a trivial solution. Our impulsive control problem may now be stated formally as follow. Subject to the dynamical system (1b), find an impulsive control law $\{\tau_k, I_k(y(\tau_k))\}$ such that the impulsive control system (2) is stable, uniformly stable, asymptotically stable or uniformly asymptotically stable.

Here, we denote K the class of continuous functions $\varphi(s) : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\varphi(s)$ is strictly increasing and $\varphi(0) = 0$; $\sum$ the class of functions $V(t,x(t))$: $\mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ such that $V(t,x(t))$ is positive definite, locally Lipschitzian in $x(t)$, continuous everywhere except possibly at a sequence of point $\{\tau_k\}$ at which $V(t,x(t))$ is left continuous and the right limit $V(\tau_k^+, x(\tau_k^+))$ exists for all $x(t) \in \mathbb{R}^n$. We assume that there exists a $\rho_1 \in (0,\rho)$ such that $x(t) \in s(\rho_1)$ implies $x(t) + I_k(x(t)) \in s(\rho)$. The following theorem gives sufficient conditions for various stability criteria. For more details see [10].

Theorem 1 (Liu et al. 错误!未找到引用源。[10]). Assume that

i) There exist $\mathcal{J}_k \in \mathbb{R}$ and $c_k \in K$ such that

$D^+V(t,x(t)) \le \frac{\mathcal{J}_k}{\Delta\tau_k} c_k\left(V(t,x(t))\right), (t,x(t)) \in (\tau_{k-1},\tau_k) \times s(\rho)$,

ii) There exist $v_k \in \mathbb{R}$ and $d_k \in K$ such that

$V\left(\tau_k^+, x(t) + I_k(x(t))\right) \le V(\tau_k, x(t)) + v_k d_k\left(V(\tau_k, x(t))\right)$, $x(t) \in s(\rho)$,

iii) $\mathcal{J}_k + v_k \le 0$, for $s \in (0,\rho), c_k(s) \le d_k(s)$ if $v_k < 0$ and $d_k(s) \le c_k(s)$ if $\mathcal{J}_k < 0$.

Then the system (1b) is stable. In addition to all above condition, suppose further that $V(t,x(t))$ is deceasing and

iv) For any $\eta > 0$, there exist a $\sigma > 0$ such that

$$s + |v_k| d_k(s) < \eta, \qquad \forall s \in (0,\sigma), \quad \forall k = 1,2,\cdots,$$

Then the system (1b) is uniformly stable. Also if (i), (ii) and (iii) and the following condition hold

$$\sum_{k=1}^{\infty} (\mathcal{J}_k + v_k)\, e_k(\beta) = -\infty, \qquad \forall\, \beta > 0\,,$$

where $e_k(s) = max\{c_k(s), d_k(s)\}$, then system (1b) is asymptotically stable. Finally, if (i), (ii), (iii), (iv) hold and there exist positive integer N such that

$$\sum_{k=q+1}^{q+N} (\mathcal{J}_k + v_k)\, l_k(\beta) < -C, \qquad \forall\, q \ge 0, \ \ \beta, C > 0,$$

where $l_k(s) = max\{c_k(s), d_k(s)\}$, and the sequence $\{\Delta\tau_k\}$ is bounded for any $\beta, C > 0$. Then, system (1b) is uniformly asymptotically stable. $\Delta\tau_k = \tau_k - \tau_{k-1}$, and

$$D^+V(t,x(t)) = \lim_{\delta\to 0^+} sup \frac{1}{\delta}\left[V(t+\delta, x(t)+\delta) f(t,x(t)) - V(t,x(t))\right]$$

If $V(t,x(t))$ is continuously differentiable, then $D^+V(t,x(t)) = \frac{\partial}{\partial t}V(t,x(t)) + \frac{\partial}{\partial x}V(t,x(t)) \cdot f(t,x(t))$. ■

Remark 1. If $c_k(s) = d_k(s) = s$, then the condition (iii) is reduced to

$$\mathcal{J}_k + v_k \le 0 \quad \text{and} \quad v_k \ge -1.$$ ■

III. Robust Stability Subject to Parametric Uncertainties

Consider an affine T-S fuzzy impulsive control system under discussion, which has bounded parametric uncertainties as

IF p_1 *is* M_1^i *and* $\cdots$ *and* p_s *is* $M_{\wp}^i$

THEN $\begin{cases} \dot{x} = \hat{A}_i(t)x + \varpi_i \hat{e}_i(t) + \hat{B}_i(t)u \\ u = \sum_{k=1}^{\infty} \delta(t-\tau_k) I_{ik}(x) \end{cases}$, $(i = 1,2,\cdots,r)$ (3)

where r is the number of fuzzy rules, $p(t)$ is vector of premise variables such that

$$p(t) = \left[p_1(t), p_2(t), \cdots, p_{\wp}(t)\right]^T = \mathcal{O}x(t),$$

$rank(\mathcal{O}) = \wp(1 \le \wp \le \mathrm{n})$,
$x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$ is state vector at time t, n is the number of states variable, $\left(\hat{A}_i(t), \hat{B}_i(t)\right)$ is controllable pair of system matrices, $\hat{e}_i(t)$ is offset terms, $u(t) = [u_1(t), u_2(t), \cdots, u_m(t)]^T \in \mathbb{R}^m$ is control input at time t with appropriate dimension, and $M_j^i(i = 1,2,,\cdots,r,\ j = 1,2,\cdots,\wp)$ stands for the fuzzy set of jth antecedent variable in the ith rule. Also,

$$\begin{cases} \hat{A}_i(t) = A_i + \Delta A_i(t)\,, \Delta A_i(t) = H_{a_i} F_{a_i}(t) L_{a_i} \ , F_{a_i}{}^T(t) F_{a_i}(t) \le R_{a_i} \\ \hat{B}_i(t) = B_i + \Delta B_i(t)\,, \Delta B_i(t) = H_{b_i} F_{b_i}(t) L_{b_i} \ , F_{b_i}{}^T(t) F_{b_i}(t) \le R_{b_i} \\ \hat{e}_i(t) = e_i + \Delta e_i(t) \ \ , \Delta e_i(t) = H_{e_i} F_{e_i}(t) L_{e_i} \ , F_{e_i}{}^T(t) F_{e_i}(t) \le R_{e_i} \end{cases}$$

and $\{\Delta A_i(t), \Delta B_i(t), \Delta e(t)\}$ are the system uncertainties satisfying the norm bounded condition. $H_i \coloneqq [H_{a_i}, H_{b_i}, H_{e_i}]$ and $L_i \coloneqq [L_{a_i}, L_{b_i}, L_{e_i}]$ are known constant matrices, $F_{a_i}(t)$, $F_{b_i}(t)$, and $F_{e_i}(t)$ belong to the following set

$\Omega \coloneqq$
$\left\{F(t) \middle| F^T(t)F(t) \le I, \text{element of } F(t) \text{ are Lebesgue measurment}\right\}$

Based on impulsive controller techniques [28], [30], the closed-loop system can be described by

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \mu_i\left(\hat{A}_i(t)x + \varpi_i \hat{e}_i(t)\right) & t \ne \tau_k \\ \Delta x = x(\tau_k{}^+) - x(\tau_k{}^-) = I_k(x) = \sum_{i=1}^{r} \mu_i \hat{B}_i(t) I_{ik}(x) & t = \tau_k \end{cases} \qquad (4)$$

The rule set of the system is divided into I_0 and I_1. I_0 represents the rules which contain the origin and I_1 is the remaining rules which do not contain the origin. So, if $i \in I_0$, we get $\hat{e}_i = 0$ which guarantee the trivial solution for $\dot{x}(t) \equiv 0$. By noting mentioned description we define ϖ_i as $\varpi_i = \begin{cases} 0 & ,if\ i \in I_0 \\ 1 & ,if\ i \in I_1 \end{cases}$.

Our task is to design an impulsive control law $\{\tau_k, I_{ik}(x(\tau_k))\}$ such that the overall affine T-S fuzzy impulsive control system is stabilized (asymptotically, uniformly). For stability analysis of (4), the following theorem is given in which impulsive function $(I_{ik}(x))$ is considered as $I_{ik}(x) = D_{ik}x + \varpi_i g_{ik}(x)$ and $\|g_{ik}(x)\| \le b_{ik}\|x\|$ in which b_{ik} is a positive scalar.

Theorem 2: The origin of the affine T-S fuzzy impulsive control system (4) is stable if there exist positive definite matrix T, scalars $\mathcal{I}_k$'s, ν_k's, positive scalars $\Delta\tau_k$'s, τ_i's, ϵ_{ik}'s, ϵ_i^n's, and matrices v_i^k's such that the following conditions hold

$$\mathcal{I}_k + \nu_k \le 0 \quad , \quad \nu_k \ge -1, \tag{5a}$$

for $i \in I_0$

$$\begin{pmatrix} \mathcal{M}_{ik}^n & * & * \\ \mathcal{L}_n & -\epsilon_i^n T_n & * \\ \epsilon_i^n \mathcal{H}_n{}^T & 0 & -\epsilon_i^n I_n \end{pmatrix} \le 0, n = 1,2, \tag{5b}$$

for $i \in I_1$

$$\begin{pmatrix} \mathcal{M}_{ik}^n & * & * \\ \mathcal{L}_n & -\epsilon_i^n T_n & * \\ \epsilon_i^n \mathcal{H}_n{}^T & 0 & -\epsilon_i^n I_n \end{pmatrix} \le 0, n = 3,4, \tag{5c}$$

where

$\mathcal{M}_{ik}^1 = TA_i^T + A_iT - \frac{\mu_k}{\Delta\tau_k}T, \mathcal{H}_1 = H_{a_i}, \mathcal{L}_1 = L_{a_i}T, T_1 = T,$

$\mathcal{M}_{ik}^2 = \begin{pmatrix} B_i v_i^k + v_i^{k^T} B_i{}^T - \nu_k T & * \\ v_i^{k^T} B_i{}^T & -T \end{pmatrix}$, $\mathcal{H}_2 = \begin{pmatrix} H_{b_i} & H_{b_i} \\ 0 & 0 \end{pmatrix}$,

$\mathcal{L}_2 = diag(L_{b_i}v_i^k, L_{b_i}v_i^k), T_2 = diag(T,T),$

$\mathcal{M}_{ik}^3 = \begin{pmatrix} TA_i^T + A_iT - \frac{\mu_k}{\Delta\tau_k}T & * \\ e_i^T + \tau_i x_{c_i}^T Q_{c_i} T & \tau_i O_i \end{pmatrix}, \mathcal{H}_3 = \begin{pmatrix} H_{a_i} & H_{e_i} \\ 0 & 0 \end{pmatrix}, \mathcal{L}_3 = diag(L_{a_i}T, L_{e_i}), T_3 = diag(T,T),$

$$\mathcal{M}_{ik}^4 = \begin{pmatrix} \tilde{\Omega}_{iik} & * & * & * & * \\ T & -\gamma_{ik} I & * & * & * \\ T & 0 & -(1+\epsilon_{ik})T & * & * \\ \tau_i x_{c_i}^T Q_{c_i} T & 0 & 0 & \tau_i O_i & * \\ B_i v_i^k & 0 & T & 0 & -T \end{pmatrix},$$

$\mathcal{L}_4 = diag(L_{b_i}v_i^k, 0,0,0,0), T_4 = diag(T,T,T,T,T)$

$$\mathcal{H}_4 = \begin{pmatrix} H_{b_i} & * & * & * & 0 \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & * \\ H_{b_i} & 0 & 0 & 0 & 0 \end{pmatrix},$$

I_n is identity matrix with appropriate dimension.

Then $D_{jk} = v_j^k T^{-1}$, $\tau_k = \Delta\tau_k + \tau_{k-1} (\tau_0 = t_0)$, and $g_{ik}(x)$ is any continuous function in respect to x such that $\|g_{ik}(x)\| \le b_{ik}\|x\|$.

Proof. Consider the Lyapunov function $V(t,x) = x^T Px$ for the uncertain affine T-S fuzzy impulsive control system (4). Now, from Theorem 1, Remark 1, and by taking the upper right-hand generalized derivative of the Lyapunov function along the trajectories of the system (4), the following is obtained

$$\begin{aligned} &D^+V(t,x) - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px \\ &= \dot{x}^T Px + x^T P\dot{x} - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px \\ &= \sum_{i=1}^r \mu_i \left(\hat{A}_i(t)x + \varpi_i \hat{e}_i(t)\right)^T Px \\ &+ \textstyle\sum_{i=1}^r \mu_i x^T P\left(\hat{A}_i(t)x + \varpi_i \hat{e}_i(t)\right) - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px \end{aligned} \tag{6}$$

If $i \in I_0$ (i.e. $\varpi_i = 0$), we can show if the following term is negative then $D^+V(t,x) - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px < 0$.

$$\begin{aligned} &T\hat{A}_i^T(t) + \hat{A}_i(t)T - \frac{\mu_k}{\Delta\tau_k}T \\ &= TA_i^T + A_iT + T\left(H_{a_i}F_{a_i}(t)L_{a_i}\right)^T \\ &+ H_{a_i}F_{a_i}(t)L_{a_i}T - \frac{\mu_k}{\Delta\tau_k}T \\ &= \mathcal{M}_{ik}^1 + \mathcal{H}_1\mathcal{F}_1(t)\mathcal{L}_1 + \mathcal{L}_1{}^T\mathcal{F}_1(t)^T\mathcal{H}_1{}^T, \end{aligned} \tag{7}$$

where $\mathcal{F}_1(t) = F_{a_i}(t), \hat{A}_i^T(t) = A_i + H_{a_i}F_{a_i}(t)L_{a_i}$. By using Lemma A.1 ($R = T^{-1}$) and Schur's complement, it is easy to see that inequality (5b) ($\forall\, n = 1$) implies (7) is negative.

In a similar manner, also if, we get

$$\begin{aligned} &\begin{pmatrix} \hat{B}_i(t)v_i^k + v_i^{k^T}\hat{B}_i(t)^T - \nu_k T & * \\ v_i^{k^T}\hat{B}_i(t)^T & -T \end{pmatrix} \\ &= \mathcal{M}_{ik}^2 + \begin{pmatrix} H_{b_i}F_{b_i}(t)L_{b_i}v_i^k + v_i^{k^T}\left(H_{b_i}F_{b_i}(t)L_{b_i}\right)^T & * \\ v_i^{k^T}\left(H_{b_i}F_{b_i}(t)L_{b_i}\right)^T & 0 \end{pmatrix} \\ &= \mathcal{M}_{ik}^2 + \mathcal{H}_2\mathcal{F}_2(t)\mathcal{L}_2 + \mathcal{L}_2{}^T\mathcal{F}_2(t)^T\mathcal{H}_2{}^T \end{aligned} \tag{8}$$

where $\mathcal{F}_2(t) = diag\left(F_{b_i}(t), F_{b_i}(t)\right), \hat{B}_i^T(t) = B_i + H_{b_i}F_{b_i}(t)L_{b_i}$. By using Lemma A.1 $\left(R = \left(diag(T,T)\right)^{-1}\right)$ and Schur's complement, we can conclude that (5b) ($\forall\, n = 2$) implies (8) is negative.

Now, consider $i \in I_1 (i.e.\ \varpi_i = 1)$; Because $D^+V(t,x) - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px$ has to be given in terms of matrix inequalities, by following 错误!未找到引用源。[31], positive scalars $\sum_{i\in I_1}\mu_i\tau_i\left(1 - x^T Q_{c_i}x + x^T Q_{c_i}x_{c_i} + x_{c_i}^T Q_{c_i}x - x_{c_i}^T Q_{c_i}x_{c_i}\right)$ are added to (6). This implies that each rule of I_1 is encircled by a hyper ellipsoid in which τ_i is a positive scalar and $\left(1 - x^T Q_{c_i}x + x^T Q_{c_i}x_{c_i} + x_{c_i}^T Q_{c_i}x - x_{c_i}^T Q_{c_i}x_{c_i}\right)$ is the definition of the hyper ellipsoid which includes the region of the ith rule ($i \in I_1$). In here, Q_{c_i} is the positive definite matrix. Therefore, we get

$$\begin{aligned} &D^+V(t,x) - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px = \dot{x}^T Px + x^T P\dot{x} - \frac{\mathcal{I}_k}{\Delta\tau_k}x^T Px \\ &= \sum_{i=1}^r \mu_i (A_i x + \varpi_i e_i)^T Px \\ &+ \sum_{i=1}^r \mu_i x^T P(A_i x + \varpi_i e_i) \end{aligned}$$

$$+\sum_{i\in I_1}\mu_i\tau_i\big(1-x^TQ_{c_i}x+x^TQ_{c_i}x_{c_i}+x_{c_i}^TQ_{c_i}x-x_{c_i}^TQ_{c_i}x_{c_i}\big) -\frac{\mathcal{J}_k}{\Delta\tau_k}x^TPx \quad (9)$$

Then for $i\in I_1$, if $\begin{pmatrix}\tilde{\psi}_{ik} & * \\ e_i^T+\tau_i x_{c_i}^TQ_{c_i}T & \tau_i O_i\end{pmatrix}<0$, we can get

$$\begin{pmatrix}T\hat{A}_i^T(t)+\hat{A}_i(t)T-\frac{\mu_k}{\Delta\tau_k}T & * \\ \hat{e}_i^T(t)+\tau_i x_{c_i}^TQ_{c_i}T & \tau_i O_i\end{pmatrix}$$
$$=\mathcal{M}_{ik}^3+\begin{pmatrix}T\big(H_{a_i}F_{a_i}(t)L_{a_i}\big)^T+H_{a_i}F_{a_i}(t)L_{a_i}T & * \\ \big(H_{e_i}F_{e_i}(t)L_{e_i}\big)^T & 0\end{pmatrix}$$
$$=\mathcal{M}_{ik}^3+\mathcal{H}_3\mathcal{F}_3(t)\mathcal{L}_3+\mathcal{L}_3{}^T\mathcal{F}_3(t)^T\mathcal{H}_3{}^T \quad (10)$$

where $\mathcal{F}_3(t)=diag\left(F_{a_i}(t),F_{e_i}(t)\right)$, $\hat{e}_i^T(t)=e_i+H_{e_i}F_{e_i}(t)L_{e_i}$. By using Lemma A.1 $\left(R=\big(diag(T,T)\big)^{-1}\right)$ and Schur's complement, it is easy to see that (5c) ($\forall\, n=3$) implies (10) is negative.

Now, for $i\in I_1$, we get $V\big(\tau_k^+,x+I_k(x)\big)-V(\tau_k,x)+v_kd_k\big(V(\tau_k,x)\big)$ and By repeating the same manner, we get

$$\begin{pmatrix}\begin{pmatrix}\hat{B}_i(t)v_i^k+v_i^{k^T}\hat{B}_i(t)^T\\ -v_kT\end{pmatrix} & * & * & * & * \\ T & -\gamma_{ik}I & * & * & * \\ T & 0 & -(1+\epsilon_{ik})T & * & * \\ \tau_i x_{c_i}^TQ_{c_i}T & 0 & 0 & \tau_iO_i & * \\ \hat{B}_i(t)v_i^k & 0 & T & 0 & -T\end{pmatrix}$$
$$=\mathcal{M}_{ik}^4+\begin{pmatrix}\begin{pmatrix}H_{b_i}F_{b_i}(t)L_{b_i}v_i^k\\ +v_i^{k^T}\big(H_{b_i}F_{b_i}(t)L_{b_i}\big)^T\end{pmatrix} & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & * \\ H_{b_i}F_{b_i}(t)L_{b_i}v_i^k & 0 & 0 & 0 & 0\end{pmatrix}$$
$$=\mathcal{M}_{ik}^4+\mathcal{H}_4\mathcal{F}_4(t)\mathcal{L}_4+\mathcal{L}_4{}^T\mathcal{F}_4(t)^T\mathcal{H}_4{}^T, \quad (11)$$

where $\mathcal{F}_4(t)=diag\big(F_{b_i}(t),0,0,0,0\big)$. Eq. (5c) ($\forall\, n=4$) implies (11) is negative.

Also based on condition (ii) of Theorem 1 and by assuming $c_k(s)=d_k(s)=s$, we get

$$V\big(\tau_k^+,x+I_k(x)\big)-V(\tau_k,x)-v_kd_k\big(V(\tau_k,x)\big)$$
$$=\big(x+I_k(x)\big)^TP\big(x+I_k(x)\big)-V(\tau_k,x)-v_kd_k\big(V(\tau_k,x)\big)$$
$$=x^TPx+x^TPI_k(x)+I_k(x)^TPx+I_k(x)^TPI_k(x)-x^TPx-v_kx^TPx, \quad (12)$$

By considering $I_k(x)=\sum_{i=1}^r\mu_i\hat{B}_i(t)I_{ik}(x)$, and the proof of Theorems in [14] and [30], for $i\in I_0$, it can be shown that (5b) yields to $V\big(\tau_k^+,x+I_k(x)\big)-V(\tau_k,x)+v_kd_k\big(V(\tau_k,x)\big)$ is negative and similarly to , for $i\in I_1$, we can conclude that (5c) yields to $V\big(\tau_k^+,x+I_k(x)\big)-V(\tau_k,x)+v_kd_k\big(V(\tau_k,x)\big)$ is negative. The proof is now completed. ■

Now, consider a PDC based affine T-S fuzzy impulsive control system [28] as follows:

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^r\mu_i\left(\hat{A}_i(t)x+\varpi_i\hat{e}_i(t)\right), & t\neq\tau_k\\ \Delta x=x(\tau_k{}^+)-x(\tau_k{}^-)=I_k(x) \\ \quad =\sum_{i=1}^r\sum_{j=1}^r\mu_j\,\mu_i\left(\hat{B}_i(t)I_{jk}(x)\right), & t=\tau_k\end{cases} \quad (10)$$

The stability conditions of PDC based affine T-S fuzzy impulsive control system (10) with linear impulsive function $\big(I_{jk}(x)=D_{jk}x\big)$, can now be summarized by the following theorem.

Theorem 3: The origin of the PDC-based affine T-S fuzzy impulsive control system (10) is stable if there exist positive definite matrices T, scalars $\mathcal{J}_k$'s, v_k's, positive scalars $\Delta\tau_k$'s, τ_i's, ϵ_{ik}'s, ϵ_i^n's, and matrices v_i^k's such that (5a) and the following conditions hold

for $i\in I_0$

(5b), $\forall n=1$,

for $i\in I_1$

(5c), $\forall n=3$,

for $(i,j)\in\{(i,j)|1\le i\le j\le r,\}$

$$\begin{pmatrix}\mathcal{M}_{ik}^5 & * & * \\ \mathcal{L}_5 & -\epsilon_i^5T_5 & * \\ \epsilon_i^5\mathcal{H}_5{}^T & 0 & -\epsilon_i^5I_5\end{pmatrix}<0, \quad (11a)$$

$$\mathcal{T}=\begin{pmatrix}T_{11} & \cdots & T_{1n}\\ \vdots & \ddots & \vdots \\ T_{1n} & \cdots & T_{nn}\end{pmatrix}<0, \quad (11b)$$

where

$$\mathcal{M}_{ik}^5=\begin{pmatrix}\begin{pmatrix}B_iv_j^k+v_j^{k^T}B_i{}^T-2v_kT\\ +B_jv_i^k+v_i^{k^T}B_j{}^T-\eta^{-2}T_{ij}\end{pmatrix} & * & * \\ rB_jv_i^k & -T & * \\ rB_iv_j^k & 0 & -T\end{pmatrix}$$

$\mathcal{H}_5=(\mathcal{H}_{51}\quad \mathcal{H}_{51}),\mathcal{L}_5=\begin{pmatrix}\mathcal{L}_{51}\\ \mathcal{L}_{52}\end{pmatrix}$

$\mathcal{H}_{51}=\begin{pmatrix}H_{b_i} & 0 & 0\\ 0 & 0 & 0\\ rH_{b_i} & 0 & 0\end{pmatrix},\mathcal{H}_{52}=\begin{pmatrix}H_{b_j} & 0 & 0\\ rH_{b_j} & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$

$\mathcal{L}_{51}=diag\left(L_{b_i}v_j^k,0,0\right),\mathcal{L}_{52}=diag\left(L_{b_j}v_i^k,0,0\right)$, $T_5=diag(T,T,T,T,T,T)$,

then $D_{jk}=v_j^kT^{-1}$, $\tau_k=\Delta\tau_k+\tau_{k-1}(\tau_0=t_0)$. ■

All notations are similar to Theorem 2.

Proof. Similar to the proof of Theorem 2, we recall that

$$\begin{pmatrix}\begin{pmatrix}\hat{B}_i(t)v_j^k+v_j^{k^T}\hat{B}_i(t)^T-2v_kT\\ +\hat{B}_j(t)v_i^k+v_i^{k^T}\hat{B}_j(t)^T-\eta^{-2}T_{ij}\end{pmatrix} & * & * \\ r\hat{B}_j(t)v_i^k & -T & * \\ r\hat{B}_i(t)v_j^k & 0 & -T\end{pmatrix}$$
$$=\mathcal{M}_{ik}^5+$$

$$\begin{pmatrix} \begin{pmatrix} H_{b_i}F_{b_i}(t)L_{b_i}v_j^k + {v_j^k}^T\left(H_{b_i}F_{b_i}(t)L_{b_i}\right)^T \\ +H_{b_j}F_{b_j}(t)L_{b_j}v_i^k + {v_i^k}^T\left(H_{b_j}F_{b_j}(t)L_{b_j}\right)^T \end{pmatrix} & * & * \\ rH_{b_j}F_{b_j}(t)L_{b_j}v_i^k & 0 & * \\ rH_{b_i}F_{b_i}(t)L_{b_i}v_j^k & 0 & 0 \end{pmatrix}$$

$$= \mathcal{M}_{ik}^5 + \mathcal{H}_5\mathcal{F}_5(t)\mathcal{L}_5 + {\mathcal{L}_5}^T\mathcal{F}_5(t)^T{\mathcal{H}_5}^T \quad (12)$$

where $\mathcal{F}_5(t) = diag\left(\mathcal{F}_{51}(t), \mathcal{F}_{52}(t)\right), \mathcal{F}_{51}(t) = diag\left(F_{b_i}(t), 0,0\right)$, and $\mathcal{F}_{52}(t) = diag\left(F_{b_j}(t), 0,0\right)$.

By using Lemma A.1 $\left(R = \left(diag(T,T,T,T,T,T)\right)^{-1}\right)$ and Schur's complement, (11a) and (11b) imply (12) is negative. Regarding the proof of Theorem 2, the above proof is now completed. ∎

IV. Illustrative Examples

In this section, a numerical example is presented to verify the results of the proposed stabilization procedure of the affine T-S fuzzy impulsive control system. Based on proposed method, Theorem 3 is applied for a system given by the following T-S fuzzy system as follow

Rule i: $IF\ x_1\ is\ M_1^i\ and\ x_2\ is\ M_2^i$

$$THEN \begin{cases} \dot{x} = \hat{A}_i(t)x + \varpi_i\hat{e}_i(t) \\ u = \sum_{k=1}^{\infty} \delta(t-\tau_k)I_{ik}(x) \end{cases}, \quad (i = 1,2,3).$$

which yields to

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{3} \mu_i\left(\hat{A}_i(t)x + \varpi_i\hat{e}_i(t)\right), & t \neq \tau_k \\ \Delta x = x({\tau_k}^+) - x({\tau_k}^-) = I_k(x) \\ \quad = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_j\,\mu_i\left(\hat{B}_i(t)I_{jk}(x)\right), & t = \tau_k \end{cases}$$

where

$$\hat{A}_1 = \begin{bmatrix} \delta_1 & \delta_1 \\ -\delta_2 & -4 \end{bmatrix}, \hat{A}_2 = \begin{bmatrix} \delta_3 & \delta_2 \\ 0 & 2 \end{bmatrix}, \hat{A}_3 = \begin{bmatrix} \delta_1 & 3 \\ -1 & \delta_3 \end{bmatrix},$$

$$\hat{e}_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The normalized membership functions of subsystem 1 are shown in Fig. 1. The uncertainties are considered as follows. It is also assumed that $\hat{A}_i^l = A_i^l + H_{ai}^l F_{ai}^l L_{ai}^l$, where

$\delta_1 = [(1 - 0.25\%)(1 + 0.25\%)]$,

$\delta_2 = \left[\left({}^2/_3 - 10\%\right)\left({}^2/_3 - 10\%\right)\right]$,

$\delta_3 = [(0 - 0.5\%)(0 + 0.5\%)]$

and

$$A_1 = \begin{bmatrix} 1 & {}^2/_3 \\ -1 & -4 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & {}^2/_3 \\ 0 & 2 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$$

$$F_{a_1} = diag(\xi_1, \xi_2), F_{a_2} = \begin{bmatrix} 0 & \xi_1 \\ \xi_2 & 0 \end{bmatrix}, F_{a_3} = diag(\xi_1, \xi_3)$$

$$H_{a_1} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.1 \end{bmatrix}, H_{a_2} = \begin{bmatrix} 0.1 & 0.4 \\ 0 & 0 \end{bmatrix}, H_{a_3} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.4 \end{bmatrix}$$

$$L_{a_1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, L_{a_2} = L_{a_3} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $\xi_i, i = 1,\cdots,5$ are random numbers on interval $[-1,1]$. Now by using the MATLAB LMI Toolbox, Theorem 3, the following solution is obtained.

$$P = \begin{pmatrix} 2.1630 & -0.0112 \\ -0.0112 & 2.0250 \end{pmatrix}$$

$\mathcal{I}_k = 0.98, v_k = -0.998$

$\left({}^1/_{\Delta\tau_k}\right) = 1.0e+004\ \times [2.6313 \quad 2.6313]$.

$$B_1D_{1k} = \begin{bmatrix} -0.9837 & 0.0020 \\ 0.0020 & -0.9774 \end{bmatrix}$$

$$B_2D_{2k} = \begin{bmatrix} -0.8849 & 0.0031 \\ 0.0031 & -0.8776 \end{bmatrix}$$

$$B_3D_{3k} = \begin{bmatrix} -0.8042 & 0.0039 \\ 0.0039 & -0.7998 \end{bmatrix},$$

$$B_4D_{4k} = \begin{bmatrix} -0.8197 & 0.0007 \\ 0.0007 & -0.8215 \end{bmatrix}.$$

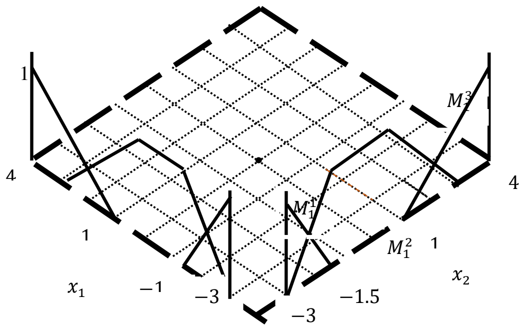

Fig. 1. Membership functions

Conclusion

In this paper, a new approach based on extended Lyapunov theory is proposed to stabilize the uncertain affine T-S fuzzy impulsive control systems. Two classes of uncertain affine T-S fuzzy impulsive control systems have been presented and it is shown that under some sufficient conditions, impulsive controller can be developed to stabilize uncertain affine T-S fuzzy systems. It has been also shown that the stabilization parameters can be determined by solving a set of matrix inequalities. Through this conditions, there is no need to set pre-defined controller parameters to solve matrix inequalities where there solutions indicates the stability (uniformly, asymptotically) of affine T-S fuzzy systems.

Appendix

Lemma A.1 (Xie [32]). Given matrices Q, H, R, E of appropriate dimensions with $Q = Q^T$, $R = R^T$ and $R > 0$ then

$Q + HFE + E^TF^TH^T < 0$,

For all F satisfying $F^TF < R$, if and only if there exists some $\epsilon > 0$ such that $Q + \epsilon HH^T + \epsilon^{-1}E^TRE < 0$ ∎

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