13th International Conference on Industrial Engineering and Industrial Management
XXIII Congreso de Ingeniería de Organización
Gijón, Spain, July 11-12, 2019

An improved mathematical model for a two-agent scheduling problem in a two-machine flow shop

Ahmadi-Darani M-H[1], Sahraeian R[2]

Abstract This paper addresses a two-agent scheduling problem on a two-machine flow shop. The objective is to minimize the total tardiness of the first agent jobs in the circumstance that no tardy job is allowed for the second agent. This problem has investigated in the literature, and a branch-and-bound algorithm and some meta-heuristic developed to solve it. In this study, first, a mixed integer mathematical programming (MIP) model is developed and then improve the efficiency of this model by using of some optimal solution properties. Experimental results show that the MIP model outperforms the branch and bound algorithm in efficiency by a significant margin and the improved MIP model reduced running time about 37.9 percentages.

Keywords: Two-agent scheduling; Two-machine flow shop; MIP model;

1 Introduction

The management problems in which multiple agents compete on the usage of a common processing resource are receiving increasing attention in different application environments and different methodological fields, such as artificial intelligence, decision theory, operations research, etc. (Agnetis et al., 2004). One major stream of research in this context is multi-agent scheduling problems that have been an active area of research for the past three decades. In this problem, each agent has a certain objective function independent of the other agents' objective. For example, in industrial management, the multi-agent scheduling problem can be formulated as a sequencing game, where the objective is to devise some mechanisms to encourage the agents to cooperate with a view to minimizing the overall cost (Curiel et al., 1989), (Hamers et al., 1995). In project scheduling, the problem can be concerned with negotiation to resolve conflicts whenever the agents find their own schedules unacceptable (Kim et al., 2000). In telecommunication services, the problem is to do with satisfying the service requirements of individual agents, who compete for the use of a commercial satellite to transfer voice, image and text files for their clients (Cheng et al., 2008). In rescheduling that can be defined as the process of updating a current production schedule in response to disruptions or other changes (Herrmann, 2006), such as the arrival of new jobs to be processed. In a supply chain, a classical problem is to minimize overall manufacturing

[1]Mohammad-Hasan Ahmadi-Darani (e-mail: Mh.AhmadiDarani@Shahed.ac.ir)
Department of Industrial Engineering, College of Engineering, Shahed University, Persian Gulf Expressway, Tehran, Iran.

[2]Rashed Sahraiean (✉e-mail: Sahraeian@Shahed.ac.ir)
Department of Industrial Engineering, College of Engineering, Shahed University, Persian Gulf Expressway, Tehran, Iran.

and distribution costs integrating production and delivery. If there are two customers with corresponding jobs that competing for using a common processing resource, then the problem implies interfering jobs (Perez-Gonzalez and Framinan, 2014).

Two-agent scheduling problems problem classified to constrained optimization problem, Pareto optimization problem and recognition problem (Agnetis et al., 2004). In the constrained optimization problem, given the job sets of the two agents with corresponding objective functions and the goal is to find a schedule such that objective function of the first agent optimized while the objective function of the second agent bounded by an upper bound. In the Pareto optimization case, all non-dominated schedules must be generated, and in the recognition problem, the goal is to find a feasible schedule such that the objective function of each agent is smaller than the predetermined values.

In this paper, a two-agent constrained optimization scheduling problem in a supply chain is considered. Each classical machine scheduling problems introduced by two important parts; objective function and machine environment. In the objective function perspective, to optimize the schedule's cost and to deliver the products to the customers on time are always among the highest priorities. Both total tardiness and total number of tardy job performance measures have a significant impact on a schedule's cost and agent's responsiveness. On the other view, in many applications in both industrial and planning areas, each job must undergo two or more basic processes in the same order implying that the jobs have to follow the same route. This environment is referred to as a flowshop. For example, in a supply chain, each product is processed in the first stage and then is delivered to a customer; in the steel industry, each job undergoes wire drawing first and annealing next; in general manufacturing industries, each job must undergo fabrication first and assembly next.

Motivated by these, the objective of considered problem is to minimize the total tardiness of the first agent with the restriction that the number of tardy jobs of the second agent is zero, where the machine environment is flowshop with two machine. This problem have been shown to be NP-hard in the strong sense by Lee et al. (2010a) and they proposed branch and bound algorithm to find exact solutions. However, their proposed algorithm couldn't efficiently solve problem instances with more than 12 jobs in size. Since this problem is NP-hard in the strong scene, solving it to optimality in a reasonable amount of time is difficult. To this end, in this study a mixed integer mathematical programming (MIP) model developed to tackle the problem. Afterward, by using literature optimal solution properties, a set of valid inequalities created for the problem to strengthen the formulation and value of some decision variables fixed to accelerate the search process of the MIP model.

In the rest of this paper, first the literature is reviewed and then the studied problem is described. MIP model is introduced in Section 4. In Section 5, some of optimal solution properties are proposed and applied in the MIP model. Performance of the MIP model and the branch and bound algorithm is compared in Section 6, and at last conclusions and future research are presented.

2 Literature Review

As mentioned in section 1, Lee et al. (2010a) considered the two-agent scheduling problem in the two machine flow shop environment. They used a branch-and-bound algorithm and simulated annealing heuristic algorithms to find the optimal and near-optimal solutions of the problem, respectively. However, they don't propose a mathematical formulation of the problem. So, in the current study a mathematical formulation of the problem will be investigated and in this section, some of the relative studies are reviewed.

Because of the importance of the single machine environment in the scheduling problems, most of the multi-agent scheduling problems are studied in the single machine environment. Also, most of the studies have dealt with the two-agent cases because increasing the number of agents increases the complexity of such problems. However, the multi-agent scheduling problems are considered in other environments such as shop and parallel machine. Agnetis et al. (2004) were the first authors that investigated the two-agent scheduling problems in the two-machine shop environment. They proved that the complexity of the constrained optimization problem with makespan criterion for both agents is NP-hard. For this problem, Luo et al. (2012) provided a dynamic programming algorithm with pseudo-polynomial time complexity and an FPTAS. Lee et al. (2011) considered a constrained optimization problem in which the objective of the first agent's is minimizing total completion time while no tardy job is allowed for the second agent's job. Fan and Cheng (2016) studied two-agent scheduling problem in a two-machine flow-shop with two distinct cost function. They proposed a pseudo-polynomial-time algorithm and an approximation algorithm for these problems. Shiau et al. (2015) studied a constrained optimization problem with learning effects. The objective is to minimize the total completion time of the jobs from one agent, given that the maximum tardiness of the jobs from the other agent cannot exceed a bound. They provide a branch-and-bound algorithm and several genetic algorithms.

In the literature, there are much multi-agent scheduling problems in the single machine environment have been investigated, but in the following, some of the most related studies will be referred to. Lee et al. (2010b) considered a single-machine scheduling problem with a linear deterioration assumption where the objective is to minimize the total weighted completion time of jobs from the first agent with the restriction that no tardy job is allowed for the second agent. Wu et al. (2011) investigated the constrained optimization problem in the single-machine environment with learning effects. The objectives are the total tardiness and the number of tardy jobs of the first and second agent jobs, respectively. Wu et al. (2013) deliberated upon a two-agent single-machine scheduling problem with deteriorating jobs. The objective of this problem was to minimize the total weighted number of tardy jobs of the first agent's jobs and the limited maximum lateness of the second agent. Cheng et al. (2011) consider a two-agent single-machine scheduling problem with deteriorating jobs and learning effects, simultaneously. The objective is to minimize the total weighted completion time of the jobs of the first agent with the restriction that no tardy job is allowed for the second agent. In all these studies, the authors developed a branch-and-bound algorithm and some heuristic algorithms to search for the optimal solution and near-optimal solutions, respectively. For more studies on this line of research, the reader may refer to Agnetis et al. (2007), Leung et al. (2010), Perez-Gonzalez and Framinan (2014) and Agnetis et al. (2014). Also for acquaintance with some new research in this context that considering more assumption of real condition, studies of Jiang et al. (2018), Li et al. (2018), Wu et al. (2017) and Wang et al. (2017) can be reviewed.

3 Problem Definition

In this paper, the two-agents constrained optimization problem in the two-machine flow shop is considered. The objective is finding a schedule that minimizes total tardiness of the first agent jobs, under the situation that the total number of tardy jobs of the second agent is zero. There are n jobs simultaneously available at the zero time for processing on a two-machine flowshop. They belong to one of two agents, namely agent A and agent B. Let J_A with n_A jobs and J_B with n_B jobs ($J_A \cap J_B = \emptyset$) be set of agents jobs and parameters a_j, b_j and d_j define the processing time on the machine 1, processing time on the machine 2 and due date for each job j, respectively. Also, for a given schedule σ, let $C_j^x(\sigma)$ and $T_j^x(\sigma) = Max\ \{(0, C_j^x(\sigma) - d_j) \forall\ j\ \in J_x\}$ are the completion time and the tardiness of job j of the agent x, respectively. Also, Let $U_j^x(\sigma)$ be a binary variable that equals to one if the job j is tardy and equal to zero otherwise. By defining these notations, the objective is finding an optimal schedule such that minimizes $\sum_{j=1}^{n_A} T_j^A(\sigma)$ where$\sum_{j=1}^{n_B} U_j^B(\sigma) \leq 0$.

4 Mathematical Programing

The considered problem in this study is NP-hard in the strong sense because of when the number of agent B jobs is zero, the considered problem is reduced to the minimizing total tardiness problem which is strongly NP-hard (Lenstra et al., 1977). By considering problem complexity, Lee et al. (2010a) have proposed the branch-and-bound algorithm and the simulated annealing heuristic algorithm to solving the considered problem. However, they haven't proposed any mathematical model for this problem. In this section, a mathematical formulation for the considered problem is proposed.

The mathematical formulation is based on the following decision variables:

x_{pj}: is 1 if the job j is scheduled in the p-th position, and zero otherwise,

$T_{[p]}$: is equal to the tardiness of job scheduled in the position p,

$C_{[p]m}$: is completion time of job scheduled at position p on the machine m.

By defining these variables, the MIP formulation is:

$$Minimize\ \sum_{p=1}^{n} T_{[p]} \tag{1}$$

$$subject\ to$$

$$\sum_{p=1}^{n} x_{pj} = 1 \qquad i = 1, 2, \dots, n. \tag{2}$$

$$\sum_{j=1}^{n} x_{pj} = 1 \qquad p = 1, 2, \dots, n. \tag{3}$$

$$C_{[p]1} = \sum_{j=1}^{n} a_j x_{1j} \tag{4}$$

$$C_{[p]1} - C_{[p-1]1} = \sum_{j=1}^{n} a_j x_{pj} \qquad p = 2, 3, \dots, n. \tag{5}$$

$$C_{[1]2} = \sum_{i=1}^{n} (a_i + b_i) x_{1j} \tag{6}$$

$$C_{[p]2} \geq C_{[p-1]2} + \sum_{j=1}^{n} b_j x_{pj} \qquad p = 2, 3, \dots, n. \tag{7}$$

$$C_{[p]2} \geq C_{[p-1]1} + \sum_{j=1}^{n} (a_j + b_j) x_{pj} \qquad p = 2, 3, \dots, n. \tag{8}$$

$$T_{[p]} \geq C_{[p]2} - \sum_{j \epsilon J_A} (d_j - M) x_{pj} - M \qquad p = 1, 2 \dots, n. \tag{9}$$

$$C_{[p]2} \leq \sum_{j \epsilon J_B} \left(d_j - M\right) x_{pj} + M \qquad p = 1, 2 \dots, n. \tag{10}$$

$$T_{[p]}, C_{[p]m} \geq 0 \text{ and } x_{jp} \in \{0, 1\} \qquad p = 1, 2, \dots, n.\ j = 1, 2, \dots, n. m = 1, 2 \tag{11}$$

In the MIP model, the objective function (1) minimizes total tardiness for jobs in each position of the sequence. According to constraint (10), the tardiness of agent B jobs have to be zero and so, equation (1) is equal to minimizing the total tardiness of agent A jobs. Equation (2) and (3) states that at each position there is sequenced only one job, and each job must be assigned to exactly one position in the sequence, respectively. Constraints (4) to (8) give the completion time of the scheduled job at each position on the machine1 and 2. Constraint (9) states that tardiness is calculated for the first agent job. Finally, according to constraint (10), if a job of the agent B scheduled at a position, its completion time is limited to the corresponding due date value. The proposed *MIP* formulation includes $O(n^2)$ binary and $O(n)$ continuous variables, and $O(n)$ constraints.

5 MIP Model Improvement

In this section, some optimal solution properties and lower bound of some MIP model variables, are applied to improve the efficiency of the model. At the first, set of jobs that satisfied these properties conditions are determined and they are used in valid inequality that is added to the MIP model. Also, the calculated lower bound of some MIP model variables is applied to set some binary variables value to zero. In the following, these procedures are described.

5.1 Valid inequality

Pan and Fan (1997) proposed a branch and bound algorithm to minimize the total tardiness of jobs in the two machine flow shop environment; they used some theorems and lower bounds in their developed algorithm. One of these theorem indicates that for given two jobs i and j, there exists an optimal schedule such that job j is processed after job i. In the following, this theorem is proposed.

Theorem 1(Pan and Fan, 1997): For any two jobs i and j, if conditions $a_i \le a_j$, $b_i = b_j$ and $d_i \le d_j$ are met, then there exists an optimal schedule such that job j is processed after job i.

It's obvious that this theorem can be generalized for the two agent scheduling problems, where both jobs i and j belong to the agent A. To formulate this property as a valid inequality, let S be a set of pair jobs that satisfied conditions of this theorem. The following valid inequalities can be added to the MIP model.

$$\sum_{p=1}^{n} kx_{pi} + 1 \le \sum_{p=1}^{n} kx_{pj} \quad \forall\ (i,j) \in S \tag{12}$$

Note that the expression $\sum_{k=1}^{n} kx_{ki}$ denotes the position index of job i and this inequality entitle that for each pair of job i and j belong to the set S, job i processed before job j.

5.2 Determining value of some binary variable

Haouari and Kharbeche (2013) developed a procedure to calculate the lower bound of completion times of jobs in different positions of the sequence. It is obvious that for some of the jobs, the lower bound of completion time in some positions of the sequence is greater than due date values. Thus Due to restriction on the completion time of the jobs of agent B (that must be lower than or equal to its due dates), these jobs can't place in these positions and so, the corresponding binary variable must be set to zero.

6 Numerical Experiment

To evaluate the performance of the MIP Model in comparison with the branch and bound algorithm, a computational experiment was conducted. The problem instances were generated following the strategy described in Pan and Fan (1997) and Lee et al. (2011). The processing times on machine 1 and machine 2 were randomly drawn from a discrete uniform distribution between [1, 100]. The due dates of agent A jobs were from a discrete uniform distribution between [δ (1-τ-R/2), δ (1-τ+R/2)] where R is the due date range parameter, τ is the tardiness factor, and δ is a simple lower bound on the makespan. The due dates of Agent B jobs were from a discrete uniform distribution [δ, δ (1+R)] as suggested by Lee et al. (2011). The values of R for both agent (R(A) and R(B)) and τ for the agent A are selected as R(A) = R(B) = {0.25, 0.75} and τ = {0.25, 0.75}, respectively. Also, three values of {0.25, 0.50 and 0.75} for the proportions of the first agent jobs (ρ) are considered. As a consequence, according to the values of τ, R and ρ, the instances were generated in 24(2×2×2×3) different groups (G01 to G24) showed in table 1 and in the each one of this groups, 10 problem instances were generated.

Table 1: Groups parameters

Group	ρ	τ	*R(A)*	*R(B)*	*Group*	ρ	τ	*R(A)*	*R(B)*	*Group*	ρ	τ	*R(A)*	*R(B)*
G01	***0.4***	***0.25***	***0.25***	***0.25***	***G09***	***0.5***	***0.25***	***0.25***	***0.25***	***G17***	***0.6***	***0.25***	***0.25***	***0.25***
G02				***0.75***	***G10***				***0.75***	***G18***				***0.75***
G03			***0.75***	***0.25***	***G11***			***0.75***	***0.25***	***G19***			***0.75***	***0.25***
G04				***0.75***	***G12***				***0.75***	***G20***				***0.75***
G05		***0.75***	***0.25***	***0.25***	***G13***		***0.75***	***0.25***	***0.25***	***G21***		***0.75***	***0.25***	***0.25***
G06				***0.75***	***G14***				***0.75***	***G22***				***0.75***
G07			***0.75***	***0.25***	***G15***			***0.75***	***0.25***	***G23***			***0.75***	***0.25***
G08				***0.75***	***G16***				***0.75***	***G24***				***0.75***

The branch and bound algorithm and MIP Model were coded in C# and the MIP were solved using CPLEX 12.6. All experiments were carried out on an Intel i5-2450 Processor 2.5 GHz with 4 GB RAM. Also, the time limit to optimally solve the instances was set to 1800 seconds. The computational result contains the result of solving the instance problems by the branch and bound algorithm (Lee et al., 2010a), the initial MIP and the modified MIP (MMIP) and The mean execution seconds for each group of the problem instance were reported in table 2. In this table, notations 'n' and 'BB' indicate the number of jobs and the branch and bound algorithm, respectively. Also, the column 'Impv' shows the improvement percentage in problems solution time that is derived from using optimal solution properties in the MIP model. In table 2, the results of the groups were reported in which all 10 problem instances could be solved.

It can be seen that the performance of the MIP model was better than the branch and bound algorithm. Also, the results show that the MMIP model can solve the problem instances up to 40 jobs in size in a reasonable amount of time while the branch and bound algorithm can solve the problem instances with 15 jobs in size. It can be seen that the performance of the MMIP model was better for the G01 but became worse for the G08 problem instance. Also, the MMIP model reduces solution time about 37.9 percentages in comparisons of the MIP model.

Results show that the due date factor (τ) of agent A and the due date range of agent B (R(B)) have a significant effect on solution time. When these parameter increasing, the solution time increases as well. Its because of when the due date factor (τ) of agent A increase, the due date value of agent A jobs generate in tighter range and close to each other and so, finding the solution in which total tardiness is minimized will be difficult. Similarly, when R(B) increasing, the due date value of agent B jobs generate in looser range and feasible solution space in which agent B jobs remain without delay, increasing. Moreover, the problems were easier to solve when the number of Agent A jobs increased specially where the tardiness factor (τ) equal to 0.75.

7 CONCLUSIONS

In this paper, the two-agent scheduling problem on the two machine flow shop was considered. The objective is to minimize the total tardiness of the first agent with the restriction that the number of tardy jobs of the second agent is zero. For this problem, the MIP model incorporating with several optimal solution properties to search for the optimal solution was developed. Results show that the proposed model produces the optimal solution in an effective manner and reasonable time. Computational results indicated that the proposed MIP model outperforms the literature branch and bound algorithm and can solve the instances of 40 jobs in a reasonable amount of time.

This MIP Model, therefore, can also be used for comparison purposes in future studies and can be extended in several points: the mathematical model can be used as an exact method for a similar two machine scheduling problem with some other scheduling criteria, devising efficient and effective dominance rules and using them in the MIP model to solve the problem with a significantly larger number of jobs, extension to different resource usage modes (concurrent usage, preemption, etc.) and different system structures (parallel machines) and finally generalization of the problem to more than two agents.

Table 2: Computational results

n	***Group***	***Mean CPU time(s)***			***Impv***	***Group***	***Mean CPU time(s)***			***Impv***	***Group***	***Mean CPU time(s)***			***Impv***
		BB	***MIP***	***MMIP***			***BB***	***MIP***	***MMIP***			***BB***	***MIP***	***MMIP***	
12	***G01***	0.34	0.73	0.29	60.27	***G09***	0.21	1.11	0.37	66.41	***G17***	0.09	0.84	0.42	50.18
15		107.77	1.24	0.81	34.68		75.14	1.19	0.45	62.56		13.81	1.18	0.49	58.21
20		-	2.05	1.13	44.88		-	2.10	1.69	19.45		-	3.74	1.44	61.48
25		-	4.75	2.46	48.21		-	6.85	2.22	67.53		-	16.02	5.34	66.65
30		-	10.51	4.33	58.80		-	23.92	4.77	80.06		-	39.44	6.16	84.38
40		-	-	35.5	-		-	-	58.69	-		-	-	146.81	-
12	***G02***	3.08	0.64	0.43	32.81	***G10***	0.66	0.71	0.34	52.50	***G18***	0.42	0.67	0.22	67.32
15		-	1.18	1.49	-26.27		184.89	1.27	1.22	4.21		120.27	1.09	0.53	79.85
20		-	13.37	9.22	31.04		-	3.33	4.16	-25.05		-	2.02	1.39	31.25
25		-	55.44	184.03	-231.94		-	25.94	24.60	5.17		-	6.38	2.83	55.65
30		-	299.19	387.76	-29.60		-	68.27	42.31	38.03		-	18.36	4.93	73.16
40		-	-	-	-		-	-	-	-		-	-	36.96	-
12	***G03***	10.22	0.78	0.35	55.13	***G11***	0.28	0.82	0.34	57.76	***G19***	0.71	0.70	0.44	37.88
15		-	1.06	0.51	51.89		172.20	1.15	0.57	50.70		77.31	1.29	1.05	18.93
20		-	2.22	1.4	36.94		-	7.68	1.65	78.51		-	3.51	1.91	45.68
25		-	4.91	2.39	51.32		-	32.33	3.84	88.11		-	170.33	14.31	91.60
30		-	14.27	57.53	-303.15		-	177.31	12.63	92.88		-	408.18	42.24	89.65
40		-	-	10.61	-		-	-	77.88	-		-	-	-	-
12	***G04***	3.39	1.03	0.47	54.37	***G12***	2.29	0.75	0.46	38.90	***G20***	1.52	0.99	0.43	57.15
15		-	2.22	1.54	30.63		226.06	1.22	1.35	-10.28		296.95	1.34	0.71	46.70
20		-	6.85	6.38	6.86		-	4.74	5.07	-7.00		-	3.38	3.11	8.05
25		-	122.76	105.59	13.99		-	92.26	57.61	37.56		-	11.51	6.83	40.65
30		-	-	-	-		-	13.96	10.39	25.59		-	80.42	113.61	-41.28
40		-	-	-	-		-	-	-	-		-	-	115.60	-
12	***G05***	0.84	0.63	0.18	71.43	***G13***	0.15	1.23	0.43	64.73	***G21***	0.04	1.55	0.35	77.59
15		411.8	1.13	0.96	15.04		25.14	2.06	1.15	44.22		2.37	2.48	1.34	46.02
20		-	4.83	2.42	49.90		-	14.53	5.08	65.04		-	10.00	2.41	75.85
25		-	-	-	-		-	-	-	-		-	104.97	7.44	92.91
30		-	-	-	-		-	-	-	-		-	-	90.07	-
12	***G06***	2.7	1.08	0.74	31.48	***G14***	0.10	1.23	1.06	14.26	***G22***	0.44	1.84	1.10	40.10
15		283.65	4.46	3.93	11.88		13.78	4.26	4.07	4.55		17.56	3.08	1.96	36.20
20		-	-	-	-		-	-	-	-		-	35.21	13.46	61.76
12	***G07***	1.18	0.65	0.24	63.08	***G15***	0.35	0.97	0.53	45.76	***G23***	0.20	1.26	0.37	70.39
15		343.09	1.72	1.02	40.70		55.66	3.27	1.17	64.22		18.29	2.44	1.22	50.08
20		-	7.82	4.56	41.69		-	12.48	3.75	69.96		-	10.67	3.87	63.74
25		-	97.8	45.16	53.82		-	56.20	17.66	68.57		-	64.96	15.82	75.65
12	***G08***	0.47	0.94	0.87	7.45	***G16***	0.91	1.38	0.86	37.66	***G24***	0.57	1.17	1.12	3.67
15		45.87	5.62	4.23	24.73		228.94	3.76	3.08	18.20		90.75	2.66	1.89	28.89
20		-	450.07	185.68	58.74		-	197.36	133.23	32.49		-	193.05	24.78	87.16

8 References

AGNETIS, A., BILLAUT, J.-C., GAWIEJNOWICZ, S., PACCIARELLI, D. & SOUHAL, A. 2014. *Multi-agent scheduling,* Berlin, Springer Berlin Heidelberg.

AGNETIS, A., MIRCHANDANI, P. B., PACCIARELLI, D. & PACIFICI, A. 2004. Scheduling Problems with Two Competing Agents. *Operations Research,* 52**,** 229-242.

AGNETIS, A., PACCIARELLI, D. & PACIFICI, A. 2007. Multi-agent single machine scheduling. *Annals of Operations Research,* 150**,** 3-15.

CHENG, T. E., NG, C. & YUAN, J. J. E. J. O. O. R. 2008. Multi-agent scheduling on a single machine with max-form criteria. 188**,** 603-609.

CHENG, T. E., WU, W.-H., CHENG, S.-R. & WU, C.-C. 2011. Two-agent scheduling with position-based deteriorating jobs and learning effects. *Applied Mathematics and Computation,* 217**,** 8804-8824.

CURIEL, I., PEDERZOLI, G. & TIJS, S. J. E. J. O. O. R. 1989. Sequencing games. 40**,** 344-351.

FAN, B. Q. & CHENG, T. C. E. 2016. Two-agent scheduling in a flowshop. *European Journal of Operational Research,* 252**,** 376-384.

HAMERS, H., BORM, P. & TIJS, S. J. M. P. 1995. On games corresponding to sequencing situations with ready times. 69**,** 471-483.

HAOUARI, M. & KHARBECHE, M. 2013. An assignment-based lower bound for a class of two-machine flow shop problems. *Computers & Operations Research,* 40**,** 1693-1699.

HERRMANN, J. W. 2006. Rescheduling strategies, policies, and methods. *Handbook of production scheduling.* Springer.

JIANG, F., ZHANG, X., BAI, D. & WU, C.-C. J. E. O. 2018. Competitive two-agent scheduling problems to minimize the weighted combination of makespans in a two-machine open shop. 50**,** 684-697.

KIM, K., PAULSON, B., PETRIE, C. J. & LESSER, V. R. Compensatory negotiation for agent-based project schedule coordination. Proceedings Fourth International Conference on MultiAgent Systems, 2000. IEEE, 405-406.

LEE, W.-C., CHEN, S.-K., CHEN, C.-W. & WU, C.-C. 2011. A two-machine flowshop problem with two agents. *Computers & Operations Research,* 38**,** 98-104.

LEE, W.-C., CHEN, S.-K. & WU, C.-C. 2010a. Branch-and-bound and simulated annealing algorithms for a two-agent scheduling problem. *Expert Systems with Applications,* 37**,** 6594-6601.

LEE, W.-C., WANG, W.-J., SHIAU, Y.-R. & WU, C.-C. 2010b. A single-machine scheduling problem with two-agent and deteriorating jobs. *Applied Mathematical Modelling,* 34**,** 3098-3107.

LENSTRA, J. K., KAN, A. R. & BRUCKER, P. 1977. Complexity of machine scheduling problems. *Annals of discrete mathematics,* 1**,** 343-362.

LEUNG, J. Y. T., PINEDO, M. & WAN, G. 2010. Competitive Two-Agent Scheduling and Its Applications. *Operations Research,* 58**,** 458-469.

LI, S.-S., CHEN, R.-X., LI, W.-J. J. A. J. F. S. & ENGINEERING 2018. Proportionate Flow Shop Scheduling with Multi-agents to Maximize Total Gains of JIT Jobs. 43**,** 969-978.

LUO, W., CHEN, L. & ZHANG, G. 2012. Approximation schemes for two-machine flow shop scheduling with two agents. *Journal of combinatorial optimization,* 24**,** 229-239.

PAN, J. C.-H. & FAN, E.-T. 1997. Two-machine flowshop scheduling to minimize total tardiness. *International Journal of Systems Science,* 28**,** 405-414.

PEREZ-GONZALEZ, P. & FRAMINAN, J. M. 2014. A common framework and taxonomy for multicriteria scheduling problems with interfering and competing jobs: Multi-agent scheduling problems. *European Journal of Operational Research,* 235**,** 1-16.

SHIAU, Y.-R., TSAI, M.-S., LEE, W.-C. & CHENG, T. C. E. 2015. Two-agent two-machine flowshop scheduling with learning effects to minimize the total completion time. *Computers & Industrial Engineering,* 87**,** 580-589.

WANG, D.-J., YIN, Y., WU, W.-H., WU, W.-H., WU, C.-C. & HSU, P.-H. J. S. C. 2017. A two-agent single-machine scheduling problem to minimize the total cost with release dates. 21**,** 805-816.

WU, C.-C., HUANG, S.-K. & LEE, W.-C. 2011. Two-agent scheduling with learning consideration. *Computers & Industrial Engineering,* 61**,** 1324-1335.

WU, W.-H., XU, J., WU, W.-H., YIN, Y., CHENG, I. F. & WU, C.-C. 2013. A tabu method for a two-agent single-machine scheduling with deterioration jobs. *Computers & Operations Research,* 40**,** 2116-2127.

WU, W.-H., YIN, Y., CHENG, T., LIN, W.-C., CHEN, J.-C., LUO, S.-Y. & WU, C.-C. J. J. O. T. O. R. S. 2017. A combined approach for two-agent scheduling with sum-of-processing-times-based learning effect. 68**,** 111-120.