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A stochastic multi-product, multi-stage supply chain design considering products waiting time in the queue

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Abstract
In this paper, we develop a supply chain network design (SCND) model considering both strategic and operational decisions. The model determines plant and DC locations as well as product shipments among components of network regarding single sourcing and capacity of plants and distribution centers (strategic level) while the shipments have to wait in the queue for transporting from plants to DCs (operational level), which result in the lead time is incorporated in model. In practice, the parameters of problem such as demand, cost and capacity are changed and aren't described as certain. Hence, we extend proposed SCND model by defining demands as different scenarios and apply the two-stage stochastic programming approach to solve it. Finally, a numerical example is given to illustrate the mentioned model and some other scenario based approaches are presented so that decision makers select one of the approaches based on their policy.

Keywords
Supply Chain Network Design, Lead time, Stochastic Programming

1. Introduction
The concept of supply chain management (SCM), which appeared in the early 1990s, has recently raised a lot of interest since the opportunity of an integrated SCM can reduce the unexpected/undesirable events through the network and can affect definitely the efficiency of all the members. A critical and momentous component of the planning activities in SCM is the efficient design of its supply chain. SCND problems include extensive scope of formulations ranged from simple single product type to complex multi-product/multi-period one, from linear deterministic models to complex non-linear stochastic ones and from customary forward networks to closed-loop ones. Many attempts have been made to model and optimize the supply chain design and these studies recently have been surveyed by Melo et al [11]. In SCM, three planning levels are usually distinguished depending on the time horizon: strategic (long term), tactical (medium term) and operational (short term). Simchi-Levi et al. [13] state that ''the strategic level deals with decisions that have along-lasting effect on the firm. These include decisions regarding the number, location and capacities of warehouses and manufacturing plants, or the flow of material through the logistics network". This statement specifies a clear link between location models and strategic SCM. In practice, strategic decisions are made by top managers, while the tactical and operational decisions are made by bottom level managers. Examples of these tactical/operational aspects are the lead time and inventory control policy, the choice of transportation mode/capacity, warehouse design and management, vehicle routing, among others. On the other hand the uncertainty of parameters such as demand, cost and capacity is an important problem. To deal with the uncertainty of parameters exist two approaches: stochastic programming or robust optimization. In stochastic programming, the objective is to minimize the expected cost. In robust optimization, no probability information is known, however, and the objective is typically to minimize the worst-case cost or regret. (The regret of a solution under a given scenario is the difference between the objective function value of the solution under the scenario and the optimal objective function value for that scenario.)

Our purpose in this paper is to present a supply chain network model considering both strategic and operational decisions. The model determines plant and DC locations as well as product shipments among components of network regarding single sourcing and capacity of plants and distribution centers (strategic level) while the shipments have to wait in the queue for transporting from plants to DCs (operational level). Then we extend proposed SCND model by defining demands as different scenarios and apply the two-stage stochastic programming approach to solve it. The rest of the paper is organized as follows. The next section briefly reviews the existing works in context of supply chain network design. Section 3 describes the proposed model and then it is illustrated by a numerical example in section 4. Finally, conclusions are given in section 5.

2. Literature review

Given a set of potential facility locations with capacity limits on the demand that can be served by each location and a set of customers, the objective of the fixed charge capacitated facility location problem (CFLP) is to locate distribution centers (DCs) among candidate locations to satisfy the demand points while minimizing the sum of fixed location and transportation costs. A number of authors [5, 2, 9] present models and solution procedures for the CFLP and its variations. Also SCND models are recently reviewed by Melo et al [11] in which identify four basic features for network structure to make it useful in strategic supply chain planning: multi-layer facilities, Single commodity/multiple commodities, single/multiple period(s) and deterministic/stochastic parameters. Usual questions to be answered are: (i) which facilities should be used (opened)? (ii) Which customers should be serviced from which facility (or facilities) so as to minimize the total costs? In addition to this generic setting, a number of constraints such as lead time are considered from the specific application domain. In literature [1, 12, 7, 14, 15] explicitly consider lead times in network design. Also in recent work [3, 4, 17, 18], the lead time at a candidate location is modeled as an explicit function of the volume of flow transport through that location.

Most real SC design problems are characterized by numerous sources of technical and commercial uncertainty, and so the assumption that all model parameters, such as cost coefficients, supplies, demand, etc., are known with certainty is not realistic. A supply chain network is supposed to be in use for a considerable time during which many parameters can change. If a probabilistic behavior is associated with the uncertain parameters (either by using probability distributions or by considering a set of discrete scenarios each of which with some subjective probability of occurrence), then a stochastic model may be the most appropriate for this situation. There are a few research works addressing comprehensive (strategic and tactical issues simultaneously) design of SC networks using stochastic models. Mir Hassani et al. [10] considered a two-stage model for multi-period capacity planning of SC networks. Tsiakis et al. [16] also considered a two-stage stochastic programming model for SC network design under demand uncertainty. Goh et al. [6] developed a stochastic model of the multi-stage global SC network problem, considering supply, demand, exchange and disruption as the uncertain parameters.

3. Problem description

We consider a supply chain network design problem with production facilities that produce multiple products for which demand occurs at geographically discrete locations (retailer) as possible scenarios. The objective is to locate plants and DCs to serve the retailers such that the sum of fixed cost of operating and opening plants and DCs plus the expected costs of transportation and inventory is minimized. In next section, we follow [14, 15] for modeling lead time at the DCs whose explanations are given below. Their model is a deterministic single product SCND problem with only one manufacturing plant while our model is a stochastic multi-plant, multi-product SCND problem with capacity restrictions for warehouses and manufacturing plants in which location of both the plant and warehouse are determined.

3.1. Modeling lead time at the DCs

We assume that products are shipped from the production facilities to DCs in full truckloads. The products incur a waiting time at the production facilities until material are accumulated adequately to fill a truck. Sending full truckloads is not necessarily optimal in all situations, but it is assumed that the DCs are far enough from the production facilities and that the shipment sizes are high enough (since we group the demands of multiple retailers) to justify it. The shipments dock at an unloading zone when they arrive at the DC and wait in a First-In-First-Out (FIFO) queue to be unloaded and sent to the retailers. This process is similar to the operation of a cross-docking facility and is shown in Fig.1 [15]. For such a replenishment process, the replenishment lead time at a DC has three components:

- Load make-up time – The time elapse in the waiting area of the production facilities before the products are transmitted to the DC. As more demand is assigned to a DC, the average load make-up time per unit decreases.
- Constant DC replenishment time (time/unit) – The replenishment lead time between the production facility and the DC due to the physical locations of facilities. We assume that it also comprises the time spent due to postponements such as material handling, and also general inefficiency and unavoidable processing, such as paperwork.
- Congestion time – The time elapse in the unloading zone. At high utilization of the resources at the unloading zone, shipments have to wait longer in the queue. This states that congestion time increases as the demand assigned to a location approaches its capacity.

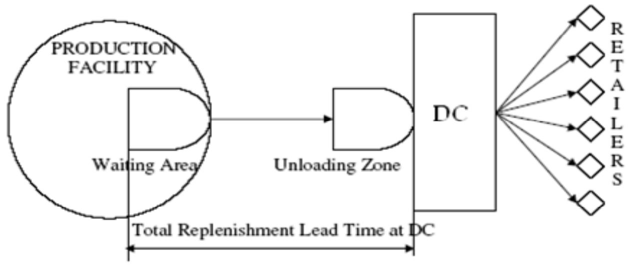

Fig.1. Supply chain structure used for modeling replenishment lead time at DCs.

The total mean demand answered by a DC specifies the expected load make-up time for a shipment to the DC. Whereas all shipments to a DC vie for the same resources at its unloading zone, the expected congestion time is also determined by the total mean demand served by a DC. Given this, we assume the replenishment lead time at a DC to be of the form:

$$\mathrm{LT} = \frac{\mathrm{P}}{\mathrm{B}} + \mathrm{q} + \frac{\mathrm{r}}{\mathrm{W} - \mathrm{B}} \tag{1}$$

Where B denotes the total mean demand allocated to the DC and W the throughput at DC. p is referred as the load make-up time parameter and r as the congestion time parameter. The motivation for this lead time model are deduced from earlier work by Eskigun [3], which used a model of this type derived from an extensive simulation study. Wang et al. [18] also use a similar model for lead times. The first term shows the average load make-up time per unit, q the constant DC replenishment time per unit, and the third term the average congestion time per unit. As the total mean demand at DC increases, load make-up time decreases and congestion time increases.

3.2. Mathematical model

In this section, we provide a mixed integer non-linear programming formulation to the single-source, multi-product, multi-stage SCND problem under uncertain demands along incorporating lead time in model. This problem is to determine the subsets of plants and DCs to be opened and to design the production and distribution network that will satisfy all capacities and demand requirement for each product while the demands of customers are stochastic. The objective function minimize sum of investment costs and the expected costs of production, transportation and inventory. The assumptions used in this problem are:

1. The number of potential plants, DCs and their maximum capacities are known.
2. Retailer demands are served from a single DC.
3. The demands are uncertain and are considered as discrete scenarios.

The following notations are used to define the mathematical model:

Indices

I	Set of customers
J	Set of warehouses
K	Set of plants
L	Set of products
S	Set of scenarios

Parameters

p_{jl}	load make-up time parameter of lead time for product l at DC j
q_{jl}	constant lead time component per unit for product l at DC j
r_{jl}	congestion parameter of lead time for product l at DC j
D_k	Capacity of plant K
W_j	throughput at DC j

$d^s{}_{ij}$	Demand for product l at customer i under scenario s
M	Maximum number of DCs
N	Maximum number of plants
o_j	Annual fixed cost for operating a DC j
g_k	annual fixed cost for operating a plant k
v_{lk}	Unit production cost for product l at plant k
h_{jl}	unit inventory cost for product l at DC j
c_{ijl}	Unit transportation cost for product l from DC j to customer i
a_{jkl}	Unit transportation cost for product l from plant k to DC j
n_l	Space requirement rate of product l on a DCType equation here.
m_l	Capacity utilization rate per unit of product l
p_s	Occurrence probability of scenario s

Variables

z_j	1 if DC j is opened, 0 o.w.
p_k	1 if plant k is opened, 0 o.w.
$y^s{}_{ij}$	1 if DC j serves customer I, 0 o.w.
$x^s{}_{lk}$	Quantity of product l produced at plant k under scenario s
$q^s{}_{ijl}$	Quantity of product l shipped from DC j to customer i under scenario s
$f^s{}_{jkl}$	Quantity of product l shipped from plant k to DC j under scenario s

Using the lead time expression from Section 3.1, the expected lead time LT_{jl} at the DC at location j in each scenario when the mean demand flow of product l through that DC is B_{jl} units is given by

$$LT_{jl}^s = \frac{P_{jl}}{B_{jl}^s} + q_{jl} + \frac{r_{jl}}{W_{jl} - B_{jl}^s} \tag{2}$$

Where

$$B_{jl}^s = \sum_i d_{il}^s \, y_{il}^s \quad \forall s \tag{3}$$

By Little's Law, the inventory between the production facility and the DC at location j in each scenario is given by

$$I_{jl}^s = LT_{jl}^s B_{jl}^s \tag{4}$$

Therefore the expected inventory cost between the production facility and DC at location j in each scenario can be obtained as:

$$Inventory\ Cost_{jl}^s = h_{jl} I_{jl}^s \tag{5}$$

Accordingly, the problem can be formulated as follows:

Min	$Z = \sum_j o_j z_j + \sum_k g_k p_k + \sum_s p_s \left(\sum_l \sum_k v_{lk} x^s{}_{lk} + \sum_i \sum_j \sum_l c_{ijl} q^s{}_{ijl} + \sum_j \sum_k \sum_l a_{jkl} f^s{}_{jkl} + \sum_j \sum_l h_{jl} \left(p_{jl} + q_{jl} \sum_i d^s{}_{il} y^s{}_{il} + \frac{r_{jl} \sum_i d^s{}_{il} y^s{}_{il}}{W_j - \sum_i d^s{}_{il} y^s{}_{il}} \right) \right)$	(6)
S.t.	$\sum_j y^s{}_{ij} = 1 \qquad \forall i, \forall s$	(7)

	$\sum_i \sum_l n_l d^s{}_{il} y^s{}_{ij} \leq W_j z_j \quad \forall j, \forall s$	(8)
	$\sum_j z_j \leq M$	(9)
	$q^s{}_{ijl} = d^s{}_{il} y^s{}_{ij} \quad \forall i, j, l, \forall s$	(10)
	$\sum_k f^s{}_{jkl} = \sum_i q^s{}_{ijl} \quad \forall j, l, \forall s$	(11)
	$\sum_l m_l x^s{}_{lk} \leq D_k p_k \quad \forall k, \forall s$	(12)
	$\sum_j f^s{}_{jkl} \leq x^s{}_{lk} \quad \forall k, l, \forall s$	(13)
	$\sum_k p_k \leq N$	(14)
	$z_j = \{0,1\} \quad \forall j$	(15)
	$p_k = \{0,1\} \quad \forall k$	(16)
	$y^s{}_{ij} = \{0,1\} \quad \forall i, j, \forall s$	(17)
	$x^s{}_{lk} \geq 0 \quad \forall l, k, \forall s$	(18)
	$q^s{}_{ijl} \geq 0 \quad \forall i, j, l, \forall s$	(19)
	$f^s{}_{jkl} \geq 0 \quad \forall j, k, l, \forall s$	(20)

The objective function minimizes the total cost of the supply chain. It consists of the fixed cost of operating and opening plants and DCs and the expected costs of production, inventory and transportation of the products from plants to DCs and from DCs to customers. Equation (7) shows the unique assignment of a DC to a customer in each scenario, (8) limits the capacity for DCs, (9) restricts the number of DCs that can be opened, (10) and (11) give the satisfaction of customers and DCs demand for all products in each scenario, (12) limits the plant capacity, (13) restricts the total quantity of product shipped from a manufacturing plant to customers through DCs that cannot exceed the amount of that product produced in that plant in each scenario, (14) limits the number of plants that can be opened, (15)–(17) imposes the integrality restriction on the decision variables Z_j, P_k, y_{ij}^s, (18)–(20) impose the non-negativity restriction on decision variables x_{lk}^s, q_{ijl}^s, f_{jkl}^s.

In order to deal with the effects of uncertainty in demands, the two-stage stochastic programming approach is applied in this paper. Decision variables, which characterize the network configuration, namely those binary variables that represent the existence and the location of plants and warehouses of the SC, are considered as first-stage variables. It is assumed that they have to be taken at the design stage before the realization of the uncertainty. On the other hand, decision variables related to the amount of products to be produced and stored in the nodes of the SC and the flows of materials transported among the entities of the network are considered as second-stage variables, corresponding to decisions taken after the uncertain parameter has been revealed.

4. Numerical Example

Consider a supply chain design network consists of plants, distribution centers and demand points. Suppose a company is willing to design its SC. This company produces two products for three customer located in three different cities A, B and C. There are four possible locations D, E, F, and G to establish the distribution centers as well as four possible locations H, I, J and K to establish the plants. The products produced await in area of the production facilities before the products are transmitted to the DCs. Also products sent to DCs have to wait in the unloading zone. For simplicity, without considering other market behaviors (e.g. novel promotion, marketing strategies of competitors and market-share effect in different markets), each market demand merely depends on the local economic conditions. Transportation costs between nodes on each stage of SCN are acquired as coefficient of their Euclidean distances. The unit inventory cost of product at warehouses and unit production cost of products at plants are generated from U[1,10]. The load make-up time parameter and the replenishment lead time come from U[140,150] and U[1,4], respectively . The congestion time parameter is equalized to capacity of the DC. The space requirement rate of products on a DC, and capacity utilization rate of products in plants are drawn according to

Table 1- product demands

Customers	Product 1						Product 2					
	Scenarios			Probabilities			Scenarios			Probabilities		
A	110	160	210	0.4	0.4	0.2	130	195		0.35	0.65	
B	125	180		0.3	0.7		115	180	245	0.3	0.4	0.3
C	170	295	380	0.15	0.35	0.5	120	210		0.22	0.78	

Table 2- The results of numerical example

Approach	Plants				warehouses				Total Cost
	H	I	J	K	D	E	F	G	
Deterministic equivalent	1	0	1	0	1	1	1	0	363358.958
Expected value problem	1	0	1	0	1	1	1	0	363282.553
Worst objective function	1	0	1	1	1	1	1	0	507576.568
Best objective function	1	0	1	0	1	0	1	0	315209.156

U[2,5]. While the fixed cost of DCs are generated from U[10000,30000], the fixed cost of plants are generated from U[50000,150000]. After calculating the total capacity of DCs as $1.5\sum_{i\in I}\sum_{l\in L} n_l d_{il}$, the capacity of DCs are determined randomly by sharing the total capacity into DCs. In a similar way, the capacities of plants are determined. The total capacity of plants is calculated as $1.5\sum_{i\in I}\sum_{l\in L} m_l d_{il}$. Finally, the demand for each type product is assumed as discrete scenarios with corresponding probabilities shown in Table 1.

This problem attempts to minimize the fixed investment costs and expected cost arising from different scenarios while making the following determinations:

- Which of the plants and warehouses to build (first-stage variables)?
- Amount of product to be produce in each plant, amount of product to be transport from plants to distribution centers and finally amount of product to be transport from distribution centers to customer centers (second-stage variables)?

Since the product demands are defined as discrete distribution, total number scenarios are obtained by multiplying possible situation of each uncertain demand is equal to $3 \times 2 \times 3 \times 2 \times 3 \times 2 = 216$. In fact the problem can be treated as a deterministic equivalent problem with $|I||S|$ customers instead of $|I|$. We use GAMS software to solve the numerical example. We also solve problem when expected value of demand is considered as deterministic demand in model and show the best and worst values of objective function over scenarios in Table 2 (1 means the plant/warehouse is built and 0 otherwise). Moreover, Fig.2 show the objective functions in every scenario. Table 2 and Fig.2 are a good tool for decision makers so that select one of approaches based on their policy.

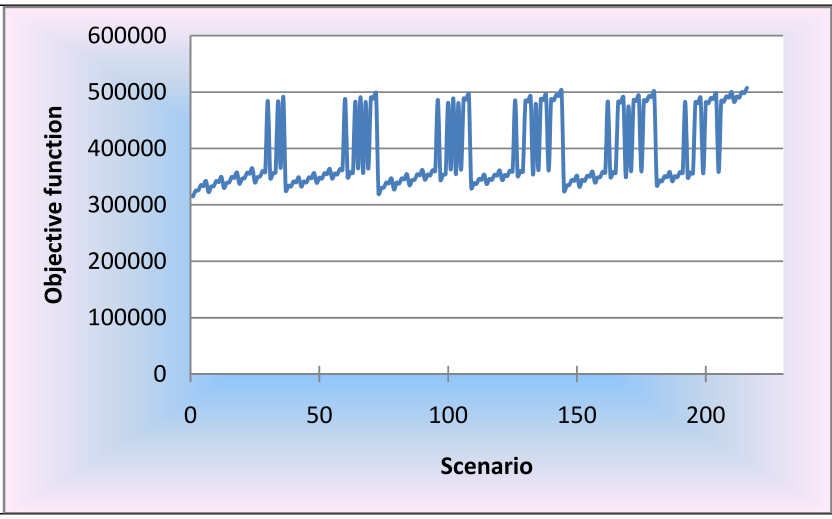

Fig.2. Values of objective functions in scenarios.

5. Conclusions

In supply chain network design, strategic decisions and tactical/operational decisions have been usually tackled in isolation from one another. Also determining the optimal SC configuration is a difficult problem since a lot of factors and conditions practically are changed in long period of time which may turn a good location to day into a bad one in the future. Hence, the proposed model in this paper presented a supply chain network model considering both strategic and operational decisions. The model determines location of plants and DCs regarding single sourcing and capacity of plants and distribution centers (strategic level) while the shipments have to wait in the queue for transporting from plants to DCs (operational level), which lead to the lead time is incorporated in model. To afford the condition change in practice, then we extended model by defining demands as different scenarios. To deal with uncertainty of parameters, the two-stage stochastic programming approach and other approaches were presented for decision maker(s) to adopt the relevant policy.

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