Iterative Hard and Soft Decision Based Detection Methods for Uplink Massive MIMO Systems

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ABSTRACT

In this paper, two low complexity iterative hard and soft decision based detection methods for massive multiuser multiple-input and multiple-output (MIMO) systems have been proposed. In soft decision based method, in each iteration, the estimated symbols are mapped to the nearest constellation points using a soft mapping operation. The zone of the soft mapping operation is optimized for the best performance in terms of convergence speed and Bit Error Rate (BER). Simulation results show that the proposed detectors outperform the conventional detection methods without any additional complexity.

Keywords: Massive MIMO systems, Detectors, Iterative Method, steepest descent, Jacobi iteration, matrix inversion.

1. INTRODUCTION

In recent years, Massive multiuser multiple-input and multiple-output (MIMO) systems have been suggested in cellular communications. In massive MIMO systems, a large number of antennas are used at the Base station (BS) to support many single (or multi) antenna(s) users. Massive MIMO is one of the promising solutions for meeting the 5G requirements such as: high spectrum efficiency, mitigation of interference and high reliability of communications [1, 2].

However, there are some challenges in these systems such as hardware implementation complexity, detection complexity, channel estimation and antenna correlation [3, 4].

In massive MIMO systems, due to the large number of antennas at the base station, linear detection algorithms, such as Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE), can achieve the BER close to optimal performance [3, 5]. However, these linear detection algorithms contain matrix inversion that imposes a considerable computational complexity on the BS side due to the large number of antennas.

Recently, different algorithms have been proposed to avoid calculation of inverse of large size matrix, such as Neumann series (NS) [5], Gauss-Seidel (GS) [6], Joint Steepest Descent and Jacobi Iteration (JSDJD) [7], Jacobi-DA [8], Jacobi [9] and Chebyshev iteration [10]. But when the number of transmitters is near to the number of receiver antennas, the performance of these methods is far away from the optimal detector. In this paper, two low complexity iterative detection methods for massive MIMO systems have been proposed, which are based on hard and soft mapping.

The proposed iterative detections improve the convergence speed and provide better BER in comparison to previous methods, especially when the number of transmitters is close to the number of receiver antennas and in high order modulations. In the proposed detectors, in each iteration, estimated symbols are mapped to the constellation points using hard or soft mapping operations.

The rest of the paper is organized as follows. Section 2 describes the system model of uplink multiuser massive MIMO system. Section 3 reviews related works. In Section 4, the proposed detectors are presented. In section 5, the simulation results and discussions about the performance of the proposed algorithm are presented and finally the paper is concluded in Section 6.

Notation: Boldface capital letters and boldface lowercase letters represent matrices and vectors, respectively. $\mathbf{I}_K$ denotes the $K \times K$ identity matrix; $(.)^H$, $(.)^{-1}$, $(.)^T$ denote conjugate transpose, inversion and transpose operations, respectively. $£^{m\times n}$ denote the set of all $m \times n$ complex matrix.

2. SYSTEM MODEL

We consider uplink multiuser massive MIMO system with n_t single antenna users which transmit data to the base station equipped with n_r receive antennas. The transmit vector, $\mathbf{x} = \left[x_1, x_2, ..., x_{n_t}\right]^T$, includes n_t data symbols that belong to the M-QAM constellation with average power of σ_x^2 per symbol with $E\left(\mathbf{x}\mathbf{x}^H\right) = \sigma_x^2 \mathbf{I}_{n_t}$. The received signal vector, $\mathbf{y} \in \mathbb{C}^{n_r \times 1}$, at the BS can be represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \qquad (1)$$

Where $\mathrm{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix between the BS and the n_t users whose entries are modeled as i.i.d complex Gaussian variables with zero mean and unit variance, and $\mathbf{n}$ is a white Gaussian noise vector with zero mean and correlation matrix $E\left(\mathbf{n}\mathbf{n}^H\right) = \sigma_n^2 \mathbf{I}_{n_r}$. It is assumed that channel matrix is perfectly known at the BS, but it is unknown at the transmitters. In BS, $\mathbf{y}$ is measured and the goal is detection of the transmitted vector of symbol, $\mathbf{x}$.

3. RELATED WORKS

3.1 MMSE Detection

The MMSE detector is a linear detector which minimizes the mean square error between the transmitted vector and the estimated one. The MMSE estimation of signal is given by

$$\hat{\mathbf{x}} = \left(\mathbf{H}^H\mathbf{H} + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}_{n_t}\right)^{-1} \mathbf{H}^H\mathbf{y} = \mathbf{A}^{-1}\mathbf{y}_{MF} \qquad (2)$$

where $\mathbf{y}_{MF} = \mathbf{H}^H\mathbf{y}$ is the matched filter estimation of transmitted signal, $\hat{\mathbf{x}}$ is the MMSE estimated vector, $\mathbf{A}^{-1}$ is the MMSE filtering matrix and $\mathbf{A} = \mathbf{H}^H\mathbf{H} + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}_{n_t}$. As mentioned before, it has been proved that the MMSE detector can perform close to the optimal detector in massive MIMO systems, because of that its performance is conventionally considered as the benchmark for the performance evaluation of the other Massive MIMO detectors [5-10].

In massive MIMO systems, dimension of the matrix $\mathbf{A}$ is large and calculation of its inverse is complex. To avoid this complexity, equation (2) can be converted to solving the following linear equation [11].

$$\mathbf{A}\mathbf{x} = \mathbf{y}_{MF} \qquad (3)$$

The equation (3) can be solved using iterative methods to find $\mathbf{x}$. Some of these methods are Neumann series (NS) [5], Gauss-Seidel (GS) [6], Jacobi [9], Joint Steepest Descent and Jacobi (JSDJD) [7] and Chebyshev iteration [10].

Among these methods JSDJD and GS have the best performances. Thus, in the following subsections these two algorithms are described briefly and in section 6 the performance of our proposed methods is compared with that of these two methods.

Since the matrix $\mathbf{A}$ is Hermitian, then it can be decomposed as $\mathbf{A} = \mathbf{D} + \mathbf{L} + \mathbf{L}^H$, where matrix $\mathbf{D}$ is the diagonal part of $\mathbf{A}$ and $\mathbf{L}$ is the lower triangular part of $\mathbf{A}$.

3.2 Joint Steepest Descent and Jacobi Detection method (JSDJD)

In this detector, Jacobi method is used to solve equation (3), using iterative computation instead of matrix inversion. The k th iteration of Jacobi Detection (JD) method can be represented as

$$\hat{\mathbf{x}}^{(k)} = \mathbf{D}^{-1}\left[\left(\mathbf{D} - \mathbf{A}\right)\hat{\mathbf{x}}^{(k-1)} + \mathbf{y}_{MF}\right] \qquad (4)$$

Where k is the iteration number and $\mathbf{D}^{-1}$ is the inverse matrix of diagonal matrix $\mathbf{D}$. The rate of the convergence of (4) is low. To overcome this problem, in [7] JSDJD was proposed. In this method, the initialization is done by $\hat{\mathbf{x}}^{(0)} = \mathbf{D}^{-1}\mathbf{y}_{MF}$. Then $\hat{\mathbf{x}}^{(1)}$ is calculated using steepest descent method as follows:

$$\hat{\mathbf{x}}^{(1)} = \hat{\mathbf{x}}^{(0)} + \beta\mathbf{r}^{(0)} - \mathbf{D}^{-1}\left(\mathbf{r}^{(0)} - \beta\mathbf{A}\mathbf{r}^{(0)}\right) \qquad (5)$$

where $\beta = \frac{(\mathbf{r}^{(0)})^H\mathbf{r}^{(0)}}{(\mathbf{A}\mathbf{r}^{(0)})^H\mathbf{r}^{(0)}}$ and $\mathbf{r}^{(0)} = \mathbf{y}_{MF} - \mathbf{A}\hat{\mathbf{x}}^{(0)}$. Then for $k \geq 2$, the iteration of Jacobi method as shown in (4) is used to obtain $\hat{\mathbf{x}}^{(k)}, k \geq 2$. In [7], it has been shown that JSDJD performance is better than Jacobi algorithm at the same number of iterations.

3.3 Gauss-Seidel (GS)

The GS method exploited to realize the MMSE algorithm iteratively. This method estimates the transmitted vector $\mathbf{x}$ as

$$\hat{\mathbf{x}}^{(k)} = (\mathbf{D}+\mathbf{L})^{-1}\left[\mathbf{y}_{MF} - \mathbf{L}^{H}\hat{\mathbf{x}}^{(k-1)}\right] \tag{6}$$

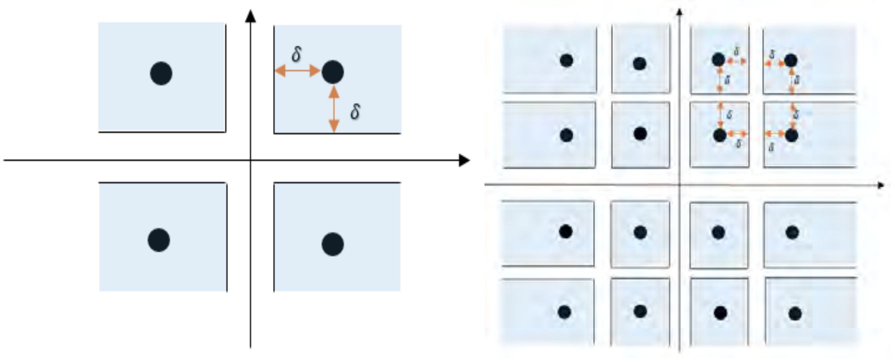
4. PROPOSED METHOD

In this section, two efficient iterative algorithms are presented which can be used for detection of the transmitted symbols in uplink of massive MIMO systems. Since the elements of the transmit vector $\mathbf{x}$, belong to M-QAM constellation points, in [8] JD algorithm was modified such that the j th data symbol in the k th iteration, $\hat{x}_j^{(k)} (1 \le j \le n_t)$, is mapped to the nearest constellation point [8] .This approach is known as the hard decision based detector. The hard decision based JD detector can be illustrated as follow [8]:

$$\hat{\mathbf{x}}^{(k)} = \mathbf{D}^{-1}\left[(\mathbf{D}-\mathbf{A})Q\left(\hat{\mathbf{x}}^{(k-1)}\right) + \mathbf{y}_{MF}\right] \tag{7}$$

that, $Q(.)$ denotes the hard decision function which means that the argument of this function is mapped to the nearest constellation point. JD algorithm with hard decision, introduced in [8] has two important performance limitations. It does not have a good performance when the number of transmitters is close to the number of receiver antennas and it does not well in high order modulations such as 16-QAM. To overcome these limitations, we propose two modifications: 1) application of the decision at the output of JSDJD method instead of JD algorithm. It means that (6) is used for the first iteration 2) using soft decision instead of hard decision. It is noteworthy that if a symbol is mapped incorrectly to a constellation point it leads to the error propagation to the other symbols. This is the main drawback of hard decision based detector of (7). Therefore, a soft decision based detector is proposed in this paper. In soft decision based detector only the estimated symbols, $\hat{x}_j^{(k)} (1 \le j \le n_t)$, which are very close to one of the constellation points are mapped and the remaining ones (which are near the borders of decision areas) are left without any decision. For example, for 4-QAM modulation, as shown in Fig.1. (a), if the symbols are in highlighted regions, then they are decided to the corresponding constellation point. The summary of these two modifications leads to an algorithm is called JSDJD-SD detector which can be summarized as

$$\begin{aligned}
&\hat{\mathbf{x}}^{(0)} = \mathbf{D}^{-1}\mathbf{y}_{MF} \\
&\beta = \frac{\left(\mathbf{r}^{(0)}\right)^{H}\mathbf{r}^{(0)}}{\left(\mathbf{A}\mathbf{r}^{(0)}\right)^{H}\mathbf{r}^{(0)}} \\
&\hat{\mathbf{x}}^{(1)} = \hat{\mathbf{x}}^{(0)} + \beta\mathbf{r}^{(0)} - \mathbf{D}^{-1}\left(\mathbf{r}^{(0)} - \beta\mathbf{A}\mathbf{r}^{(0)}\right) \\
&\hat{\mathbf{x}}^{(k)} = \mathbf{D}^{-1}\left[(\mathbf{D}-\mathbf{A})\tilde{Q}\left(\hat{\mathbf{x}}^{(k-1)}\right) + \mathbf{y}_{MF}\right] \quad ,k \ge 2
\end{aligned} \tag{8}$$



a) b)

Fig. 1. *Soft mapping areas with parameter* δ *in* a) *4-QAM* b)*16-QAM modulations*

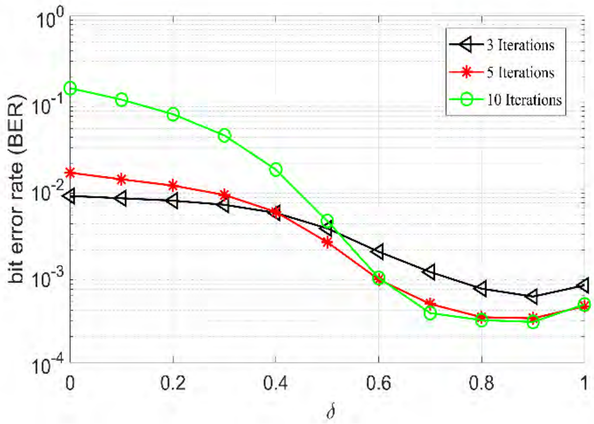


Fig. 2. *BER versus δ for $n_t \times n_r = 32 \times 64$ massive MIMO system at SNR = 8dB, 4-QAM modulation and different iterations*

where

$$\mathcal{Q}(x) = \begin{cases} C_i & \exists i\ x \in S_i, \\ x & \forall i\ x \notin S_i, \end{cases} \quad i \in \{1,2,\dots,M\} \tag{9}$$

Where $C_i, i \in \{1,2,\dots,M\}$ are the constellation points and S_i is the highlighted area around the constellation point C_i. For 4-QAM modulation the highlighted areas are defined by a parameter δ $(0 \le \delta \le d_{min}/2)$, where d_{min} is the minimum distance between two arbitrary constellation points. For instant in 4-QAM modulation and $C_i = \pm 1 \pm j$ with $d_{min} = 2,\ 0 \le \delta \le 1$.The detector performance is related to δ, thus we will find the optimum value of δ based on simulation results in section 5. It is clear that if $\delta = 1$, the soft mapping is converted to the hard decision. For 16-QAM modulation, the soft decision areas have been chosen as shown in Fig.1. (b).

When the estimated value of a symbol is in highlighted regions we can be sure that it can be decided to the corresponding constellation points. The estimations, which are near the border, have a high probability of error and thus they are kept to be improved in next iterations. Since some of the symbols are decided previously, the most of the correction power of the existing iterations is concentrated on the remained symbols.

The computational complexity of the proposed detectors can be analyzed by the number of real value multiplications. It is obvious that the proposed detectors require a similar number of real multiplications compared to the JSDJD at the same number iterations, because the complexity of hard and soft mapping procedures is negligible. It means that the order of complexity is n_t^2, the same as JSDJD [7] and GS [6], while the complexity of MMSE is $o(n_t^3)$.

5. SIMULATION RESULTS

In this section, the performance of the proposed detector in massive MIMO system with 4-QAM and 16-QAM modulations have been evaluated. Fig.2. shows BER of the proposed detector for different number of iterations versus δ when SNR is 8dB with $n_r = 64$ and $n_t = 32$. As can be seen in this figure, by increasing the number of iterations from 3 to 5, the detection performance will improve gradually but there is not a considerable performance difference when the outputs of the 5th and the 10th iterations are compared, i.e. the proposed method converges to the final result with around three iterations. In this figure, SNR has been fixed at 8dB and the parameter δ varies from 0 to 1. As it is shown in Fig.2, in 3, 5 and 10 iterations, the optimum value of δ can be considered around $\delta = .9$. In fact, for different number of iterations, the range of optimum δ is not changed considerably. Fig. 3. shows BER of different detection methods versus SNR for 4-QAM modulation with $n_r = 64$ and $n_t = 32$. In this figure HD and SD stand for hard and soft decisions, respectively and dotted, dashed and solid curves, represent the result of 3, 5 and 10 iterations of different method, respectively. For soft decision, the values of $\delta = .9$ has been chosen.

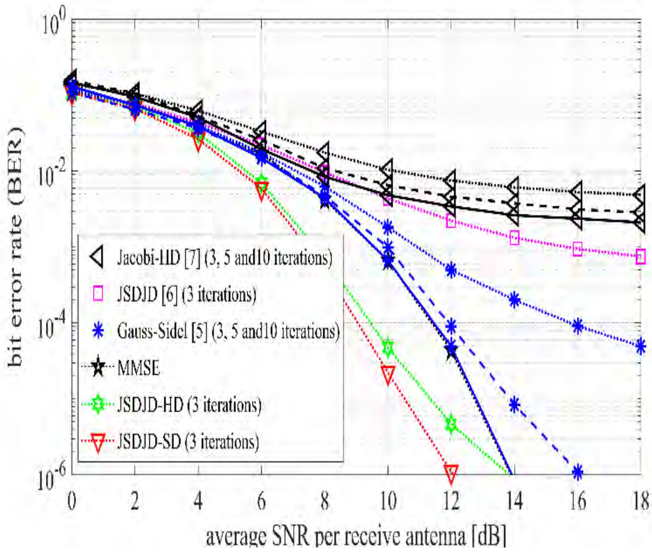


Fig.3. *BER performance comparison between the proposed and other methods for* $n_t \times n_r = 32 \times 64$ *and 4-QAM modulation with* $\delta = .9$.

As can be seen the BER performance of the Jacobi method with HD [8] is even worse than the BER of normal JSDJD method, while our proposed JSDJD-HD and SD methods outperform both of MMSE and JSDJD and GS methods. It is noteworthy that the proposed method converges after only 3 iterations, while in GS method 10 iterations are required for convergence. As can be seen from this figure, In BER= 10^{-6} JSDJD-SD based method has about 2dB performance improvement compared with JSDJD-HD algorithm.

In Fig.4 simulations have been repeated for $n_r = 90$, $n_t = 32$ and 4-QAM modulation. As it can be seen form this figure, when the number transmitters are much lower than the number of receive antenna ($n_t = n_r$), performance of the Jacobi-HD [8] gets better but still our proposed algorithms, JSDJD-HD and JSDJD-SD, outperform the other methods and their convergence speeds are also higher than the others.

Fig.5 shows the BER performance of the proposed methods for $n_r = 128$, $n_t = 32$ and 16-QAM modulation. The optimum value of $\delta = .8$ was chosen for this case (it was derived from simulation similar to Fig.2). In this case, Jacobi-HD method [8], does not have a good performance while our proposed method can achieve better BER performances than the MMSE and other methods. It is noteworthy that JSDJD-HD and JSDJD-SD methods with 3 iteration have better performance than the best other ones which is GS method with 10 iterations.

Considering figures 3, 4 and 5, it can be seen that the proposed method resolves two following limitations of the other methods: 1) performance degradation when the number of transmitters is close to the number of receiver antennas and 2) Performance degradation in high order modulations.

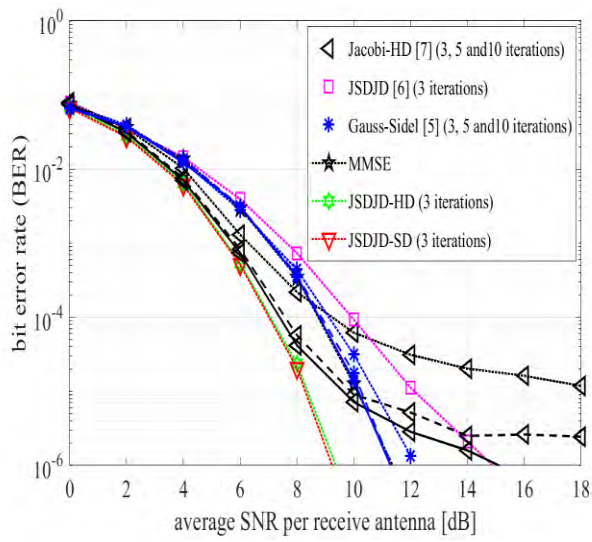


Fig.4. *BER performance comparison between the proposed and other methods for* $n_t \times n_r = 32 \times 90$ *and 4-QAM modulation with* $\delta = .9$.

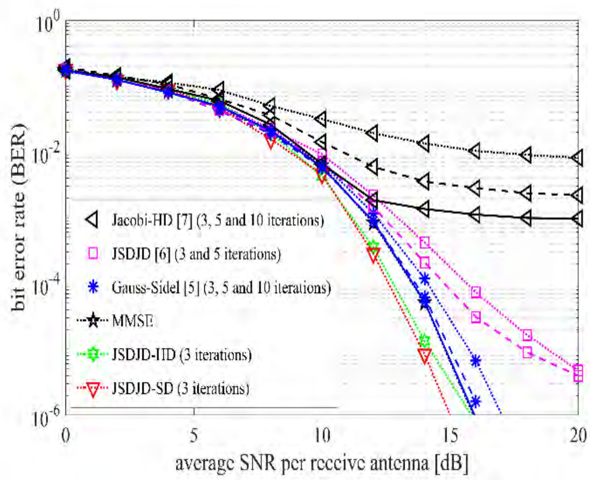


Fig.5. *BER performance comparison between the proposed and other methods for* $n_t \times n_r = 32 \times 128$ *and 16-QAM modulation with* $\delta = .8$.

6. CONCLUSION

In this paper, two low complexity iterative hard and soft decision based methods for detection of uplink massive MIMO systems have been proposed. In the proposed methods, which are based on previously proposed JSDJD algorithm, in each iteration the estimated symbols are mapped to the nearest constellation points using a hard or soft mapping operation. This approach improves the convergence speed and provides better BER with low computational complexity. Simulation results show that the proposed methods have a good BER performance regardless of how close the number of the transmitter and receiver antennas is and order of modulation.

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