Journal of Industrial and Systems Engineering
Vol. 12, No. 4, pp. 242-251
Autumn (November) 2019

Developing two variables sampling plans considering the compliance rate with the ideal OC curve

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Abstract

An essential tool for examining the quality of manufactured products is acceptance sampling. This research applies the concept of minimum angle method to extend two variables sampling plans including the variables multiple dependent state (VMDS) sampling plan and the variables repetitive group sampling (VRGS) plan on the basis of the process yield index S_{pk}. Optimal parameters of acceptance sampling plans can be determined by solving a non-linear optimization model with the following conditions: 1) The objective function of the plan is to minimize the average sample number. 2) Constraints are set in a way that the compliance rate will be satisfied with the ideal operating characteristic (OC) curve as well as the producer's and costumer's risks. The assessment of the proposed plans reveals that by increasing the rate of convergence to the ideal OC curve, the proposed VRGS plan performs better than the proposed VMDS plan in terms of the average sample number. A numerical example is considered to reveal the applicability of the proposed acceptance sampling plans.

Keywords: Acceptance sampling, minimum angle method, nonlinear optimization, operating characteristic curve, yield index

1- Introduction

Acceptance sampling plans are implemented to judge whether each submitted lot should be accepted or rejected. By data type, acceptance sampling plans are classified into attributes and variables. In some situations, sample sizes needed by attributes sampling plans become impractically large e.g. when the inspection cost is significantly high. Another situation is when required quality levels are very high and at the same time testing is destructive. Therefore, it is advisable to use variable sampling plans to reduce the costs. Based on the process capability indices, several sampling schemes have been extended. Examples include Pearn and Wu (2007), Arizono et al. (2014), Liu et al. (2014), Wu et al. (2015), Lee et al. (2018), Wang and Wu (2019), Tamirat and Wang (2019).

Balamurali and Jun (2007) broadened the concept of attributes multiple dependent state (MDS) sampling plan to the variables multiple dependent state (VMDS) sampling plan. Wu et al. (2015) extended the VMDS sampling plan based on the indexS_{pk}. They found out that the sample required by VMDS sampling plan for inspection is smaller than the one required by variables single sampling (VSS) plan. Aslam et al. (2019) developed a new VMDS sampling plan based on the process capability index C_{pk} which comprises the features of the existing MDS sampling plan and repetitive group sampling (RGS) plan. Further studies on the VMDS plan can be found in Aslam et al. (2014), Wu and Wang (2017).

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Balamurali and Jun (2006) developed a variables repetitive group sampling (VRGS) for a normally distributed quality characteristic. According to their research, the proposed VRGS plan performs better than the VSS and the double sampling plans in terms of the average sample number (ASN). Wu et al. (2015) presented a VRGS plan based on the C_{pk} index. The results showed that the sample size which should be inspected for the proposed VRGS plan is less than the VSS plan under the same conditions. Wu and Chen (2019) provided a modified VRGS plan with an adjustable mechanism (Adj-VRGSP) based on the index S_{pk}. They concluded that the Adj-VRGSP is better than the VRGS and VSS plans in terms of sample size as it can meet the required quality standards and retain the permitted risks. Further studies on the VRGS plan can be found in Wu and Liu (2018), Sherman (1965).

There is a method in acceptance sampling called minimum angle method, in which both risks will be considered. In this method, the ideal operating characteristic (OC) curve is achieved by optimizing the producer's and the customer's risk simultaneously in acceptance sampling plans. Niaki and Fallahnezhad (2012) provided a new approach for acceptance sampling plan using Markov chain and minimum angle method. Fallahnezhad and Yazdi (2016) considered the concept of minimum angle method in acceptance sampling plan based on the run length of conforming items. Fallahnezhad et al. (2018) presented four variables acceptance sampling plans by considering not only the total expected loss to the producer and customer but also the conformity to the ideal OC curve as an objective function.

To the best of author's knowledge, no sampling plan has been designed so far with the concept of the ideal OC curve and ASN minimization based on the S_{pk} index. In this paper, we develop the VRGS and VMDS sampling plans based on the S_{pk} index, examine the minimum angle method as a constraint in the proposed plans and then analyze its impact on the model results. A nonlinear optimization model is used to determine the optimal parameters by considering the ideal OC curve convergence, acceptable quality level (p_{AQL}) for the producer's risk, and rejectable quality level (p_{LQL}) for the customer's risk. The remainder of the paper is organized as follows: In Section 2, we introduce the concept of the process yield index S_{pk}. The concept of minimum angle method is briefly explained in Section 3. Both the procedure and the formulation of the proposed plan as a nonlinear programming problem are presented in Section 4. The plans analyses and discussion are mentioned in Section 5. A numerical example is provided in Section 6 to illustrate the proposed plan. In the last section, conclusions are remarked.

2- Process yield index

The process yield index S_{pk} was firstly proposed by Boyles (1994) to specially reflect the exact process yield that can be used as an important criterion for judging the process performance. Using equation (1), the process yield index S_{pk} can be derived. Moreover, the index S_{pk} can be expressed as a function of two process capability indices C_p (for reflecting the process precision) and C_a (for reflecting the process centering). In addition, if $S_{pk} = S_0$, then the process yield can be calculated by $\%\text{Yield} = [2\phi(3S_0) - 1] \times 100\%$ and the nonconformities in parts per million (NCPPM) can be derived by $\text{NCPPM} = 2[1 - \phi(3S_0)] \times 10^6$.

$$S_{pk} = \frac{1}{3}\phi^{-1}\left[\frac{1}{2}\phi\left(\frac{USL-\mu}{\sigma}\right) + \frac{1}{2}\phi\left(\frac{\mu-LSL}{\sigma}\right)\right] = \frac{1}{3}\phi^{-1}\left[\frac{1}{2}\phi\big(3C_pC_a\big) + \frac{1}{2}\phi\big(3C_p(2-C_a)\big)\right], \quad (1)$$

Where

$$C_p = \frac{USL - LSL}{6\sigma},$$
$$C_a = 1 - (|\mu - M|/d).$$

In the above equations, $M = (USL + LSL)/2$, $d = (USL - LSL)/2$, USL and LSL are the upper and lower specification limits, $\phi(.)$ denotes the cumulative distribution function of the standard normal distribution, and $\phi^{-1}(.)$ is the inverse function of $\phi(.)$.

The estimation of the performance index S_{pk}, ($\hat{S}_{pk}$), is obtained using equation (2) as below.

$$\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{USL-\bar{x}}{S}\right)+\frac{1}{2}\Phi\left(\frac{\bar{x}-LSL}{S}\right)\right] = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi(\hat{C}_p\hat{C}_a)+\frac{1}{2}\Phi\left(3\hat{C}_p(2-\hat{C}_a)\right)\right], \quad (2)$$

where$\bar{x} = \sum_{i=1}^{n} x_i/n$, $S = [\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)]^{1/2}$. Furthermore, Lee et al. (2002) elicited an approximate distribution of the yield index S_{pk} via first-order Taylor expansion and expressed the $\hat{S}_{pk}$ as

$$\hat{S}_{pk} \approx N\left(S_{pk}, [a^2+b^2]\{36n\left[\phi\left(3S_{pk}\right)^2\right]\}^{-1}\right), \quad (3)$$

where ϕ denotes the probability density function of the standard normal distribution $N(0,1)$. Also, a and b are functions of C_a and C_p and are defined by

$$a = \frac{1}{\sqrt{2}}\left\{3C_pC_a\,\phi\left(3C_pC_a\right)+3C_p(2-C_a)\,\phi\left(3C_p(2-C_a)\right)\right\},$$
$$b = \phi\left\{3C_p(2-C_a)\right\}-\phi\left(3C_pC_a\right).$$

The probability density function of $\hat{S}_{pk}$ can be expressed as:

$$f_{\hat{S}_{pk}}(x) = \sqrt{\frac{18n}{\pi}}\frac{\phi(3S_{pk})}{\sqrt{a^2+b^2}}\,exp\left[-\frac{18n(\,\phi(3S_{pk}))^2}{a^2+b^2}\times(x-S_{pk})^2\right], \quad -\infty < x < \infty. \quad (4)$$

3- Minimum angle method

In this method, the aim is to provide a new approach, implementing the points p_{AQL} and p_{LQL} so that the OC curve converges to its ideal. In an acceptance sampling plan with the ideal OC curve, the probability of accepting a good lot is equal to one and the probability of accepting a poor lot is equal to zero. Obviously, to reach the ideal OC curve, the risk of producer and customer must be minimized. On the other hand, by decreasing α and β the sample size increases which may lead to higher costs. Therefore, the sampling plan that results in the reduction of the inspected sample size is more desirable. Based on Soundararajan and Christina's research (1997), we present equation (5) to achieve the ideal OC curve based on the process yield.

$$\pi_a(p_{AQL})-\pi_a(p_{LQL}) \geq W, \quad (5)$$

Where the values of $1-\pi_a(p_{AQL})$ and $\pi_a(p_{LQL})$ represent producer's and consumer's risks, respectively. Therefore, it is desirable to maximize the value of $\pi_a(p_{AQL})$ and to minimize the value of $\pi_a(p_{LQL})$ in order to reduce producer's and consumer's risks. For this reason, maximizing the value of $\pi_a(p_{AQL})-\pi_a(p_{LQL})$ can be justified. It is worth noting that in equation (5), W as a plan parameter can take values less than one and ideally its value is one. The larger the discrepancy of W and one, the further away it is from the ideal.

4- Developing variable sampling plans by comprising the minimum angle method and the yield index S_{pk}

Two conditions should be satisfied in sampling plans: (1) the probability of accepting a lot at the acceptable quality level (AQL) must be more than the producer's confidence level $1-\alpha$ and (2) the probability of accepting a lot at the rejectable quality level (RQL) must be lower than the customer's risk β. Thus, the OC curve of a sampling plan must pass through those two specified points (AQL, 1- α) and (RQL, β). In real practice, we are faced with limitations for each sampling plan, so it is necessary to consider the constraints in the proposed model in order to have an optimal and efficient plan. To use the concept of minimum angle method and to show the rate of similarity of the proposed OC curve to the ideal OC curve based on the index S_{pk}, we consider equations (8) and (19) in the proposed model.

4-1- Multiple dependent state sampling plan

The operating procedure of the proposed VMDS plan based on the index S_{pk} is described as follows:

Step 1: Determine the values of p_{AQL}, p_{LQL} and α , β and W.

Step 2: Select a random sample of size n from the lot and calculate the $\hat{S}_{pk}$ value.

Step 3: (Come to a decision). Accept the lot if $\hat{S}_{pk} \geq k_a$, and reject the lot if $\hat{S}_{pk} < k_r$, where k_a and k_r are critical values; if $k_r < \hat{S}_{pk} < k_a$, start step 4.

Step 4: Accept the lot if previous m lots were accepted in the terms of $\hat{S}_{pk} \geq k_a$. Otherwise, reject the lot.

On the basis of the index S_{pk}, the OC function of the proposed VMDS sampling plan can be established as:

$$\pi_a\left(S_{pk}\right) = P(\hat{S}_{pk} \geq k_a) + P(k_r < \hat{S}_{pk} < k_a) \times \left[P\left(\hat{S}_{pk} \geq k_a\right)\right]^m, \tag{6}$$

where the first part in the right-hand side shows the probability of accepting the lot related to the current sample, and the probability of accepting the lot based on the states of previous lots has been shown by the second part.

The average sample number for the VMDS sampling plan is given as $ASN = n$.

By solving the following optimization problem, we can obtain the plan parameters of the proposed VMDS sampling plan based on the yield index S_{pk}:

$$\text{Min } ASN_{VMDS} \tag{7}$$

Subject to:

$$\pi_a\left(p_{AQL}\right) - \pi_a\left(p_{LQL}\right) \geq W, \tag{8}$$

$$\pi_a\left(p_{AQL}\right) = P\left(\hat{S}_{pk} \geq k_a\right) + P(k_r < \hat{S}_{pk} < k_a) \times \left[P\left(\hat{S}_{pk} \geq k_a\right)\right]^m \geq 1 - \alpha, \tag{9}$$

$$\pi_a\left(p_{LQL}\right) = P\left(\hat{S}_{pk} \geq k_a\right) + P(k_r < \hat{S}_{pk} < k_a) \times \left[P\left(\hat{S}_{pk} \geq k_a\right)\right]^m \leq \beta, \tag{10}$$

$$k_a > k_r > 0, \tag{11}$$

$$p_{AQL} < p_{LQL}, \tag{12}$$

$$n \geq 2. \tag{13}$$

The above constraints denote the conformity to the ideal OC curve and also satisfying the first and second type error probabilities.

4-2- Repetitive group sampling plan

The operating procedure of the proposed VRGS plan based on the index S_{pk} is described as follows:

Step 1. Determine the values of p_{AQL}, p_{LQL}, α , β and W.

Step 2. Select a random sample of size n from the lot and calculate the $\hat{S}_{pk}$value.

Step 3. (Come to a decision). Accept the lot if $\hat{S}_{pk} \geq k_a$, and reject the lot if $\hat{S}_{pk} < k_r$, where k_a and k_r are critical values; otherwise, we should take a new sample for further judgment (i.e. repeat Step 2).

Note that if $k_a = k_r$, the VRGS plan will reduce to the VSS plan.

On the basis of the index S_{pk}, the probability of accepting the lot can be defined as:

$$P_a(p) = P\left(\hat{S}_{pk} \geq k_a\right). \tag{14}$$

Similarly, the probability of rejecting the lot based on the index S_{pk} can be defined as:

$$P_r(p) = P(\hat{S}_{pk} < k_r). \tag{15}$$

So, the OC function of the proposed VRGS plan based on the index S_{pk} can be established as:

$$\pi_a(S_{pk}) = \frac{P_a(\hat{S}_{pk} \geq k_a)}{P_a(\hat{S}_{pk} \geq k_a) + P_r(\hat{S}_{pk} < k_r)}. \tag{16}$$

The usual selection is to minimize the ASN at p_{AQL} or p_{LQL} or to minimize the average value of ASN at both p_{AQL} and p_{LQL}. In this paper, we considered the ASN value based on p_{LQL}. The ASN for the VRGS plan is given as:

$$ASN = \frac{n}{P_a(\hat{S}_{pk} \geq k_a) + P_r(\hat{S}_{pk} < k_r)}. \tag{17}$$

By solving the following optimization problem, we can determine the plan parameters of the proposed VRGS plan based on the yield index S_{pk}:

Minimize ASN_{VRGS} (18)

Subject to:

$$\pi_a(p_{AQL}) - \pi_a(p_{LQL}) \geq W, \tag{19}$$

$$\pi_a(p_{AQL}) = \frac{P_a(\hat{S}_{pk} \geq k_a)}{P_a(\hat{S}_{pk} \geq k_a) + P_r(\hat{S}_{pk} < k_r)} \geq 1 - \alpha, \tag{20}$$

$$\pi_a(p_{LQL}) = \frac{P_a(\hat{S}_{pk} \geq k_a)}{P_a(\hat{S}_{pk} \geq k_a) + P_r(\hat{S}_{pk} < k_r)} \leq \beta, \tag{21}$$

$$k_a > k_r > 0, \tag{22}$$

$$p_{AQL} < p_{LQL}. \tag{23}$$

The above constraints denote the conformity to the ideal OC curve and also satisfying the first and second type error probabilities.

5- Analysis and discussion

This paper calculates and tabulates the plan parameters, (n, k_a, k_r) and ASN of the proposed VRGS and VMDS sampling plans by considering the minimum angle method based on the process yield index S_{pk} with different combinations of α, β, p_{AQL} and p_{LQL}. Note that the values of the parametersp_{AQL},p_{LQL}, W, α and β are determined by the producer's and the customer's predetermined contract. The values of the parameters, in this paper, are taken from the literature (Wu and liu, 2018 and Wu et al., 2015).

In this paper, we use a grid search algorithm in MATLAB R2018 software for solving the optimization models to obtain the optimal parameters of the proposed sampling plans where n = 2(1)1000, k_r = 0.001(0.001)2.0 and k_a= 0.001(0.001)2.0. The optimal parameters of the proposed sampling plans are obtained according to the optimization model in Sections 4-1 and 4-2. Tables 1-3 show the optimal plans parameters (n, k_a, k_r) and the objective function, ASN, for (α, β) = (0.01, 0.01), (0.01, 0.05), (0.05, 0.05); p_{AQL}=1, 100, 200, 500, and p_{LQL}= 100, 200, 500, 1000, 3000 and W = 0.95.

The following results are obtained from tables 1-3: As the levels of producer's and customer's risks increase, the sample size to be inspected decreases. For instance, suppose the quality levels are set equal to p_{AQL}= 100 PPM, p_{LQL}= 1000 PPM, W = 0.95, and (α, β) = (0.01, 0.01). Then, the inspected sample size of the VRGS plan is n_{VRGS} = 172. Setting (α, β) = (0.05, 0.05) while all other values are the same, then n_{VRGS} = 112. This is also true for the proposed VMDS sampling plan. When the difference between the quality levels of the producer and customer is reduced, larger sample sizes are required to avoid misunderstanding the quality of information and judgment errors. For instance,

suppose the quality levels are set to (p_{AQL}, p_{LQL}) = (1 PPM, 100 PPM), $W = 0.95$, and (α, β) = (0.01, 0.01). Then, the inspected sample size of the VMDS plan ($m = 2$) is $n_{VMDS} = 132$. Setting (p_{AQL}, p_{LQL}) = (1 PPM, 3000 PPM) while all other values are kept as the same, then $n_{VMDS} = 27$. This is also true for the proposed VRGS sampling plan.

Table 1. The plan parameters for the VMDS sampling plan ($m = 2$) under various p_{AQL}, p_{LQL}, α and β.

		$\alpha = 0.01$			$\alpha = 0.01$			$\alpha = 0.05$		
		$\beta = 0.01$			$\beta = 0.05$			$\beta = 0.05$		
p_{AQL}	p_{LQL}	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a
1	100	132	0.001	1.483	99	0.001	1.460	92	0.001	1.477
	200	92	0.001	1.453	69	0.001	1.426	64	0.001	1.445
	500	59	0.001	1.410	45	0.001	1.376	41	0.001	1.404
	1000	43	0.001	1.373	33	0.001	1.335	30	0.001	1.367
	3000	27	0.001	1.303	21	0.001	1.258	19	0.001	1.290
100	500	574	1.145	1.240	418	1.107	1.231	395	0.001	1.239
	1000	250	0.001	1.212	184	0.001	1.198	173	0.001	1.209
	3000	94	0.001	1.158	71	0.001	1.136	65	0.001	1.157
200	500	1635	0.001	1.208	1190	1.145	1.202	1123	1.132	1.207
	1000	472	1.063	1.180	347	1.060	1.170	326	0.001	1.179
	3000	136	0.865	1.129	102	0.001	1.112	95	0.001	1.124
500	1000	2253	0.001	1.135	1669	1.084	1.130	1562	1.076	1.134
	3000	276	0.001	1.088	203	0.001	1.076	191	0.001	1.086

Table 2. The plan parameters for the VMDS sampling plan ($m = 3$) under various p_{AQL}, p_{LQL}, α and β.

		$\alpha = 0.01$			$\alpha = 0.01$			$\alpha = 0.05$		
		$\beta = 0.01$			$\beta = 0.05$			$\beta = 0.05$		
p_{AQL}	p_{LQL}	n	k_r	k_a	n	k_r	k_a	n	k_r	k_a
1	100	139	0.001	1.478	105	0.001	1.453	98	0.001	1.473
	200	97	0.001	1.447	73	0.001	1.419	68	0.001	1.444
	500	63	0.001	1.401	48	0.001	1.367	44	0.001	1.395
	1000	46	0.001	1.363	36	0.001	1.321	33	0.001	1.344
	3000	29	0.001	1.292	23	0.001	1.242	21	0.001	1.267
100	500	603	0.001	1.238	444	0.001	1.229	420	0.001	1.236
	1000	264	0.001	1.208	195	0.001	1.194	185	0.001	1.203
	3000	99	0.001	1.153	75	0.001	1.131	70	0.001	1.147
200	500	1741	0.001	1.206	1249	0.001	1.201	1195	0.001	1.205
	1000	495	0.001	1.178	365	0.001	1.168	347	0.001	1.176
	3000	144	0.001	1.125	108	0.001	1.107	101	0.001	1.121
500	1000	2374	0.001	1.134	1754	0.001	1.129	1661	0.001	1.133
	3000	289	0.001	1.085	215	0.001	1.073	204	0.001	1.081

Table 3. The plan parameters and ASN value for the VRGS plan under various p_{AQL}, p_{LQL}, α and β.

		$\alpha = 0.01$				$\alpha = 0.01$				$\alpha = 0.05$			
		$\beta = 0.01$				$\beta = 0.05$				$\beta = 0.05$			
p_{AQL}	p_{LQL}	n	k_r	k_a	ASN	n	k_r	k_a	ASN	n	k_r	k_a	ASN
1	100	96	1.348	1.527	134.35	90	1.343	1.481	126.44	63	1.349	1.582	92.53
	200	70	1.301	1.496	96.15	68	1.301	1.440	90.94	46	1.306	1.564	65.51
	500	47	1.230	1.453	64.63	47	1.235	1.385	61.51	31	1.241	1.537	43.45
	1000	37	1.181	1.407	49.14	36	1.179	1.339	46.88	24	1.194	1.506	32.65
	3000	25	1.086	1.329	32.77	25	1.090	1.249	31.41	16	1.105	1.451	21.33
100	500	366	1.180	1.266	534.89	329	1.176	1.248	496.73	252	1.182	1.289	376.45
	1000	172	1.128	1.242	242.97	162	1.126	1.213	227.48	112	1.126	1.278	169.35
	3000	69	1.032	1.197	98.43	66	1.030	1.154	92.98	48	1.045	1.242	67.06
200	500	984	1.171	1.225	1477.27	909	1.170	1.213	1364.46	700	1.173	1.236	1049.81
	1000	311	1.119	1.205	444.43	281	1.115	1.186	413.46	213	1.121	1.229	312.44
	3000	99	1.028	1.162	138.22	91	1.022	1.130	130.02	67	1.034	1.208	95.04
500	1000	1380	1.106	1.149	2042.09	1228	1.104	1.140	1882.29	966	1.107	1.158	1452.32
	3000	189	1.016	1.115	267.24	171	1.011	1.092	249.54	124	1.015	1.146	186.36

An important measure of the sampling plan performance is the operating characteristic (OC) curve, which quantifies the risks of producers and customers. Figure 1 displays the OC curves of the proposed VRGS and the VMDS sampling ($m = 2, 3$) plans under $p_{AQL} = 100$ PPM, $p_{LQL} = 1000$ PPM and $\alpha = 0.01$, $\beta = 0.05$. It can be observed that the VRGS plan has a better OC curve than the

VMDS sampling plan and the probability of acceptance will increase as the value of p decreases for the two proposed sampling plans.

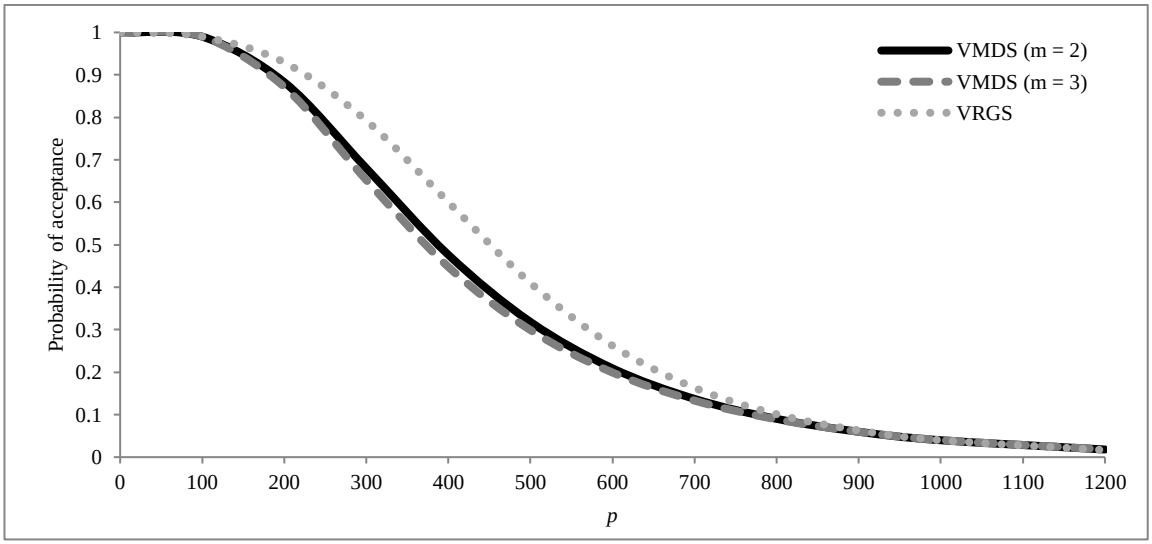

Fig 1. The OC curves of the proposed VRGS and VMDS sampling plans ($m = 2, 3$) under $(p_{AQL}, p_{LQL}) = (100, 1000)$.

We consider the minimum angle method in the proposed plan as a constraint and examine it. In figure 2, the ASN curves of the proposed VMDS and VRGS plans are plotted with different values of p_{AQL} under $p_{LQL} = 3000$ PPM, $(\alpha, \beta) = (0.01, 0.05)$, and different values of W. The results show that by increasing$W$, the value of the objective function also increases. The VRGS plan is more economical because by increasing the resemblance of the OC curve to the ideal OC curve, the VRGS plan is more efficient than the VMDS sampling plan with smaller ASN. In addition, as the quality level of the lot increases, the performance of the two proposed sampling plans gets close to each other.

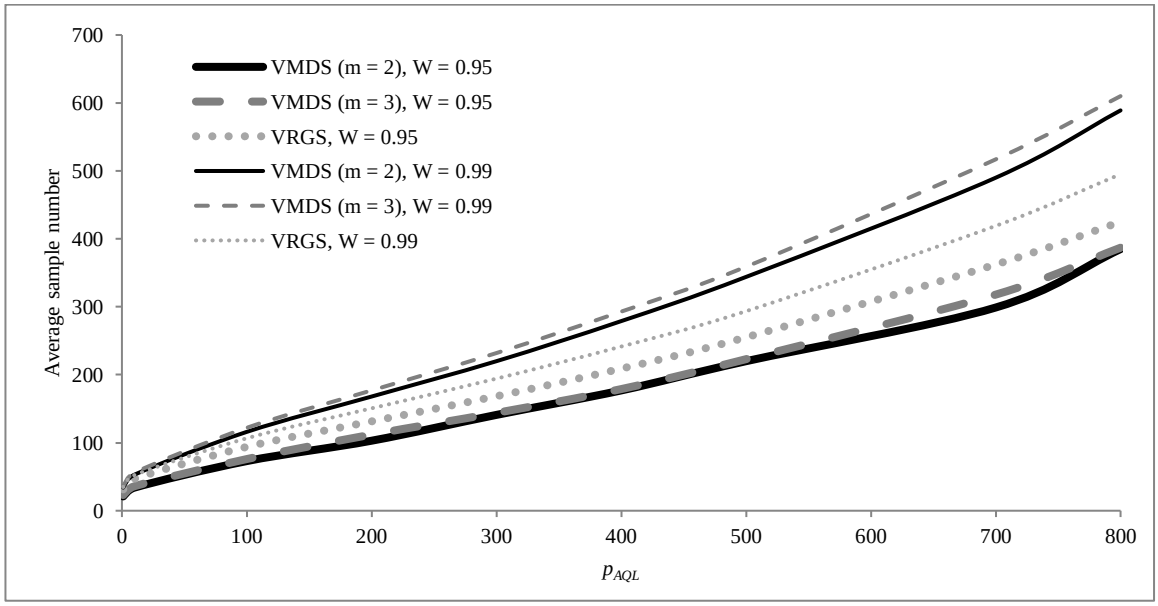

Fig 2. The ASN curves of the proposed VRGS and VMDS sampling plans ($m = 2, 3$) under different values of p_{AQL} under $p_{LQL} = 3000$ PPM and $(\alpha, \beta) = (0.01, 0.050)$.

6- Application example

We consider a numerical example of indium tin oxide (ITO)-conductive thin films scrutinized in Wu and Liu (2018) to reveal the applicability of the proposed plan. Suppose the contract between the producer and the customer is the VMDS sampling plan with $USL = 88\%$ and $LSL = 92\%$. The quality levels are set to p_{AQL}= 100 PPM, p_{LQL}= 3000 PPM, $W = 0.95$, and $(\alpha, \beta) = (0.01, 0.01)$. Then, the optimal parameters of VMDS sampling plan with $m = 2$ are (94, 0.001, 1.158) following table 1 according to the steps outlined in Section 4-1 for the VMDS sampling plan and the corresponding optimization model. This guides us to take a random sample of size 94 from the lot, then accept the lot if $\hat{S}_{pk} \geq 1.158$, and reject the lot if $\hat{S}_{pk} < 0.001$. If $0.001 < \hat{S}_{pk} < 1.158$, accept the lot provided that the previous $m = 2$ lots were accepted in the terms of $\hat{S}_{pk} \geq 1.158$. Otherwise, reject the lot. Thus, 94 samples are measured and summarized in table 4. $\hat{S}_{pk} = 1.2293 \geq k_a$ =1.158 and therefore, the customer would accept the lot.

Table 4. The 94 collected samples taken from the lot. (unit:%)

89.70	90.70	89.63	90.53	88.91	90.93	90.86	90.96
90.66	89.45	90.23	91.02	90.04	90.27	90.24	90.71
90.32	89.58	90.51	90.07	91.23	89.69	90.93	90.07
90.72	90.64	89.71	91.39	89.68	90.88	89.70	89.80
90.83	90.81	90.69	89.93	90.04	90.38	89.52	89.70
90.30	90.93	89.41	90.43	90.21	90.49	90.38	89.42
90.12	90.25	89.47	90.92	90.47	90.05	89.27	89.49
90.03	89.47	90.40	90.56	90.01	89.93	89.79	89.79
90.81	90.24	89.88	90.37	89.74	89.80	90.07	90.45
90.12	89.76	89.81	90.04	91.34	89.89	90.47	90.36
90.45	89.74	89.60	90.38	89.72	91.03	90.05	
89.93	89.51	89.52	90.56	90.38	89.92	90.21	

7- Conclusion and a suggestion for future research

Sampling, one of the primeval aspects of quality assurance, is used to inspect and decide on products. The difference between the quality level of the provided lot and the required lot can be significantly reduced by a well-designed sampling plan. It is apparent that in an acceptance sampling plan achieving the ideal OC curve is desirable. However, the ideal OC curve is achieved only by 100% inspection, provided that the inspection is error-free. In addition, 100% inspection leads to higher costs and is usually inefficient and impractical. In this paper, we developed two new acceptance sampling plans by considering the constraints on the convergence to the ideal OC curve as well as satisfaction of the producer's and customer's risks, simultaneously. In the proposed plans, we considered the minimum angle method as a constraint and the results showed that the performance of two sampling plans approaches each other as the quality level of the lot increases. Moreover, by increasing the similarity of the proposed plans OC curves to the ideal OC curves, the proposed VRGS plan performs better than the proposed VMDS sampling plan in terms of ASN.

It must be stressed that the proposed VRGS and VMDS sampling plans are developed for normally distributed processes. The development of the proposed sampling plans for non-normal distributions can be of interest for further studies because in real practice the product quality characteristic may follow a non-normal distribution.

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