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Dear Dr. Waezi,

I am pleased to inform you that your paper co-authored with Mohammad Amin Raoof entitled, “A Semi-Analytic Method to Estimate the Response Spectrum of the Synthetic Acceleration Records with Evolutionary Spectrum” (Ref. No: SCI-2005-4474), has been accepted as an "Article" for publication in one of the issues of Scientia Iranica, Transactions A: Civil Engineering.

Sincerely yours,

S. T. A. Niaki

[Signature]
A Semi-Analytic Method to Estimate the Response Spectrum of the Synthetic Acceleration Records with Evolutionary Spectrum

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Abstract

In this paper, we introduce a semi-analytic procedure for deriving the response spectrum of the synthetic acceleration records generated using the Double Frequency Model (DFM). DFM is a filtered white noise method for fully non-stationarity synthetic acceleration records. The proposed semi-analytic procedure is based on the theory of the first passage problem, which precludes time and computationally extensive methods e.g. Monte Carlo simulations. Assuming a slowly-varying envelope and evolutionary transfer functions, the procedure for estimating the elastic response of a structure is implemented in both time and frequency domains. Comparing the results of our model with previous models and approximations, we conclude that for a set of 10000 realizations of the DFM model, the semi-analytic model produces less than 10\% error for 92\% of the realizations. The accuracy of estimations is higher in the short-period compared to the long-period ranges of the response spectrum. Comparing the accuracy of approximations used to arrive at peak factors, results show that Michaelov et al’s approximation executed in the frequency domain yields the best results compared to Poisson or Vanmarcke’s procedures.

Keywords: Priestley’s Evolutionary Process, First Passage Problem, Synthetic ground motion, Double-Frequency Model (DFM), power spectral density

1 Introduction
In recent years the tendency toward the development of synthetic ground motions has grown extensively. There are generally physical, stochastic, and hybrid models to generate artificial earthquake motion. The popularity of stochastic methods which aim to regenerate the statistical characteristics of the recorded motions has soared in recent years. Stochastic ground-motion models are generally of two types: ‘source-based’ and ‘site-based’ [1]. Although the source-based models have the advantage of using the physical parameters obtained through simulation processes, with due attention to the seismological nature of the region, these parameters vary significantly from region to region. On the other hand, as site-based models do not need detailed seismological information, they are more favorable where the number of instrumental recordings is limited [2]. Due to the capability of the stochastic methods in generating high-frequency signals [3, 4] and the availability of fast computers, pure “physical models” have evolved to the "hybrid models" using stochastic methods and thus formed powerful tools to simulate ground motions for scenario-based earthquake simulation.

Depending on the time variation of its amplitude and frequency content, an earthquake accelerogram as a signal can be categorized either as stationary or non-stationary. The amplitude non-stationarity is defined as the change in the amplitude or intensity of the acceleration record versus time, while the frequency non-stationarity indicates the change of its “instantaneous power spectrum” [1]. Frequency non-stationarity of signals results from the dynamic nature of the ground motion that is mainly due to the faster propagation speed of high-frequency waves in the soil media [5]. Many studies indicate the significance of the frequency content change on the seismic-induced response of linear and nonlinear structures [6]. The coincidence of stiffness degradation with the arrival of low-frequency surface waves could lead to disastrous results and even the collapse of these structures [7]. Contrary to the amplitude non-stationarity, it is hard to simulate or even capture the frequency non-stationarity of the signals. Some approaches have been introduced for considering the non-stationarity of the records, especially in the frequency domain, with some of them having a large number of parameters involved [2, 8-12]. Accurate detection of the frequency content evolution demands the utilization of complex time-frequency distributions (such as quadratic distributions) or multi-resolution analyses (such as wavelet or Hilbert-Huang transforms). On the other hand, the proper representation of these variations requires a considerable amount of data. Two main methods have been used in recent studies to generate fully nonstationary earthquake records: 1) filtered white-noise models and 2) spectral representation model. The filtered white noise model is generally described as a convolution integral of an input white noise and an evolutionary impulse response function [13]. The spectral representation is generally stated as the sum of the modulation of the different harmonics with a random phase but extended versions of the model are also proposed [14].

After the development of the models proposed by Rezaeian and Der Kiureghian [1, 15] multiple studies have been published with a similar approach that incorporated the ground motion variability in their models through paying special attention to the correlation of the model parameters with the earthquake scenario parameters.

Waezi and Rofooeei [16, 17] introduced a new site-based method for the stochastic generation of non-stationary acceleration records. The proposed model is capable of considering two large and one small dominant frequency for efficient capturing of recorded strong motion’s power spectrum. They also proposed a high-pass filtered, time-varying, double-frequencies model (DFM) with time-variant parameters to simulate a frequency-wise non-stationary process. Moreover, they included a method for scenario-specific record simulation using DFM [18].
Having the synthetic record, it’s desirable to estimate the distribution of the response of the structures subjected to these excitations given that their model parameters are fixed. This could help to develop methods that produce acceleration records whose response spectrum is compatible with those that seismic design codes prescribe [19]. This is important because the common methods that generate spectrum-compatible records generally use a seed acceleration time-history and change their frequency-content through a try-and-error scheme to achieve the best compliance. However, this approach neglects the correlation between different harmonics present in the time-history of ground motion and only captures the response spectrum compliance.

One of the methods to determine the response of the structures to non-stationary excitation is through the 'first passage problem' which may be referred to as extreme value distribution (EVD) theory or peak-factor problem. In recent decades, obtaining peak factors in earthquake engineering has been very active due to an increasing interest in performance-based design [20]. Derivation of analytical equations for structures is straightforward when the response of the structure is stationary, but assuming the oscillator response to be non-stationary, calculating the peak response turns to an extensive process since several convolution multiplications should be conducted to determine the spectral specifications of the structure response process, and also a nonlinear integral equation should be solved to evaluate the peak response [21].

Since the peak response is a random variable, it can be comprehensively described by its cumulative distribution function or probabilistic distribution function. While using the average value of peak responses is common for design purposes, since the structural response may exceed them, it is not a conservative approach. Nevertheless, using a direct method, in which the design level is determined by a response level of a certain non-exceedance probability, seems to be more rational; this level is called “quantile (percentile) level”. It can be proved that calculating the quantile level is easier than calculating the mean and the standard deviation of peak response [22]. The major problem in estimating the 3 parameters of the value of the mean, standard deviation, and the quantile level of response, is the mathematical form of the CDF of peak value for structure response being unknown. Traditionally, this task has been addressed thanks to the computation power of modern computers and numerical methods; however, the complexity of these problems and the amount of time needed to obtain the exact result is a disadvantage which might prevent designers from using such methods [22].

Lots of studies have been performed on the evaluation of the reliability of structures subjected to the non-stationary excitations but most of them have been focused on the excitations generated using the spectral representation method. Barbato and Vasta [23] developed a method for estimation of the evolutionary parameters of the non-classically damped MDOF linear systems subjected to the time-modulated colored noise excitation. The concept of bandwidth factor was generalized by Barbato and Conte [24] to complex-valued non-stationary processes from which they evaluated the reliability of non-classically damped MDOF structures subjected to Kanai-Tajimi [25] excitations with amplitude non-stationarity. Barbato and Conte [26] developed a method to find the reliability of the non-classically-damped MDOF subjected to fully nonstationary stochastic excitation described by spectral representation. Alderucci et al [27] proposed a method to generate a specific type of fully non-stationary synthetic records compatible with a design code based on the solution of the ‘first passage problem’. Yu et al [28] derived the close-form solution of the stochastic response of the linear MDOF systems in the time domain under fully nonstationary excitation described using the spectral representation method. To reduce the computation cost of the evaluation of the stochastic response of MDOF large-scale structures based on random vibrations theory, Zhao and Huang [29] proposed an approach involving FFT. They also used a spectral
representation to describe the nonstationary random excitation. Alderucci et al [27] evaluated analytically the time-frequency response function of non-classically damped MDOF structures subjected to correlated fully non-stationary excitations. Xu and Feng [30] proposed a new point selection method for higher-order variate-space to numerically obtain the reliability of the nonlinear systems using the probability density evolution method subjected to fully non-stationary ground motions represented by spectral representation method. Xu et al [31] used kernel density distribution along with Latin hyper-cube simulation to find the small probability failure of nonlinear structures subjected to the nonstationary ground motions represented by the spectral representation method.

In this paper, the approximate maximum oscillation response of the structures, subjected to a non-stationary excitation generated by the DFM model, a filtered white noise model, is studied. The purpose of this paper is to determine a semi-analytic relationship to describe the response of the linear elastic structure under non-stationary excitations simulated by DFM. At first, the modulating function and the evolutionary power spectrum are expressed in terms of the model parameters using some simplifications. Then the maximum peak-factors are computed by a semi-analytic method which precludes the need for computationally extensive simulations and the efficiency of the proposed method is evaluated.

2 DFM model
The double frequency model (DFM), is a site-based filtered white noise stochastic method proposed by Waezi and Rofooei [16, 18, 32], which is capable of generating both amplitude and frequency non-stationary records, with 13 model parameters. It was shown that the 13 model parameters can be regressed against source-path-site characteristics which enables the model to generate an ensemble of records conditioned on a specific scenario. The DFM method, which can incorporate both the amplitude and frequency non-stationarity, can be stated as:

$$a(t) = q(t) - \int \frac{1}{\sigma(t)} h(t - \tau)w(\tau)d\tau$$  \hspace{1cm} (1)

Where $a(t)$, $q(t)$, $h(t, \tau)$ and $\sigma(t)$ represent the synthetic acceleration record, envelope function (EF), evolutionary impulse response function (IRF) of the model, and standard deviation of the process resulted from the integral in Equation (1), respectively. Considering that $w(\tau)$ is a stationary white noise with constant power spectral density (PSD) equal to $S_0$, the evolutionary variance of the resulted non-stationary process $a(t)$ can be described as:

$$\sigma^2(t) = 2\pi S_0 \int_0^T h(t, \tau)^2 d\tau$$  \hspace{1cm} (2)

The evolutionary IRF of DFM is defined according to the inverse Fourier transform of a complex function called evolutionary transfer function (ETF). ETF of DFM is outlined as follows:

$$H(\omega, t) = \frac{-2i \zeta_y(t) \omega}{\omega_y(t)} \times \frac{-2i \zeta_y \omega_y}{\omega \omega_y(t)}$$

$$= \frac{1- \frac{2i \zeta_y(t) \omega}{\omega_y(t)}}{1- \frac{2i \zeta_y \omega_y}{\omega^2}} \times \frac{1- \frac{2i \zeta_y \omega_y}{\omega \omega_y(t)}}{1- \frac{2i \zeta_y \omega_y}{\omega^2}} \times \frac{1- \frac{2i \zeta_y \omega}{\omega_y(t)}}{1- \frac{2i \zeta_y \omega_y}{\omega^2}} \times \frac{1- \frac{2i \zeta_y \omega_y}{\omega \omega_y(t)}}{1- \frac{2i \zeta_y \omega_y}{\omega^2}}$$  \hspace{1cm} (3)

Where $\zeta_y(t)$ and $\omega_y(t)$ represent the damping and the frequency parameter of the time-variant part of the ETF while $\zeta_y$ and $\omega_y$ denote their counterpart for the stationary part of ETF. This model is capable of encompassing two distinct PSD peaks, which is not uncommon in the recorded ground motions. The DFM is an
adjustment of the modified Kanai-Tajimi (MKT) model for capturing the low-frequency region of the PSD of the resulted synthetic records. It’s assumed here that $\xi_f(t)$ and $f_f(t) = \frac{\omega_f(t)}{2\pi}$ are linear functions of time as:

$$f_f(t) = f_{f_0} + \frac{f_{f_n} - f_{f_0}}{T_d} t$$

$$\xi_f(t) = \xi_{f_0} + \frac{\xi_{f_n} - \xi_{f_0}}{T_d} t$$

(4)

Where $T_d$ represents the effective duration of the records and "0" and "n" subscripts indicate the value of the time-variant parameters (i.e. $\xi_f$ and $f_f$) at the beginning and the end of the effective duration of the acceleration time-history. For the sake of simulation, the effective length of the record is adopted as the duration between the times corresponding to 0.1% and 99.9% of the maximum Arias intensity.

The envelope function $q(t)$ in Equation (1) is used to curb the evolutionary variance of the simulated process independently from the frequency content. Because of its higher flexibility, the Amin and Ang’s [33] envelope function is used as

$$q(t) = \begin{cases} 
0 & t \leq T_0 \\
\alpha_1 \left(\frac{T - T_0}{T_1 - T_0}\right)^2 & T_0 \leq t < T_1 \\
\alpha_1 & T_1 \leq t < T_2 \\
\alpha_1 \exp[-\alpha_2 (t - T_2)]^{\alpha_3} & t \geq T_2 
\end{cases}$$

(5)

The EF parameters, listed as $\alpha = \{T_0, T_1, T_2, \alpha_1, \alpha_2, \alpha_3\}$, are determined according to the best fit of Arias intensity of Amin and Ang’s modulating function to the target Arias intensity curve of the record. The authors have determined the range of variation of EF parameters according to the values obtained for non-pulse-like near-field ground motions for the previous study [18, 32]. It should be mentioned that, since $T_0$ only shifts the modulating function in time, it can be set equal to zero, and consequently $T_1$ and $T_2$ are shifted as $T_1' = T_1 - T_0$ and $T_2' = T_2 - T_0$.

To make the velocity time-histories obtained from the DFM model approach zero at the end of the time series, the components having near-zero frequencies should be carefully filtered out without disrupting the low-frequency components. For this purpose, the relative acceleration response of an SDOF $\ddot{u}(t)$, hereafter referred to as the high-pass filter (HPF), subjected to the resulted DFM acceleration record $\dot{a}(t)$ is used whose equation may be stated as follows:

$$\ddot{u} + 2\xi_f\omega_f \dot{u} + \omega_f^2 u = -\dot{a}(t)$$

(6)

Where $u$, $\ddot{u}$, and $\dot{a}(t)$ denote the SDOF’s relative displacement, acceleration, and excitation process derived from Equation (6), respectively, and $\xi_f$ and $\omega_f = 2\pi f_f$ represent the damping and frequency of the HPF. $\ddot{u}$ is hereafter used as the record simulated using DFM and will be referred to as $a_{mod}(t)$. Therefore, the evolutionary power spectral density (EPSD) of DFM’s records depend on 13 parameters including 8 IRF parameters $\{\xi_f, f_f, \xi_f', f_f', \xi_f'', f_f''\}$ and 5 EF parameters (assuming $T_0 = 0$) $\{T_1', T_2', \alpha_1, \alpha_2, \alpha_3\}$. 

5
3 DFM’s evolutionary power spectrum

According to Priestley [34], a real-valued stochastic evolutionary process can be defined as in the general form of Fourier-Stieltjes as follows:

\[ X(t) = \int_{-\infty}^{\infty} A(t,\omega) \exp(i\omega t) d\omega \]  

(7)

where \( A(t,\omega) \) is a deterministic complex-valued modulating function and \( Z(\omega) \) is a random complex-valued function. \( X_s(t) \) is the “Embedded” stationary process which is defined based on its spectrum as follows:

\[ X_s(t) = \int_{-\infty}^{\infty} \exp(i\omega t) d\omega \]  

(8)

In which:

\[ E[dZ(\omega_1)dZ(\omega_2)] = S_{XX}(\omega_1)\delta(\omega_1 + \omega_2) d\omega_1 d\omega_2 \]  

(9)

where \( S_{XX}(\omega) \) is representative of the PSD of the stationary process \( X_s(t) \) and \( \delta(.) \) is Dirac delta. If the Fourier Transform of \( A(t,\omega) \exp(i\omega t) \) on \( \omega \) axis is available, Equation (7) can be stated in the time domain by means of a convolution integral as:

\[ \psi(t,\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(t,\omega) \exp[i\omega(t-\tau)] d\omega \]  

(10)

\[ A(t,\omega) \exp(i\omega t) = \int_{-\infty}^{\infty} \psi(t,\tau) \exp[i\omega \tau] d\tau \]

Then final expression for \( X(t) \) is:

\[ X(t) = \int_{-\infty}^{\infty} \psi(t,\tau) X_s(\tau) d\tau \]  

(11)

For \( X(t) \), EPSD is stated as follows:

\[ G_{XX}(t,\omega) = |A(t,\omega)|^2 S_{XX}(\omega) \]  

(12)

Differentiating Equation (7) for \( j \) times gives:

\[ X^{(j)}(t) = \int_{-\infty}^{\infty} A^{(j)}(t,\omega) \exp(i\omega t) d\omega \]  

(13)

Where \( X^{(j)} \) is the \( j \)th derivative of \( X(t) \) and \( A^{(j)} \) is the modulating function for \( X^{(j)}(t) \) that can be obtained using the following recursive formula:

\[ A^{(j)}(t,\omega) = \dot{A}^{(j-1)}(t,\omega) + (i\omega)A^{(j-1)}(t,\omega) \]  

(14)

Comparing Equation (11) with Equation (1), one can express \( \psi(t,\tau) \) corresponding to DFM as:

\[ \psi(t,\tau) = \alpha(t) U(\tau) h(t - \tau,\tau) \]  

(15)

Where \( U(\tau) \) is the unit step function to count for the lower bound of the integral used in (1) and Substituting the above equation in Equation (10) gives:

\[ A(t,\omega) = \alpha(t) \int_{-\infty}^{\infty} U(\tau) h(t - \tau,\tau) \exp[-i\omega(t - \tau)] d\tau \]  

(16)

And by a change of integration variable \( \tau \) to \( s \) it gives:
\[ A(t,\omega) = \alpha(t) \int_0^\infty h(s; t-s) \exp[-i\omega s] ds \]  

(17)

Furthermore, \( h(t-\tau, \tau) \) in DFM is calculated from ETF, \( H(\tau, \omega) \), as follows:

\[ h(s, \tau) = \int_\omega^\infty H(\tau, \omega) \exp(i\omega s) d\omega \]  

(18)

\[ H(\tau, \omega) = \frac{1}{2\pi} \int_0^\infty h(s, \tau) \exp(-i\omega s) ds \]  

Comparing this equation with Equation (8), it reveals large differences between \( H(\tau, \omega) \) and \( A(t, \omega) \).

\[ h(s, t-s) = \int_\omega^\infty H(t-s, \sigma) \exp(i\sigma s) d\sigma \]  

(19)

\[ A(t, \omega) = \alpha(t) \int_\omega^\infty \exp[-i(\omega-\sigma)] \int_0^\infty H(\tau, \sigma) \exp[i(\omega-\sigma)\tau] d\tau d\sigma . \]

Having obtained the complex modulating function of the DFM records, their EPSD can be calculated applying Equation (12). However, another model can be proposed with a given \( H(t, \omega) \) in a way that in this model it can be proved that, if \( H(\tau, \omega) \) has a slow variation vs time:

\[ \psi(t, \tau) = \frac{1}{2\pi} \int_\omega^\infty H(t, \omega) \exp[i\omega(t-\tau)] d\omega \]  

(20)

\[ h(t-\tau, t) = \int_\omega^\infty H(t, \omega) \exp[i\omega(t-\tau)] d\omega \]

In such a case, we have:

\[ \psi(t, \tau) = \alpha(t) U(\tau) h(t-\tau, t) \]  

(21)

It should be noted that the second variable of \( h \) in Equation (21) is different from that of Equation (15). The difference springs from \( h(t-\tau, t) \) which indicates that the response of an impulse exerted at \( \tau \) and recorded at \( t \) is dependent on the parameters at \( t \). However, in the DFM case, the response if impulse exerted at \( \tau \) and recorded at \( t \) is dependent on \( \tau \). It can be proved that, if \( H(\tau, \omega) \) has a slow variation vs time:

\[ h(t-\tau, \tau) \equiv h(t-\tau, t) \]

\[ A(t, \omega) \equiv H(t, \omega) \]  

(22)

Fig. 1 shows a record generated using \( h(t-\tau, t) \) in Equation (1) for DFM, which is called Priestly Double-Frequency, with those of original DFM with \( h(t-\tau, \tau) \) in Equation (1) which is denoted as Double-Frequency. The results show that only when the variation rate of model parameters is high, the differences are noticeable.

### 4 Peak Value Problem

Let’s assume that \( X(t) \) is the displacement response of an elastic SDOF with natural frequency \( \omega_0 \) and damping ratio \( \xi_0 \), subjected to a random excitation generated by the DFM method. A technique to formulate the peak response of the SDOF is defining a new random process \( Y(t) \), which is the peak value of \( X(t) \) up to \( t \). In other words:

\[ Y(t) = \max_{0 \leq s \leq t} X(s) \]  

(23)
Peak value distribution for $X(t)$ is similar to the distribution of the random parameter $Y(t)$. It should be noted that for a stationary random process $\{X(t)\}$, it should be expected for $\{Y(t)\}$ to be non-stationary, since, if the study time is extended, higher values of $X(t)$ will be noticed. If $L_x(u,t)$ is considered as the cumulative distribution function of $Y(t)$:

$$L_x(u,t) = F_y(u) = P[Y(t) \leq u] = P[X(s) \leq u : 0 \leq s \leq t]$$

(24)

The last term indicates that $X(s)$ is less than or equal to $u$ for all values of $s$. $L_x(u,t)$ is sometimes called the survival probability. Often, expressing the survival probability as a form of the time-exponential function is convenient. In certain conditions one can write:

$$L_x(u,t) = L_x(u,0) \exp \left( -\int_0^t \eta_x(u,s) ds \right)$$

(25)

Where $\eta_x(u,t)$ is regarded as the conditional rate of up-crossing from level $u$, given that no earlier up-crossing occurrence has occurred. If the up-crossing from level $u$ is taken as the failure in a system, thus $\eta_x(u,t)$ is called the risk function in the reliability field.

There is no exact analytical relationship for $\eta_x(u,t)$, however, some simplifying assumptions are made to determine it. In this study, three assumptions are used to evaluate the response spectrum of the DFM records. These methods are summarized in the following subsections.

4.1 Poisson’s Assumption

The simplest method is to assume that $b$ level crossing occurs independently in time $t$. For an upper bound level of $b$, it can be proved that assuming the crossing occurrences to be independent, especially for broadband processes, is sufficiently proper [22, 35]. The Poisson approximation of $\eta_x(u,t)$ is shown as $\eta_x^p(u,t)$, which is represented for a symmetrical and two-sided barrier process of $b$ is represented as below:

$$\eta_x^p(b,t) = 2 \nu^*(b,t)$$

(26)

Where $\nu^*(b,t)$ is equal to the expected exceedance rate in $b$ level by $X(t)$ at any time $t$. It can be proved that this approximation for a short time interval of $t$ or a narrow-band process is too conservative. It is shown that the exceedances of narrow-band processes usually tend to cumulate in a cluster-like form, and this fact contradicts the independency assumption [22]. Shinozuka and Yang have proposed an approximation for the mean and the standard deviation value of peak responses for random processes in wide time intervals [36]. Since in this paper, records of limited length are being studied, these equations are out of acceptance. To evaluate $\nu^*(b,t)$, the following classic result can be used:

$$\nu^*(b,t) = \left[ \frac{\sigma_x (1 - \rho_{xx}^2)^{0.5}}{2 \pi \sigma_x} \right] \exp \left[ -\frac{b^2}{2 \sigma_x^2 (1 - \rho_{xx}^2)} \right] + \left[ \frac{\rho_{xx} b \sigma_x}{(2 \pi)^{0.5} \sigma_x^2} \right] \exp \left( \frac{-b^2}{2 \sigma_x^2} \right) \Phi \left[ \frac{\rho b}{\sigma_x (1 - \rho_{xx}^2)^{0.5}} \right]$$

(27)
Where $\sigma^2_x(t)$, $\sigma^2_s(t)$, $\rho_{xx}(t)$ and $\Phi$ are the variances of the process and its derivative as well as their correlation coefficient and Gaussian (normal) cumulative distribution function respectively.

### 4.2 Vanmarcke Approximation

Among the modification suggested for Poisson’s method, “Two-State Process” assumption by Markov, or Vanmarcke [37] method has received more acceptance and is proper for narrow-band processes.

In this method, the independency assumption of "eligible" crossings is implemented by the envelope process. A crossing, with a positive slope, of level $b$ by $X(t)$ envelope is called "eligible", only if it accompanies at least one positive passage of $|X(t)|$ from the same level. For a random, zero-mean, stationary Gaussian process of $X(t)$, Vanmarcke has proposed an approximate relation to be used instead of Equation (26) [37]:

$$
\eta^V_x(b,t) = 2\nu^V(b,t) \left\{ \frac{1 - \exp \left( -\left( \frac{\pi}{2} q^{0.5} \frac{b}{\sigma_s} \right)^2 \right)}{1 - \exp \left( -\frac{1}{2} \frac{b^2}{\sigma_x^2} \right)} \right\}
$$

(28)

In this method, the stationary bandwidth factor is calculated according to the spectral moments of the process, and that these values may be unbounded for the non-stationary process. Corotis et. al [38] have proposed an experimental method to calculate this value, which is only applicable to a certain case of the response of an SDOF system under a stationary excitation.

### 4.3 MLS Method

Using Vanmarcke’s approximation, Michaelov et. al [39] showed that for a more general non-stationary response, the crossing rate of the positive slope of the qualified envelope can be stated as below[40]:

$$
\eta^V_x(x,t) = \frac{1 - F_v(x,t)2\nu^V(0,t)}{F_v(x,t)} \left\{ \frac{1 - \exp \left[ -\frac{V^V_v(x,t)}{1 - F_v(x,t)2\nu^V(0,t)} \right]}{1 - \exp \left( -\frac{1}{2} \frac{b^2}{\sigma_x^2} \right)} \right\}
$$

(29)

wherein, $F_v(x,t)$ and $V^V_v(x,t)$ are equal to the transient CDF and the unconditional rate of up-crossing of the envelope process $V(t)$. Considering that PDF of envelope process $p_v(x,t)$ and its joint distribution with $V(t)$ for an evolutionary Gaussian process, it can be shown that the following relationship expresses the upcrossing rate of the qualified envelope[39]:

$$
\eta^V_x(x,t) = \frac{1}{\pi} \sqrt{1 - \rho^2_{xx}(t)} \frac{\sigma_s(t)}{\sigma_x(t)} x 
$$

$$
1 - \exp \left[ -\frac{\pi}{2} \frac{x}{\sigma_s(t)} \sqrt{q_x(t) - \rho_{xx}^2(t)} \right] \frac{\rho_{xx}(t) x}{\sqrt{q_x(t) - \rho_{xx}^2(t)}} \frac{1}{\sqrt{1 - \rho^2_{xx}(t)}} \Psi \left( \frac{\rho_{xx}(t) x}{\sqrt{q_x(t) - \rho_{xx}^2(t)}} \sigma_s(t) \right) \right]
$$

(30)

where $q_x(t)$ is the bandwidth factor and is derived in a way that it holds for a non-stationary process and

$$
\Psi(x) = \exp(-x^2/2) + \sqrt{2\pi} \Phi(x).
$$

It can be observed that except for the experimental power of 1.2, for the case
in which $\rho_{xx}(t) = 0$, equations Equation (28) and Equation (30) yield identical results. This method hereafter will be denoted as “MLS”.

5 Analytical Response Spectrum of DFM model in the frequency domain

If a linear elastic SDOF is subjected to non-stationary excitation record $a_{DFM}(t)$, derived from DFM, we have:

$$\ddot{x} + 2\zeta_0 \omega_0 \dot{x} + \omega_0^2 x = -a_{DFM}(t)$$

(31)

in this case, the response $x(t)$ can be rewritten in form of the convolution integral as below:

$$x(t) = -\int_0^t h_0(t - \tau) a_{DFM}(\tau) d\tau$$

(32)

Substituting Equation (11) in Equation (32) gives:

$$x(t) = -\int_0^\infty \alpha(\tau_2) \int_0^\infty h_0(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) U(\tau_1) W(\tau_1) d\tau_2 d\tau_1$$

$$= -\int_0^\infty U(\tau_1) W(\tau_1) \int_0^\infty \alpha(\tau_2) h_0(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) d\tau_2 d\tau_1$$

(33)

Equation (33) can be interpreted as the convolution of a white noise and a combined IRF, $h_{x,comb}$, which can be stated as below:

$$h_{x,comb}(t - \tau_1, \tau_1) = U(\tau_1) \int_0^\infty \alpha(\tau_2) h_0(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) d\tau_2$$

(34)

for $x(t)$ function, which is a non-stationary modulated process, the combined modulating function, $A_{x,comb}(t, \omega)$, can be stated as below:

$$A_{x,comb}(t, \omega) = \int_0^\infty \exp[-i \omega(t - \tau_1)] U(\tau_1) \int_0^\infty \alpha(\tau_2) h_0(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) d\tau_2 d\tau_1$$

$$= \int_0^\infty h_0(t - \tau_2) A(\tau_2, \omega) \exp[-i \omega(t - \tau_2)] d\tau_2$$

$$\approx \bar{A}(t, \omega) H_0(\omega)$$

(35)

where $\bar{A}(t, \omega)$ is the mean value of $A(\tau, \omega)$ obtained from Equation (19) for $0 \leq \tau \leq t$.

A similar procedure can be done for $\dot{x}(t)$ to achieve $\ddot{A}(t, \omega)$ which is its modulating function.

$$\dot{x}(t) = -\int_0^t h_0(t - \tau) a_{DFM}(\tau) d\tau$$

(36)

Substituting Equation (11) in Equation (32) gives:

$$h_{x,comb}(t - \tau_1, \tau_1) = A(\tau_1) \int_0^\infty \alpha(\tau_2) h_0(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) d\tau_2$$

(37)

For $\dot{x}(t)$, a non-stationary modulated process, $A_{x,comb}(t, \omega)$ the modulating function can be stated as below:

$$A_{x,comb}(t, \omega) = \int_0^\infty \exp[-i \omega(t - \tau_1)] U(\tau_1) \int_0^\infty \alpha(\tau_2) h_0(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) d\tau_2 d\tau_1$$

$$= \int_0^\infty h_0(t - \tau_2) A(\tau_2, \omega) \exp[-i \omega(t - \tau_2)] d\tau_2 \approx \bar{A}(t, \omega) H_0(\omega)(i \omega)$$

(38)

To determine the peak factor with any method prescribed in section 4, one could evaluate $\sigma_x(t), \sigma_y(t), \rho_{xy}(t)$ according to the following formulas [39]:

**Note:** The formulas and equations are written in LaTeX format for clarity and precision in the text.
\[
c_{00}(t) = 2 \int_0^\infty G_{XX}(t, \omega) d\omega = \sigma^2(t),
\]

\[
c_{11}(t) = 2 \int_0^\infty G_{XX}(t, \omega) d\omega = \sigma^2(t)
\]

\[
c_{01}(t) = c_{01}^*(t) = -2i \int_0^\infty G_{XX}(t, \omega) d\omega
\]

\[
\rho_{XX}(t) = -\frac{\text{Im}[c_{01}(t)]}{\sqrt{c_{00}(t)c_{11}(t)}}
\]

\[
p(t) = \sqrt{1 - \left(\frac{\text{Re}[c_{01}(t)]}{c_{00}(t)c_{11}(t)}\right)^2}
\]

### 6 Time-Domain Equations for Spectrum Estimation

Since computing \( A_{x,\text{comb}}(t, \omega) \) and \( A_{x,\text{comb}}(t, \omega) \), in the frequency domain is very hard and it seems that using the time-domain method is easier, equations may be derived in the time domain. Having \( h_{x,\text{comb}}(t, \tau) \) of SDOF response from Equation (34), the response variance can be stated as:

\[
c_{00}(t) = E\left[X^2(t)\right] = 2\pi \int_0^\infty A^2(\tau) h^2_{x,\text{comb}}(t, \tau) d\tau
\]

Similarly, for \( x(t) \), the variance may be calculated as:

\[
c_{11}(t) = E\left[X^2(t)\right] = 2\pi \int_0^\infty A^2(\tau) h^2_{x,\text{comb}}(t, \tau) d\tau
\]

and \( E\left[X(t)X(t)\right] \) can be expressed as:

\[-\text{Im}[c_{01}(t)] = E\left[X(t)X(t)\right] = 2\pi \int_0^\infty A^2(\tau) h_{x,\text{comb}}(t, \tau) h_{x,\text{comb}}(t, \tau) d\tau
\]

Now, \( Y(t) \) which is the “auxiliary” process for evaluation of the bandwidth factor should be defined employing the following relation:

\[
Y(t) = -\frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\tau_1) A(\tau_2) h_{h}(t - \tau_2) h(\tau_2 - \tau_1, \tau_1) W\left(\frac{u}{\tau_1 - \tau_2}\right) du d\tau_1 d\tau_2
\]

\[
= -\frac{1}{\pi} \int_{-\infty}^{\infty} A(\tau) h_{x,\text{comb}}(t, \tau) \int_{-\tau - u}^{\infty} W\left(\frac{u}{\tau - u}\right) du d\tau
\]

Now, to calculate \( Y(t) \) for the auxiliary process of displacement response, it can be also said that:

\[
Y(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} A(\tau) h_{x,\text{comb}}(t, \tau) \int_{-\tau - u}^{\infty} W\left(\frac{u}{\tau - u}\right) du d\tau
\]

Based on which, \( K_{XY}(t, t) \) can be calculated as:

\[
\text{Re}[c_{01}(t)] = K_{XY}(t, t) = E\left[X(t)Y(t)\right]
\]

\[
= 2\pi \int_{-\infty}^{\infty} h_{x,\text{comb}}(t, \tau_1) h_{x,\text{comb}}(t, \tau_2) \frac{A(\tau_1) A(\tau_2)}{\tau_2 - \tau_1} d\tau_1 d\tau_2
\]

To calculated the above integration, \( \tau_2 = \tau_1 + u \) change of variable can be used:
\[
\text{Re}\left[c_{ij}(t)\right] = K_{xy}(t, t) = 2S_0 \int_{u}^{t} h_{\text{comb}}(t, \tau - u)h_{x,\text{comb}}(t, \tau)A(t)A(\tau - u)d\tau du
\]

\[
= 2S_0 \int_{u}^{t} h_{\text{comb}}(t, \tau - u)h_{x,\text{comb}}(t, \tau)d\tau du
\]

(46)

Also, to calculate \( q(t) \), \( E[Y^2(t)] \) should be calculated by the equations in previous sections which is equal to \( \sigma_X^2(t) \). In the next section, we compare the results from both time-domain and frequency-domain estimations for different approximations and Monte Carlo Simulation (MCS).

7 Results of the simulation

In this case, the results obtained from our semi-analytical methods have been compared with the 10000 Monte Carlo simulations. For this, Equation (34) has been evaluated numerically at \( \tau_c \) with \( n\Delta t \) apart which \( n \) is assumed to be 100. The sensitivity analysis shows that this \( n \) value leads to fast simulations as well as not compromising the accuracy of the estimation. Therefore the Equation (35) can be expressed as the following expression:

\[
A_{x,\text{comb}}(jn\Delta t, \omega) = n\Delta t \sum_{k=0}^{j-1} h_{X}(n\Delta t (j - k))A(kn\Delta t, \omega)\exp[-i\omega n\Delta t (j - k)]
\]

(47)

Fig. 2 compares the results of different methods for different values of the DFM model parameters. The plots indicate that difference between time-domain based formulas and the frequency-domain based formulations is not significant. It's observed that they haven't been successful in estimating the peak of spectral acceleration at long periods. It is also seen that the frequency-domain calculations are more successful than time-domain calculations in predicting the response spectra for the very short-period region. Also, Fig. 3 depicts the cumulative distribution function of the maximum response of elastic SDOFs with different natural periods for Vanmarcke’s formulation evaluated in the time domain. It’s seen that the compatibility of the analytic and MCS increases as the period increases.

To investigate the effect of error in the estimation of \( \sigma_X^2(t) \), \( \sigma_Y^2(t) \), \( \rho_{XY}(t) \) and \( q(t) \) on SD error, for some cases these values are estimated using the Monte Carlo simulations and they are inserted into the equations based on Poisson’s assumption (Equation (27)), Vanmarcke’s crossing rate (Equation (28)), and Michaelov et. al. (Equation (30)), to predict the median values of the peak response. Fig. 4 shows the estimation error for a specific ensemble of DFM parameters. The results show that even using the evolutionary characteristics resulted from the simulation, does not lead to the best approximation of the peak response factor compared to the cases where these values are obtained from rigorous analytic functions. This indicates that the difference between the MLS and the proposed semi-analytical relationships does not stem from the inaccuracy of analytic estimation of \( \sigma_X^2(t) \), \( \sigma_Y^2(t) \), \( \rho_{XY}(t) \) and \( q(t) \).

It should be mentioned that both MLS and Vanmarcke’s methods need the bandwidth factor \( q(t) \) for the peak factor to be evaluated. However, the valid evaluation of the \( q(t) \) needs more complex computations needed for other parameters such as the variance of the response displacement and velocity as well as their correlation coefficient. Moreover, it is very unlikely to get the bandwidth factor of a single output by applying the definition in Equation (46). However, given the evolutionary characteristics of the process according to MCS the MLS formula yields the best results for the cases where the other cases show erroneous outputs. It can be deduced that the MLS is very sensitive to the evolutionary characteristics of the results and if it’s fed with proper characteristics,
it can be the most proficient method for evaluating the distribution of the peak values. This is not that easy to be implemented since the values of bandwidth $q(t)$ are not easy to be determined beforehand using only the model parameters.

It is seen that the results that are highly dependent on the correlation coefficient of the $\rho_{xx}$ lead to poor results. For most of the cases, the analytic expressions are unable to predict the maximum SPA value correctly. The results indicate that even a few percent error in the estimation of $\rho_{xx}(t)$ would lead to drastic differences for the methods involving the correlation coefficient term. Fig. 5 shows the variation of SD estimation inaccuracy vs the difference of the analytical values of $\rho_{xx}(t)$ from MCS and SDOF period using Vanmarcke’s method. It is seen that even as much as 0.04 change in $\rho_{xx}$ has increased the mean SD error by 15% for the MLS method. This justifies the significance of using MLS method which incorporates $\rho_{xx}$ more rigorously into its relationship for determining the upcrossing rate.

It should be pointed out that’s the Monte Carlo based estimation of parameters are not given in most of the cases and it is not easily calculated; therefore, the problem should be solved using analytic estimation of input parameters. Surprisingly, it is seen that there is a significant difference between the MLS method when input parameters of simulation or estimation are used. Therefore, the analytic estimation of spectral displacements is repeated given that among 4 input parameters, 3 of them are given according to analytic expressions and one of them is adopted from simulation methods. Among these parameters, using the MLS method, it can be easily seen that when the correlation coefficient parameter is calculated using the simulation methods the results are significantly improved. This shows that even a small amount of error in the estimation of the correlation coefficient will lead to disastrous results for MLS or any other method that is dependent on the correlation coefficient parameter. To circumvent this problem, two approaches can be made: 1) where the errors of estimation for correlation coefficient are lowered using more investigation in the errors of estimation and 2) using different methods for short and long period regions.

To evaluate the efficiency of each of the peak-factor methods, 10000 Monte Carlo simulations were completed. 13 model parameters were randomly generated to simulate artificial record accelerations, and then their response spectra are compared against the results obtained from analytical formulations. The empirical distribution function for the relative error of the median response spectrum and their analytical counterpart is shown in Fig. 6. It’s seen that MLS’s method in the frequency-domain has exceeded every other method and yields much better results. The results indicate that this method can produce results with an error less than 10% for 92% of the considered synthetic acceleration records. The semi-analytical method generates a mean SD error between 10 and 40% for the other 8% of the records. However, MLS’s method using the time-domain formulation has the same number of cases as much as those of the frequency-domain method with a mean SD error less than 10%. On the other hand, the frequency-domain formulation has provided fewer cases with a mean SD error between 10% and 20%. The time-domain method yields $SD$ estimations with the maximum mean error as much as 20% while the frequency-domain case may even lead to mean $SD$ results as large as 40%.

Fig. 7 illustrates the dependence of $SD$ estimation error on the natural period of SDOF on which DFM record is applied. It’s found that as the period increases higher than 1s, the accuracy of all of the methods decreases. Also, the frequency-based formulation leads to better results as the period falls. It is seen that for most of the period ranges MLS is the most efficient method for estimating the results of the spectrum compared to Poisson's and
Vanmarcke's methods. Given that the evolutionary properties of the time histories are generated according to frequency-domain results, MLS produces the best estimations for periods lower than 2s. On the contrary, for the long-period region, there is a good agreement between the results of MCS and those obtained from the MLS method using the evolutionary characteristics obtained from time-domain analytic expressions.

It's also worth noting that Vanmarcke's methods deviate rapidly (>5% estimation error) for periods higher than 1-1.5 s. Given that for these cases MLS method yields much better results, it can be deduced that assuming zero correlation coefficient for long-period SDOF’s response to DFM excitation contains a drastic error. This outcome can be further investigated by comparing the poor results of Poisson's method with other methods for all period ranges. This is due to the fact that in the estimation of crossing level by Poisson's method, compared to Vanmarcke’s and MLS, the bandwidth factor is not considered. This signifies the importance of considering bandwidth factor in the accuracy of semi-analytic estimation.

Fig. 8 demonstrates the correlation between the mean SD errors estimated from MLS’s frequency-domain formulation and the DFM parameters using the moving average of the data with a window size of 100. It’s seen that as $\xi_g$, $f_{f0}$ or $f_{f_0}$ decrease the results of SD estimation get inferior. Also, the semi-analytic estimations generate slightly more erroneous results as $f_c$ or $a_c$ decreases. There is no discernable relationship between mean SD error and other DFM parameters.

The same procedure is repeated for the mean SD error for different period ranges and the results are depicted in Fig. 9. It’s seen that as $\xi_g$, $f_{f0}$ and $f_{f0}$ decrease, the error in estimating the spectral response of longer periods increases significantly. It's also seen that as $f_c$ decreases, the mean SD error for longer periods (>2.5s) rises whereas the general trend of the SD error with DFM parameters isn't changed that much. Also, it is seen that as $\xi_c$ or $\xi_f$ decrease to values lower than 10%, the semi-analytic procedure for the period values corresponding to their frequency counterpart, i.e. $f_g$ and $f_{f0}$, yields inferior estimations.

8 Conclusion
This paper presents a semi-analytical method for estimating the elastic response spectra of acceleration records generated from the site-based DFM method. Therefore, different estimation procedures based on Vanmarck, Poisson, and Michaelov et al.’s assumptions are implemented to solve the ‘first passage problem’. For this, the variance of the displacement and velocity responses of an elastic SDOF as well as their correlation coefficient and bandwidth factor subjected to the DFM acceleration records have been determined analytically in time and frequency domains. It’s assumed that the envelope function of the DFM is a slowly varying function of time. Also, the evolutionary transfer function of the system is considered to be varying slow enough that that the instantaneous EPSD can be estimated according to the mean modulating function of the random response process from the beginning up to any time. The foretold statistical parameters required for analytical methods are numerically determined from the definition of the DFM method and their accuracy is evaluated via MCS. Thereafter, the median values of the displacement maximum response are evaluated for 10000 realizations of the DFM acceleration records to determine their elastic response spectrum. The estimation procedure is implemented in both time and frequency domains. The results indicate that using MLS assumption in frequency-domain can predict the response spectra with an error less than 10% for 92% of the considered synthetic acceleration records. The semi-analytical method generates a mean SD error between 10% and 40% for the other 8% of the records. Also, MLS
formulation executed on the frequency domain yields the best results compared to other procedures. This indicates that incorporating the correlation coefficient of the process and its derivative as well as their bandwidth into analytical estimation procedure is necessary. However, the time-domain estimations result to less compatible response spectra at lower periods. Based on the slow-varying assumptions on the modulating function, as the time-varying parameters of the DFM model, i.e. $f(t)$ and $\xi(t)$, change faster with time, the efficiency of the model will be decreased. Also, it is seen that damping values corresponding to the peak frequencies of the model decrease to values lower than 10%, the semi-analytic procedure for the period values corresponding to their frequency counterpart yields inferior estimations.
9 References


Figure Captions

Fig. 1 Comparison of Time History Records of Double-Frequency Model in two Cases of 1) Priestley’s Formulation and 2) DFM’s Formulation

Fig. 2 Comparison of different methods of median response spectrum approximation for records resulted from double-frequency method, using Poisson and Vanmarcke’s approximations using frequency-domain methods adjusted

Fig. 3 The cumulative distribution function of the maximum elastic response of SDOF with different natural period subjected to DFM excitation resulted from analytical method compared to Monte Carlo simulations

Fig. 4 The efficiency of MLS method for estimation of the response spectrum given that evolutionary characteristics are calculated using Monte Carlo simulation

Fig. 5 The variation of SD estimation inaccuracy vs. the deviation of the numerical results of $p_\text{(XX')}$ (t) from Monte Carlo resulted values and SDOF period for Vanmarcke’s method

Fig. 6 The empirical distribution function of the relative error of the analytical methods for estimating the response spectrum of the acceleration records generated from DFM

Fig. 7 The variation of the mean relative error of SD estimated using analytic methods vs period of SDOF subjected to the acceleration records generated from DFM

Fig. 8 Correlation between the mean SD errors estimated from MLS frequency-domain formulation and the DFM parameters

Fig. 9 Correlation between the mean SD errors for different period ranges estimated from MLS frequency-domain formulation and the DFM parameters
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\[ \xi_{g0} = 50\%, \xi_{f0} = 88\%, \xi_{fn} = 47\%, F_{g0} = 5.958(\text{Hz}) \]
\[ F_{f0} = 10.4581(\text{Hz}), F_{fn} = 4.1873(\text{Hz}), \xi_c = 43\%, F_c = 0.16279(\text{Hz}) \]

Fig. 4 The efficiency of MLS method for estimation of the response spectrum given that evolutionary characteristics are calculated using Monte Carlo simulation.
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