Iranian Journal of Science & Technology, Transaction B, Vol. 27, No. B4
Printed in Islamic Republic of Iran, 2003

"Research Note"

TWO-LEVEL FUZZY CONTROL OF LARGE-SCALE SYSTEMS AND ITS APPLICATION INTO PSS DESIGN*

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Abstract – This paper proposes a two-level suboptimal control using fuzzy prediction to control large-scale systems. A class of large-scale linear systems composed of interconnected subsystems is investigated. The overall control problem that is posed as a minimization of overall objective function, which is considered to be of quadratic form, is reduced to some optimization problems of lower order (sub) systems. The control input of each subsystem is composed of two signals. The first represents the local control signal (first level) and the second is the prediction signal (second level). In fact, the second signal is the prediction of interaction of other subsystems. It applies to each subsystem at every specified sample time (coordination sample times). The fuzzy logic theory is used for interaction prediction, where the prediction signal is constructed by a set of fuzzy sets with respect to state variables in an appropriate inference engine manner. The number of fuzzy sets and their interval deviations vary with time. Finally, the proposed method is applied to a three-area power system for designing a power system stabilizer (PSS).

Keywords – Decentralized control, large-scale systems, fuzzy logic theory, power system stabilizer

1. INTRODUCTION

Hierarchical structures in large-scale systems such as complex industrial systems, management systems and power systems are theoretically investigated by Mesarovic *et al.* [1]. The development of hierarchical control has grown by leaps and bounds in recent decades ([2-5]). This paper proposes a two-level suboptimal control using fuzzy prediction to control large-scale systems. It is a special case of multilevel control where the complexity of large-scale control problems can be relaxed by solving a family of sub-problems that are of smaller dimensions and are more easily handled. Most of the above hierarchical control schemes do not have the capability of on-line implementation. The control scheme used in this paper can be applied to the whole system in an on-line fashion. Because the fuzzy logic theory can be used to coordinate the large-scale systems [6], this paper employs the fuzzy logic theory to predict the interactions at the second level [7-10].

Consider the following large-scale linear system composed of N interconnected subsystems of the form

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + z_i(t) \\ z_i(t) &= \sum_{j\neq i}^{N} A_{ij} x_j(t) \end{aligned} \qquad i=1,..,N \qquad (1)$$

*Received by the editors May 30, 2000 and in final revised form July 3, 2002
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where, $u_i \in R^{p_i}, x_i, z_i \in R^{n_i}$ are respectively the control vector, state vector and interaction input signal, and A_i, B_i and A_{ij} are matrices of appropriate dimensions.

Assume that the overall performance index is of a quadratic form

$$J = \frac{1}{2}\int_0^\infty \left(\|x(t)\|_Q^2 + \|u(t)\|_R^2\right)dt \tag{2}$$

where Q and R are symmetric matrices of appropriate dimensions. The main problem can be stated as follows:

Find a state feedback control law such that the objective function J be minimized subject to (1), i.e.;

$$\begin{aligned} &\min \ J = \frac{1}{2}\int_0^\infty \left(\|x(t)\|_Q^2 + \|u(t)\|_R^2\right)dt \\ &s.t. \ \ \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + z_i(t) \\ &\qquad z_i(t) = \sum_{j\neq i}^{N} A_{ij} x_j(t) \qquad\qquad for\ i = 1, \ldots, N. \end{aligned} \tag{3}$$

2. DECOMPOSITION

Write the Lagrangian of overall optimization problem (3) as

$$\begin{aligned} L = \sum_{i=1}^{N}\Bigg\{\int_0^\infty \Bigg[&\frac{1}{2}\|x_i(t)\|_{Q_i}^2 + \frac{1}{2}\|u_i(t)\|_{R_i}^2 + \rho_i(t)^T\left(z_i(t) - \Sigma_{j\neq i}^N A_{ij}x_j(t)\right) \\ &+ P_i(t)^T\left(-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + z_i(t)\right) \Bigg]dt\Bigg\} \end{aligned} \tag{4}$$

where $\rho_i(t)$ (i=1, 2, ..., N) is Lagrangian multiplier vector and $P_i(t)$(i=1, 2, ..., N) is the adjoint vector. The Lagrangian in (4) can be decomposed as the sum of N sub-Lagrangians, L_i , i. e.

$$\begin{aligned} L_i = \int_0^\infty \Bigg[&\frac{1}{2}\|x_i(t)\|_{Q_i}^2 + \frac{1}{2}\|u_i(t)\|_{R_i}^2 + \rho_i(t)^T z_i(t) - \sum_{j\neq i}^{N}\rho_j(t)^T A_{ji} x_i(t) \\ &+ P_i(t)^T\left(-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + z_i(t)\right) \Bigg]dt \end{aligned} \tag{5}$$

for i=1, 2, ..., N. It is clear that the vectors $z_i(t)$ and $\rho_i(t)$ play the coupling role among the N sub-Lagrangians. At this moment assume that these vectors represent constant vectors; therefore all the N sub-Lagrangian problems can be solved independently. By defining the Hamiltonian of the ith subsystem as

$$H_i = \frac{1}{2}\|x_i(t)\|_{Q_i}^2 + \frac{1}{2}\|u_i(t)\|_{R_i}^2 + \rho_i(t)^T z_i(t) - \sum_{j\neq i}^{N}\rho_j(t)^T A_{ji} x_i(t) + P_i(t)^T\left(-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + z_i(t)\right) \tag{6}$$

The necessary conditions for the solution to be optimal imply that

$$\begin{aligned} &\dot{P}_i(t) = -Q_i x_i(t) - A_i^T P_i(t) + \sum_{j\neq i}^{N} A_{ji}^T \rho_j(t) \\ &u_i(t) = -R_i^{-1} B_i^T P_i(t) \end{aligned} \tag{7}$$

The above problem is similar to a tracking problem in optimal control theory. Assume that the adjoint vector $P_i(t)$ is of the following form

$$P_i(t) = K_i x_i(t) + g_i \tag{8}$$

By substituting (7) into (8), we have

$$-K_i A_i - A_i^T K_i + K_i B_i R_i^{-1} B_i^T K_i - Q_i = 0 \tag{9a}$$

$$-\left(A_i^T - K_i B_i R_i^{-1} B_i^T\right) g_i - K_i z_i(t) + \sum_{j \neq i}^{N} A_{ji}^T \rho_j(t) = 0 \tag{9b}$$

3. FIRST AND SECOND LEVEL PROBLEM FORMULATION

As one may see in the last section, the control input of the ith subsystem is

$$u_i(t) = -R_i^{-1} B_i^T \left(K_i x_i(t) + g_i\right) \tag{10}$$

where g_i and K_i come from (9). The necessary conditions for the optimality also imply that

$$z_i(t) = \sum_{j \neq i}^{N} A_{ij} x_j(t) \ , \qquad \rho_i(t) = -P_i(t) = -\left(K_i x_i(t) + g_i\right) \tag{11}$$

In [2, 3], the bounds [0, T] were considered for the integral of cost function (2); g was assumed to vary with time, then an off-line control procedure was proposed. It should be noted that if g were varying with time then Eq. (9) would be rewritten as

$$\dot{g}_i(t) = -\left(A_i^T - K_i B_i R_i^{-1} B_i^T\right) g_i(t) - K_i z_i(t) + \sum_{j \neq i}^{N} A_{ji}^T \rho_j(t) \tag{12}$$

Since the $-\left(A_i^T - K_i B_i R_i^{-1} B_i^T\right)$ is an unstable matrix, using the above equation in the on-line control structure will cause the overall instability. Therefore, in this paper g is considered fixed for each coordination sample time and Eq. (9b) is used in an on-line control structure. Equation (9b) can be simplified as follows:

By substituting $z_i(t)$ and $\rho_i(t)$ from (11) into (9b), we have

$$\left(A_i^T - K_i B_i R_i^{-1} B_i^T\right) g_i + K_i \sum_{j \neq i}^{N} A_{ij} x_j(t) + \sum_{j \neq i}^{N} A_{ji}^T \left(K_j x_j(t) + g_i\right) = 0$$

and then

$$\left(A_i^T - K_i B_i R_i^{-1} B_i^T\right) g_i + K_i \sum_{j \neq i}^{N} A_{ij} x_j(t) + \sum_{j \neq i}^{N} A_{ji}^T g_i + \sum_{j \neq i}^{N} A_{ji}^T K_j x_j(t) = 0 \ , \ i = 1, \ldots, N.$$

By combining the above equations, we may derive the following equation

$$\left(A - BR^{-1} B^T K\right)^T g + \left(A_z^T K + K A_z\right) x = 0 \tag{13}$$

where

$$A = \begin{bmatrix} A_1 & A_{12} & \ldots & A_{1N} \\ A_{21} & A_2 & \ldots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \ldots & A_N \end{bmatrix}, \ A_z = \begin{bmatrix} O & A_{12} & \ldots & A_{1N} \\ A_{21} & O & \ldots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \ldots & O \end{bmatrix}, \ g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}, \ x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix},$$

$$K = diag\{K_i\} \text{ and } B = diag\{B_i\}$$

From the above it is easily seen that vector g is related to the state vector x as

$$g = -\left(A - BR^{-1}B^T K\right)^{-T}\left(A_z^{\,T} K + KA_z\right)x \tag{14}$$

It is clear that g is dependent on time varying state vector x, however it is assumed that g is fixed in each coordination sample time. It is not practical to implement Eq. (14) because x is not available. Therefore the prediction of x , $\hat{x}$, is used in (14), i. e. ;

$$g = -\left(A - BR^{-1}B^T K\right)^{-T}\left(A_z^{\,T} K + KA_z\right)\hat{x} \tag{15}$$

So prediction of x is done at the second level, then vector g is constructed and will be sent to the local units. Local controllers generate the control input of each subsystem by Eq. (10). This structure is shown in Fig. 1.

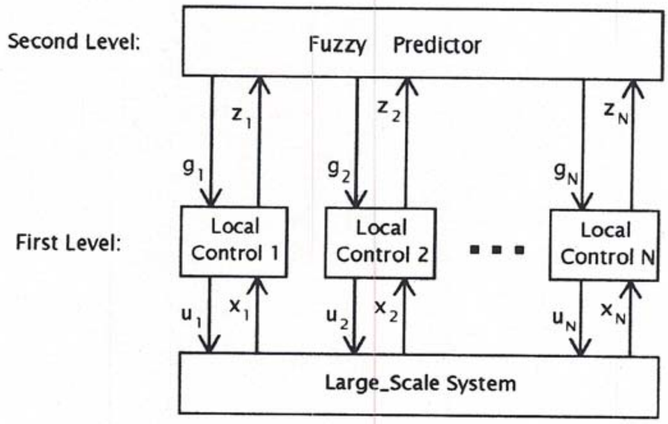

Fig. 1. Hierarchical control structure of the proposed scheme

4. FUZZY PREDICTION

Assume that x_t is the state vector at time t. The goal is to find a fuzzy-based prediction $\hat{x}_{t+1}$ from x_t and x_{t-1} . For $i = -k, ..., 0, ..., k$, let the fuzzy terms E_i , F_i and G_i be considered for variables x_t , x_{t-1} and x_{t+1}, respectively with the following triangular membership functions shown in Fig. 2.

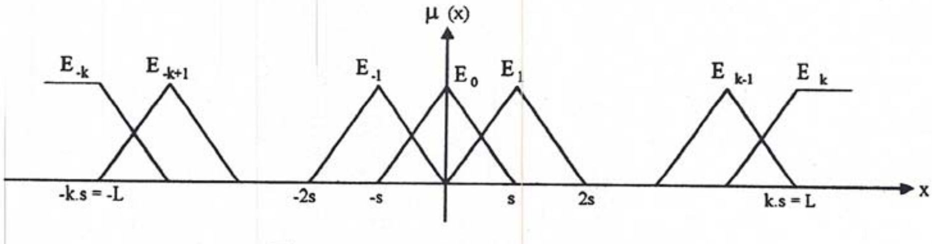

Fig. 2. Membership functions of x_t

Let x_t and x_{t-1} belong to the interval [$-L$ L]. If we assume that the values of vector x vary approximately linearly, we may estimate of x as

$$\hat{x}_{t+1} = 2x_t - x_{t-1}$$

In the fuzzy case, if x_t belongs to E_i and x_{t-1} belongs to F_j then $\hat{x}_{t+1}$ will belong to G_{2i-j} . The areas R_0 to R_8 with respect to x_t and x_{t-1} may be partitioned as shown in Fig. 3.

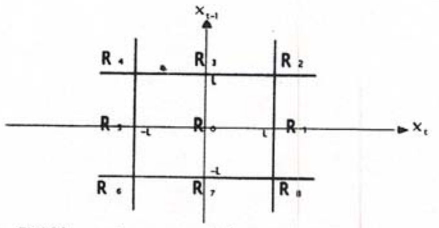

Fig.3. Different areas with respect to x_t and x_{t-1}

a) Area R_0

In this area it is obvious that any x_t (or x_{t-1}) intersects with two fuzzy sets E_i and E_{i+1} (or F_j and F_{j+1}). Therefore only the following four rules are used in this area

1. If x_t is E_{i+1} and x_{t-1} is F_{j+1} then x_{t+1} is G_{2i-j+1}
2. If x_t is E_{i+1} and x_{t-1} is F_j then x_{t+1} is G_{2i-j+2}
3. If x_t is E_i and x_{t-1} is F_{j+1} then x_{t+1} is G_{2i-j-1}
4. If x_t is E_i and x_{t-1} is F_j then x_{t+1} is G_{2i-j}

For convenience, consider two functions f_1 and f_2 as follows:

$$f_1(x,i)=\frac{x}{s}+1-i \quad \text{and} \quad f_2(x,i)=-\frac{x}{s}+1+i$$

If the membership value of x_t to E_i is equal to μ_1 and the membership value of x_{t-1} to F_j is equal to μ_2, then the corresponding output fuzzy set, G_{2i-j}, will be multiplied by $\alpha=\min(\mu_1,\mu_2)$ and the prediction is done by defuzzification of the overall output fuzzy set. Let

$$\begin{aligned}\alpha_1&=\min\{f_1(x_t,i+1), f_1(x_{t-1},j+1)\}\\ \alpha_2&=\min\{f_1(x_t,i+1), f_2(x_{t-1},j)\}\end{aligned} \quad \text{and} \quad \begin{aligned}\alpha_3&=\min\{f_2(x_t,i), f_1(x_{t-1},j+1)\}\\ \alpha_4&=\min\{f_2(x_t,i), f_2(x_{t-1},j)\}\end{aligned}$$

then the output fuzzy set G_{out} becomes

$$G_{out}=\alpha_1 G_{2i-j+1}\oplus\alpha_2 G_{2i-j+2}\oplus\alpha_3 G_{2i-j-1}\oplus\alpha_4 G_{2i-j}$$

where $\oplus$ represents the sum operation in fuzzy sets. After applying the center of gravity defuzzification to the fuzzy set G_{out}, we will have

$$\begin{aligned}x_{t+1}&=\frac{\alpha_1(2i-j+1)s^2+\alpha_2(2i-j+2)s^2+\alpha_3(2i-j-1)s^2+\alpha_4(2i-j)s^2}{\alpha_1 s+\alpha_2 s+\alpha_3 s+\alpha_4 s}\\ &=\left(2i-j+\frac{\alpha_1+2\alpha_2-\alpha_3}{\alpha_1+\alpha_2+\alpha_3+\alpha_4}\right)s\end{aligned}$$

b) Other areas

Similar to section a the following results can be derived for other areas

- In area R_1: $x_{t+1}=2L-x_{t-1}$
- In area R_2: $x_{t+1}=L$
- In area R_3: $x_{t+1}=-L+2x_t$
- In area R_4: $x_{t+1}=-3L$
- In area R_5: $x_{t+1}=-2L+2x_{t-1}$
- In area R_6: $x_{t+1}=-L$
- In area R_7: $x_{t+1}=L+2x_t$
- In area R_8: $x_{t+1}=3L$

Therefore, the fuzzy prediction of x_{t+1} is obtained from the above equations. As stated before, for $i=-k,...,0,...,k$, the fuzzy terms E_i, F_i and G_i are considered for variables x_t, x_{t-1} and x_{t+1}, respectively. k is an arbitrary number which specifies the fuzzy sets, for example k=4 specifies the fuzzy sets of zero, small, medium, large, and very large. Also, the value of L is considered as a multiplicative of x_t (i.e., $L=\alpha x_t$, where α is an arbitrary scalar, for example $\alpha=1.2$), this consideration reduces the error of prediction. The reason for the error reduction can be stated as follows. If the value of L is very high (with respect to x_t and x_{t-1}) then the prediction error will increase because the variation of x_t, with respect to fuzzy set domains is very low and its effect will not be observed for error reduction. Note that the above fuzzy inference is applied independently to each element of vector x.

In addition to the above rules, the following two improving rules are considered as follows:

R1: If $|x_t|<\varepsilon$ and $|x_{t-1}|<\varepsilon$ then x_{t+1} is 0

R2: If $\|x_t - x_{t-1}\|<\eta$ then $x_{t+1}=\gamma x_{t+1}$

Rule **R1** prevents the prediction of states from oscillation about zero when the system tends to its steady state (ε is an arbitrary small positive real number), and rule R2 conducts the system to its steady state by applying a descent factor γ to the state variables.

5. SIMULATION RESULTS

The proposed scheme is used to design a PSS for a three-area power system. The specifications of each machine are listed in Table 1.

Table 1. Power system data

Machine 1	Machine 2	Machine 3	Parameters
0.1	0.3	0.2	Damping constant D
64.56	55.2	70.46	Inertia constant M ($J.s$)
100	100	100	AVR gain
0.02	0.02	0.02	AVR time constant (s)
0.175	0.3030	0.1675	Synchronous reactance x_d ($J.s$)
0.044	0.056	0.0208	Transient reactance x'_d (pu)
0.1023	0.282	0.1675	Synchronous reactance x_q (pu)
0.2969	0.1969	0.195	Transient reactance x'_q (pu)
6.1	5.5	9	Transient time constant T'_{do} (s)
0.01	0.01	0.01	Governor time constant T_v(s)
0.01	0.01	0.01	Governor time constant T_s(s)

Network admittance matrix is considered as [11] in the form

$$Y=\begin{bmatrix} 0.846-2.988j & 0.287+1.513j & 0.210+1.226j \\ 0.287+1.513j & 0.420-2.724j & 0.213+1.088j \\ 0.210+1.226j & 0.213+1.088j & 0.277-2.368j \end{bmatrix}$$

The state variables of each machine are $x_1=\Delta\omega$, $x_2=\Delta\delta$, $x_3=\Delta e'_q$, x_4 equal to output deviation of AVR, and x_5 and x_6 represent the state variables of the governor. Our goal is the design of PSS where its inputs are frequency deviation ($\Delta\omega$) of each machine and its outputs are control signals into governor and AVR. The system is linearized as suggested by Yu [12]. Then the proposed two level controller is constructed as described in the last sections.

Two controllers are also used for comparison to the proposed method:

1. Centralized *PSS* (using optimal control for overall system) [13, 14]
2. Decentralized *PSS* (using optimal control for each isolated subsystem)[13]

The Figs. 4 to 6 compare the torque angle deviations for three kinds of *PSS* for the following initial state X_0

$$X_0 = [-.03\ .05\ -.02\ .020\ -.02\ -.05 -.02\ .05\ .05\ .03\ .05\ .04\ .025\ -.05\ .03\ -.05\ .02]$$

Note that the coordination sample time (Δt) is 100 ms.

The simulation results demonstrate the very effectiveness of the proposed method compared to the decentralized *PSSes*. It should be noted that, although the centralized *PSS* is an optimal controller for the overall system, it is useless in the large-scale power system applications because of the computational and practical problems. However, the proposed controller achieves a good performance and even it is comparable with the centralized *PSS*.

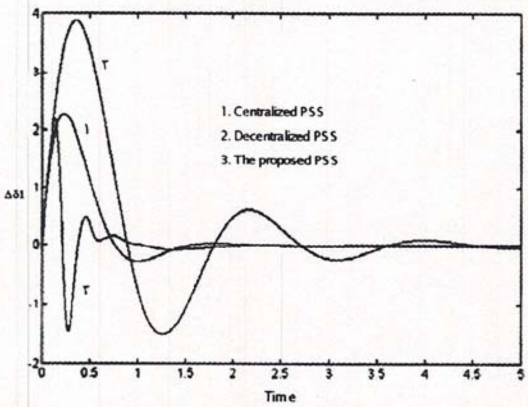

Fig. 4. Torque angle deviation of subsystem 1 for three kinds of PSS for $\Delta t = 100\,\text{ms}$

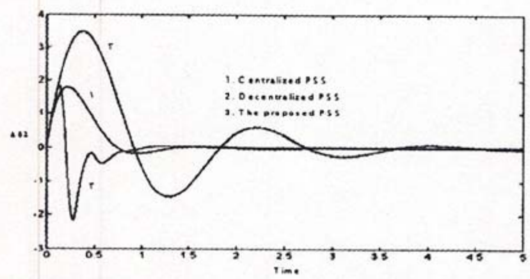

Fig. 5. Torque angle deviation of subsystem 2 for three kinds of PSS for $\Delta t = 100\,\text{ms}$

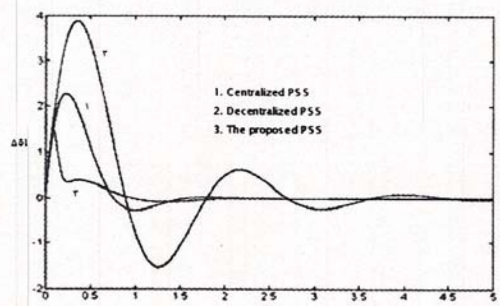

Fig. 6. Torque angle deviation of subsystem 1 for three kinds of PSS for $\Delta t = 50\,\text{ms}$

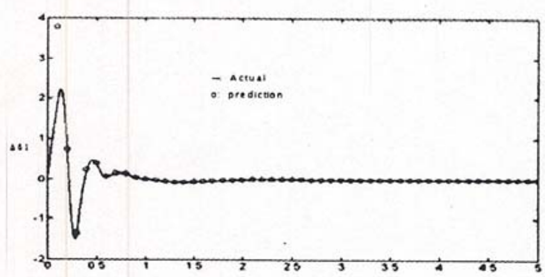

Fig. 7. Prediction of load angle of subsystem 1

As we can see from simulation results (compare Fig. 4 and Fig. 6) the performance of the proposed controller may be enhanced by reducing the coordination sample time ($\Delta t = 50$ ms). Figure 7 shows how good the actual state can be predicted by the proposed fuzzy estimator. The same results have been obtained for the other two subsystems and for the sake of brevity we didn't show them here.

6. CONCLUSION

In this paper a two-level sub-optimal control using fuzzy prediction was developed to control large-scale systems. The proposed controller has been applied to a three-area power system. The results of our proposed controller have been compared with those of the other two PSS controllers frequently cited in the literature known as decentralized and centralized controller. Simulation results easily highlight the merit of our method.

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