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Decentralized robust adaptive-output feedback controller for power system load frequency control

Mohammad Hosein Kazemi, Mehdi Karrari, Mohammad Bagher Menhaj

Abstract In this paper, a new model reference-decentralized robust adaptive-output feedback controller is proposed for the load frequency control (LFC) of large-scale power systems with unknown parameters. This control strategy requires only local input–output data and can follow random changes in the operating conditions. The controller is designed such that the trajectory errors and the control gains of each area remain uniformly bounded. In the proposed method, firstly an adaptive observer is designed to estimate the state variables and system parameters using local data only. Then a locally linear combination of the estimated states and the model reference states are used to design a robust adaptive-output feedback controller for each area. Simulation results for a three-area power system show that the proposed controller achieves good performance even in the presence of plant parameter changes and system non-linearities.

Keywords Model reference adaptive control, Large-scale systems, Power systems, Load-frequency control

1 Introduction

Load frequency control (LFC) is of importance in electric power system design and operation. The objective of the LFC in an interconnected power system is to maintain the frequency of each area and to keep tie-line power flows within some pre-specified tolerances by adjusting the MW outputs of the LFC generators so as to accommodate fluctuating load demands.

There has been increasing interest for designing load frequency controllers with better performance during past years and many control strategies have been developed [1–2] for LFC.

The first proposed control strategy was a proportional integrator (PI) controller, which is widely used in the industry nowadays. Due to the non-linearities of various components of power systems, a linear model obtained by linearization around an operating point is usually adopted for the controller design [3–5]. However, because of the inherent characteristics of changing loads, the operating points of a power system may change very much randomly during a daily cycle. As a result, a fixed controller is certainly not suitable. Therefore, some authors have suggested variable structure and robust control to make the controller insensitive to changes in the plant parameters [6–9]. However, these methods require information about the system states, which are not usually known or available.

Recently, to overcome the mentioned shortcomings, various adaptive control techniques have been proposed to deal with large parameter variations [10–12]. The main aim of such control strategies is to extend the margins of the regions of stability and therefore to satisfy increasingly complex control requirements in power systems. However, because of the long transmission lines used in the operation of power systems, it is quite important to aim for decentralization of control action for individual areas. Therefore, decentralized control has been suggested for multi-area power system control [4, 6, 12–15].

From the control strategy point of view, the extension of single-input single-output robust adaptive control algorithms to multivariable and large-scale systems has been considered by several authors [16–22]. Later attempts were about various modifications such as σ-modification [23], e_1-modification [24] and a switching approach (E. Kosmatopoulos et al., personal communication, 2001) for robust adaptive control strategy.

In this paper, we address a new decentralized robust adaptive-output feedback control scheme for load frequency control of large-scale power systems. In the proposed method, for each area, firstly an adaptive observer is introduced to estimate the state variables and system parameters using only local input–output data. Then, a linear combination of these estimates and model reference states with adjustable gains are fed back as a local stable robust adaptive control, such that the outputs of the plant track the outputs of the desired reference model asymptotically. A suitable transformation matrix is introduced which transforms the initial reference model to an equivalent reference model such that the convergence of the trajectory errors is guaranteed. In fact, this transformation matrix transforms the reference model such that some convergence conditions are satisfied. An appropriate adaptive law is derived to adjust this transformation matrix.

This paper is organized as follows. Section 2 describes the model under study. Section 3 states the "control strategy" used in this paper for load-frequency control.

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M.H. Kazemi (✉), M. Karrari, M.B. Menhaj
Electrical Engineering Department,
Amir Kabir University, 424,
Hafez Ave., Tehran, 15914, Iran
e-mail: Mhkazemi@yahoo.com
Tel: +98-21-6466009
Fax: +98-21-6406469

In Section 4, the effectiveness of the proposed scheme is demonstrated by some simulation results. Section 5 concludes the paper.

2
The system under study

Power systems are inherently non-linear. There are different complicated and non-linear models for large-scale power systems. For load-frequency control, however, a simplified and linearized model is usually used. In advanced control strategies (such as the one considered in this paper), the error caused by the simplification and linearization are considered as parameter uncertainties and unmodeled dynamics.

A three-area power system shown in Fig. 1 is taken as an example system in this paper [12]. Figure 2 shows the block diagram of area 1. Referring to Fig. 2, state vector **x**, control vector **u**, and disturbance vector **d** can be defined as follows:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \Delta P_{e1} \\ \Delta P_{e2} \\ \Delta P_{e3} \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} \Delta P_{d1} \\ \Delta P_{d2} \\ \Delta P_{d3} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$y_1 = [\Delta P_{\mathrm{tie1}} \;\; \Delta f_1]^T \,, \; y_2 = [\Delta P_{\mathrm{tie2}} \;\; \Delta f_2]^T \,,$$
$$y_3 = [\Delta P_{\mathrm{tie3}} \;\; \Delta f_3]^T$$

$$x_1 = \left[\Delta P_{\mathrm{tie1}} \;\; \Delta f_1 \;\; \Delta P_{t1} \;\; \Delta P_{\mathrm{g1}}\right]^T$$
$$x_2 = \left[\Delta P_{\mathrm{tie2}} \;\; \Delta f_2 \;\; \Delta P_{t2} \;\; \Delta P_{\mathrm{g2}}\right]^T$$
$$x_3 = [\Delta P_{\mathrm{tie3}} \;\; \Delta f_3 \;\; \Delta P_{t3} \;\; \Delta P3]^T$$

where Δf_i is the incremental frequency deviation of area i, ΔP_{gi} is the incremental governor valve position change of generator in area i, ΔP_{ci} is the control input of area i, ΔP_{ti} is the incremental output of generator in area i, $\Delta P_{\mathrm{tie}\;ij}$ is the incremental change in tie-line power between areas i and j, ΔP_{di} is the disturbance of area i, M_i is the equivalent inertia constant for area i, D_i is the equivalent damping coefficient for area i, T_{gi} is the governor time constant for area i, T_{ti} is the turbine time constant for area i, T_{ij} is the synchronizing coefficient in normal operating conditions between areas i and j, a_{ij} is the ratio between the rated MW capacity of areas i and j, and R_i is the drooping characteristic for area i. The system parameters are listed in Table 1.

The total real power exported from area i equals the sum of all out-flowing line powers $P_{\mathrm{tie}\;ij}$ to adjoining area j, i.e.

$$P_{\mathrm{tie}\;i} = \sum_j P_{\mathrm{tie}\;ij}$$

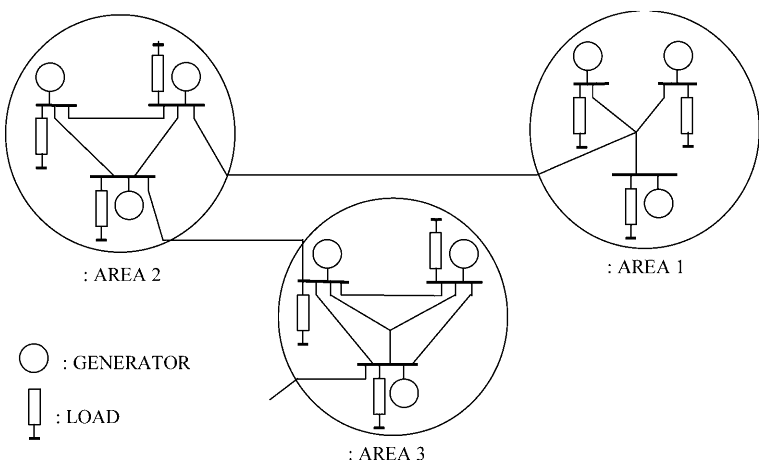

Fig. 1. Multi-area power system

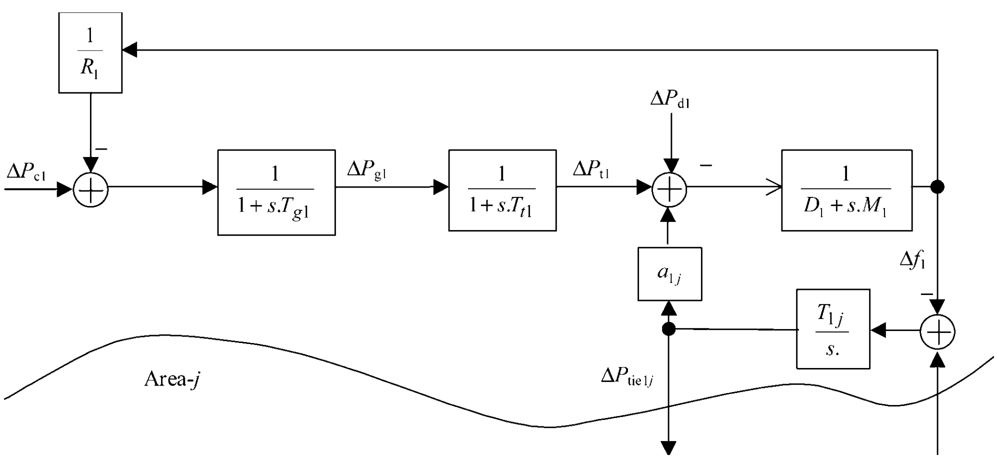

Fig. 2. Block diagram of area 1

Table 1. System parameters

Area 1	Area 2	Area 3
D_1 = 0.006	D_2 = 0.0083	D_3 = 0.008 (p.u.Mw/Hz)
M_1 = 0.2	M_2 = 0.167	M_3 = 0.15 (p.u.Mw)
T_{t1} = 0.2	T_{t2} = 0.24	T_{t3} = 0.25 (s)
T_{g1} = 0.08	T_{g2} = 0.08	T_{g3} = 0.08 (s)
R_1 = 2.2	R_2 = 2.23	R_3 = 2.2 (Hz/pu Mw)
a_{12} = 0.2		a_{23} = 5.0
T_{12} = 0.272		T_{23} = 0.109 (pu Mw/Hz)

The real power in per unit transmitted across a lossless line of reactance X_{ij} is

$$P_{\text{tie } ij} = \frac{|V_i||V_j|}{X_{ij}P_{ri}} \sin(\delta_i - \delta_j)$$

where P_{ri} is the rated power of area i, and

$$V_i = |V_i|e^{j\delta_i} \quad , \quad V_j = |V_j|e^{j\delta_j}$$

where $V_i = |V_i|e^{j\delta_i}$ and δ_i are the amplitude and the angle of the terminal voltage in area i.

3
The control strategy

In this section, we firstly state the control problem. The first part of the proposed control strategy is to construct an adaptive observer, which estimates the state vector $\mathbf{x}_i$ and system parameters (B_i and G_i) using only local input–output data. This is discussed in the next subsection. Then the adaptive control using output feedback is discussed in the following subsection.

3.1
Problem statement

Consider a large-scale power system S, composed of N local area power systems S_i (i = 1,2,...,N) described by

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i + h_i + B_iu_i \\ y_i &= C_ix_i \end{aligned} \tag{1}$$

where h_i is the interaction from other areas,

$$h_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}x_j \tag{2}$$

where $x_i \in R^{n_i}$ is the state vector of ith area and $u_i \in R^{p_i}$ is its control function. A_{ii}, B_i and C_i describe the dynamics of the isolated ith area, A_{ij} describes the interaction matrix from the jth subsystem, which is assumed to have appropriate dimensions. It is assumed that (C_i, A_{ii}) is observable and (A_{ii}, B_i) is controllable.

Each area can be represented by: [25]

$$\begin{aligned} \dot{x}_i &= (A_i + G_iC_i)x_i + B_iu_i + h_i \\ y_i &= C_ix_i \end{aligned} \tag{3}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$ and $y_i \in R^{m_i}$ are the state vector, control input vector and output vector, respectively. It is assumed that (A_i, B_i) is controllable, and the known pair (C_i, A_i) are in observable canonical form. The matrices B_i and G_i are unknown.

The goal is to design a robust adaptive-output feedback control for each area of the power plant such that the outputs, y_i, track the outputs of the following desired reference model asymptotically.

$$\begin{aligned} \dot{x}_{mi} &= T_iA_{mi}T_i^{-1}x_{mi} + T_iB_{mi}r_i \\ y_{mi} &= C_ix_{mi} \end{aligned} \tag{4}$$

where $x_{mi} \in R^{n_i}$ is the model state vector, $r_i \in R^{m_i}$ is a uniformly bounded reference input, and $y_{mi} \in R^{m_i}$ is the desired output. The matrices A_{mi} and B_{mi} are arbitrary matrices with A_{mi} stable. Matrix T_i is an unknown non-singular equivalent transformation. When the desired model reference is defined, the matrix T_i in (4) is not usually defined. It was added in the control strategy described in this paper to ensure the robustness of the proposed controller. It converges to the desired T_i^* under a suitable adaptive law such that some conditions for stability of the overall system are satisfied.

3.2
The adaptive observer

Considering the local power plant (3), the aim of this section is to construct an adaptive observer such that the unknown matrices B_i, G_i and the state vector $\mathbf{x}_i$ are estimated asymptotically.

Since (C_i, A_i) is assumed to be observable, an $n_i \times m_i$ matrix G_{0i} exists such that A_i + $G_{0i}C_i$ is asymptotically stable. Hence (3) can be expressed as:

$$\begin{aligned} \dot{x}_i &= (A_i + G_{0i}C_i)x_i + (G_i - G_{0i})C_ix_i + B_iu_i + h_i \\ y_i &= C_ix_i \end{aligned} \tag{5}$$

where A_i+$G_{0i}C_i$ is an asymptotically stable matrix.

An adaptive observer can then be introduced as:

$$\begin{aligned} \dot{\hat{x}}_i &= (A_i + G_{0i}C_i)\hat{x}_i + (\hat{G}_i - G_{0i})y_i + \hat{B}_iu_i + \hat{h}_i + v_i \\ \hat{y}_i &= C_i\hat{x}_i \end{aligned} \tag{6}$$

where $\hat{x}_i$ and $\hat{h}_i$ are the estimates of the states and interactions of ith area, $\hat{G}_i$ and $\hat{B}_i$ are the estimates of the parameters B_i and G_i respectively, and v_i is an auxiliary signal that must be chosen to ensure stability of the estimation.

From (5) and (6), the state estimation error equation is obtained as:

$$\begin{aligned} \dot{e}_i &= (A_i + G_{0i}C_i)e_i + (\hat{G}_i - G_i)y_i + (\hat{B}_i - B_i)u_i \\ &\quad + \hat{h}_i - h_i + v_i \\ e_{1i} &= C_ie_i \end{aligned} \tag{7}$$

where $e := \hat{x}_i - x_i$, $e_{1i} := \hat{y}_i - y_i$.

Define $\phi_i := \hat{G}_i - G_i$, $\Psi_i := \hat{B}_i - B_i$,, and $\tilde{h}_i = \hat{h}_i - h_i$, then the equation (7) can be summarized as:

$$\begin{aligned} \dot{e}_i &= (A_i + G_{0i}C_i)e_i + \begin{bmatrix}\Psi_i & \phi_i & \tilde{h}_i\end{bmatrix}\begin{bmatrix}u_i \\ y_i \\ 1\end{bmatrix} + v_i \\ e_{1i} &= C_ie_i \end{aligned} \tag{8}$$

Our concern now is to determine the signal $\tilde{v}_i$ and the adaptive laws for adjusting ϕ_i, Ψ_i, and $\tilde{h}_i$ such that the error e_i tends to zero asymptotically. Obtaining such adaptive laws requires the proof of the following theorem:

Theorem 1. Let (C, K) be an observable pair where $K \in R^{n\times n}$ is asymptotically stable. Given bounded piecewise-continuous functions of time $z : R^+ \rightarrow R^m$ and $\dot{\phi} : R^+ \rightarrow R^{n\times m}$, then we can find matrix $D \in R^{n\times m}$ and functions $v : R^+ \rightarrow R^n$ and $X : R^+ \rightarrow R^{n\times mn}$ such that the systems

$$\dot{e} = Ke + \phi z + v \ , \quad e_1 = Ce \tag{9}$$

and

$$\dot{\varepsilon} = K\varepsilon + DCX\ell \ , \quad \varepsilon_1 = C\varepsilon \tag{10}$$

have the same outputs as t→∞, where

$$v = X\dot{\ell} \ , \quad \ell = Vec(\phi) \ , \tag{11}$$

$Vec(\phi)$ denotes the vector formed by stacking the columns of ϕ into one long vector and the matrix X consists of m matrices $X_i : R^+ \rightarrow R^{n\times n}$ such that

$$\begin{aligned} &X = [X_1, X_2, \ldots, X_m] \\ &\dot{X}_i = KX_i + Iz_i \ , \quad i = 1, \ldots, m \end{aligned} \tag{12}$$

For the proof, see the Appendix.

Now using Theorem 1, let us define the matrix X as:

$$\begin{aligned} &X = [X_{i1}, X_{i2}, \ldots, X_{2m_i}, X_{2m_i+1}] \\ &\dot{X}_{ij} = (A_i + G_{0i}C_i)X_{ij} + Iu_{ij} \ , \quad j = 1, \ldots, m_i \\ &\dot{X}_{i(j+m_i)} = (A_i + G_{0i}C_i)X_{i(j+m_i)} + Iy_{ij} \\ &\dot{X}_{i(1+2m_i)} = (A_i + G_{0i}C_i)X_{i(1+2m_i)} + I \end{aligned} \tag{13}$$

Then, error equation (8) and the adaptive law

$$\dot{\ell}_i = -X_i^T C_i^T (\hat{y}_i - y_i) \tag{14}$$

where

$$\ell_i = Vec\left(\left[\hat{B}_i \ \hat{G}_i \ \hat{h}_i\right]\right) \tag{15}$$

with the following auxiliary signal

$$v_i = -X_i X_i^T C_i^T (\hat{y}_i - y_i) \tag{16}$$

results in the uniform stability and $\lim_{t\rightarrow\infty} e_{1i} = 0$.

If, in addition, u_i is persistently exciting with sufficient number of frequencies, it can be shown that ϕ_i, Ψ_i, $\tilde{h}_i$, and therefore e tends to zero asymptotically [26].

3.3 Adaptive control using output feedback

In Sect. 3.2, an adaptive observer (6) was derived to estimate the states of the plant. Now let us choose the following control law for the ith local area power system:

$$u_i = K_{ei}\hat{e}_i + K_{ri}r_i + K_{0i} \tag{17}$$

where $K_{ei} \in R^{m_i\times n_i}$ is the feedback gain, $K_{ri} \in R^{m_i\times m_i}$ is the feedforward gain, $K_{0i} \in R^{m_i}$ is an auxiliary input signal and $\hat{e}_i = \hat{x}_i - x_{mi}$.

The state-error equation can now be obtained as:

$$\begin{aligned} \dot{e}_i = {} & T_iA_{mi}T_i^{-1}e_i + \left(A_{pi} - T_iA_{mi}T_i^{-1} + B_iK_{ei}\right)x_i \\ & + B_iK_{ei}(\hat{x}_i) + (B_iK_{ri} - T_iB_{mi})r_i + B_iK_{0i} \\ & - B_iK_{ei}x_{mi} + h_i \end{aligned} \tag{18}$$

where $A_{pi} := A_i + G_iC_i$, and $e_i = x_i - x_{mi}$.

It is quite clear that the state-error equation of $\dot{e}_i = T_iA_{mi}e_iT_i^{-1}$ is desired. This can only happen if the following equations have unique solutions:

$$\begin{aligned} &A_{pi} - T_i^*A_{mi}T_i^{*-1} + B_iK_{ei}^* = 0 \\ &B_iK_{ri}^* - T_i^*B_{mi} = 0 \\ &B_iK_{0i}^* - B_iK_{ei}^*x_{mi} + h_i = 0 \end{aligned} \tag{19}$$

The controllability of (A_{pi}, B_i) is not sufficient to solve the first condition of (19) for K_{ei}^*. However it guarantees that there exist a K_{ei}^*, such that $A_{pi} + B_iK_{ei}^*$ and desired matrix A_{mi} have unique eigenvalues. If A_{mi} contains the desired eigenvalues, any equivalent transformation $T_iA_{mi}T_i^{-1}$ also contains those desired eigenvalues and it can be a good candidate for the reference model. The matrix T_i converges to the desired T_i^* under a suitable adaptive law such that the conditions of (19) are satisfied.

By using definitions

$$\begin{aligned} &\eta_i := \begin{bmatrix} \hat{x}_i - x_{mi} \\ r_i \\ 1 \end{bmatrix} = \begin{bmatrix} \eta_{i_1} \\ \vdots \\ \eta_{i_{(m_i+n_i+1)}} \end{bmatrix}, \\ &\theta_i : [K_{ei} \ \ K_{ri} \ \ K_{0i}] \\ &\varphi_i := \theta_i - \theta_i^* \\ &\Delta A_i := T_i^*A_{mi}T_i^{*-1} - T_iA_{mi}T_i^{-1} \\ &\qquad \tilde{T}_i := T_i - T_i^* \end{aligned} \tag{20}$$

the error equation (18) can be summarized as:

$$\begin{aligned} \dot{e}_i = {} & T_iA_{mi}T_i^{-1}e_i + \Delta A_ix_i - \tilde{T}_iB_{mi}r_i + B_i\varphi_i\eta_i \\ & + B_iK_{ei}^*(\hat{x}_i - x_i) \\ e_{1i} = {} & C_ie_i \end{aligned} \tag{21}$$

The following theorem is required to derive the adaptive control law.

Theorem 2. Let (C, K) be an observable pair where $K \in R^{n\times n}$ is asymptotically stable. Given bounded piecewise-continuous functions of time $z:R^+\rightarrow R^m$ and $\dot{\phi} : R^+ \rightarrow R^{p\times m}$, then a matrix $D \in R^{n\times m}$ and functions $v:R^+\rightarrow R^n$ and $X:R^+\rightarrow R^{n\times mn}$ can be determined such that the systems

$$\dot{e} = Ke + B\phi z + v \ , \quad e_1 = Ce \tag{22}$$

and

$$\dot{\varepsilon} = K\varepsilon + DCXB_d\ell \ , \quad \varepsilon_1 = C\varepsilon \tag{23}$$

have the same outputs as $t\rightarrow\infty$, where

$$v = XB_d\dot{\ell} \ , \quad \ell = Vec(\phi) \ , \quad B_d = I_m \otimes B \tag{24}$$

and $\otimes$ denotes the Kronecker product and the matrix X consists of m matrices $X_i:R^+\rightarrow R^{n\times n}$ such that

$$X = [X_1, X_2, \ldots, X_m] \;\; \dot{X}_i = KX_i + Iz_i, \quad i = 1, \ldots, m \tag{25}$$

Proof. The proof of the above theorem is quite similar to that of Theorem 1. Furthermore, it can be shown that the following adaptive law stabilizes the error system (22):

$$\dot{\ell} = -B_d^T X^T C^T Ce \tag{26}$$

According to Theorem 2, the error equation (21) and the following error system have the same outputs as $t \to \infty$.

$$\begin{aligned} \dot{\varepsilon}_i &= T_i A_{mi} T_i^{-1} \varepsilon_i + D_i C_i Y_i B_{di} \tilde{\theta}_i - D_i C_i Y_{Ti} \tilde{\theta}_{Ti} \\ &\quad + B_i K_{ei}^* (\hat{x}_i - x_i) + \Delta A_i x_i - v_i v_{Ti} \;\; \varepsilon_{1i} = C_i \varepsilon_i \end{aligned} \tag{27}$$

where

$$\begin{aligned} &\dot{Y}_{ij} = T_i A_{mi} T_i^{-1} Y_{ij} + I\eta_{ij}, && j = 1, \ldots, n_i + m_i + 1 \\ &Y_i = \left[Y_{i1}, Y_{i2}, \ldots, Y_{i(n_i+m_i+1)}\right] \\ &\dot{Y}_{Tij} = T_i A_{mi} T_i^{-1} Y_{Tij} + I\rho_{ij}, && j = 1, \ldots, n_i \\ &Y_{Ti} = [Y_{Ti1}, Y_{Ti2}, \ldots, T_{Tin_i}] \end{aligned} \tag{28}$$

ρ_{ij}'s are elements of $B_{mi} r_i$, i.e. $B_{mi} r_i = \left[\rho_{i1} \ldots \rho_{in_i}\right]^T$

$$v_i = Y_i B_{di} \dot{\tilde{\theta}}_i, \;\; \tilde{\theta}_i = Vec(\varphi_i), \;\; B_{di} = I_{m_i+n_i+1} \otimes B_i \tag{29}$$

$$v_{Ti} = -Y_{Ti} \dot{\tilde{\theta}}_{Ti}, \;\; \tilde{\theta}_{Ti} = Vec(\tilde{T}_i) \tag{30}$$

Now we define:

$$Y_{iF} = [Y_i B_{di} - Y_{Ti}], \;\; \tilde{\theta}_{iF} = \begin{bmatrix} \tilde{\theta}_i \\ \tilde{\theta}_{Ti} \end{bmatrix} \tag{31}$$

The system (27) can be summarized as:

$$\begin{aligned} \dot{\varepsilon}_i &= T_i A_{mi} T_i^{-1} \varepsilon_i + D_i C_i Y_{Fi} \tilde{\theta}_{Fi} - Y_{Fi} \dot{\tilde{\theta}}_{Fi} + B_i K_{ei}^* (\hat{x}_i - x_i) \\ &\quad + \Delta A_i x_i \;\; \varepsilon_{1i} = C_i \varepsilon_i \end{aligned} \tag{32}$$

The following theorem proves the convergence property of the above system.

Theorem 3. The error system (32) with the adaptive law

$$\dot{\tilde{\theta}}_{iF} = -\hat{Y}_{iF}^T C_i^T C_i \varepsilon_i - \sigma_{iF} \theta_{iF} \tag{33}$$

where

$$\sigma_{iF} = \begin{cases} \sigma_{0i} \|\varepsilon_{i1}\| & \text{if} \quad \|\theta_{iF}\| \geq \theta_{0i}, \quad \theta_{iF} = Vec([\theta_i \;\; T_i]) \\ 0 & \text{if} \quad \|\theta_{iF}\| < \theta_{0i} \end{cases}$$

$$\hat{Y}_{iF} = \left[Y_i \hat{B}_{di} - Y_{Ti}\right], \;\; \hat{B}_{di} = I_{m_i+n_i+1} \otimes \hat{B}_i \tag{34}$$

converges to the residual set

$$\begin{aligned} \Re_{\sigma i_F} = \Big\{ \left(\varepsilon_i, \tilde{\theta}_{iF}\right) &\lambda_{\max}\left(L_i L_i^T\right) \|\varepsilon_i\|^2 + \sigma_{iF} \left\|\tilde{\theta}_{iF}\right\|^2. \\ &< 2\tilde{\theta}_{iF} \left(Y_{iF} - \hat{Y}_{iF}\right)^T C_i^T C_i \varepsilon_i + 2(\hat{x}_i - x_i)^T K_{ei}^* B_i^T P_i \varepsilon_i \\ &\quad + 2x_i^T \Delta A_i^T P_i \varepsilon_i + \sigma_{iF} \left\|\theta_{iF}^*\right\|^2 \Big\} \end{aligned} \tag{35}$$

For the proof, see the Appendix.

Now, using (28) with the adaptive law

$$\dot{\tilde{\theta}}_{iF} = -\hat{Y}_{iF}^T C_i^T (y_i - y_{mi}) - \sigma_{iF} \theta_{iF} \tag{36}$$

where

$$\begin{aligned} &\sigma_{iF} = \begin{cases} \sigma_{0i} \|e_{1i}\| & \text{if} \quad \|\theta_{iF}\| \geq \theta_{0i} \\ 0 & \text{if} \quad \|\theta_{iF}\| < \theta_{oi} \end{cases} \\ &\hat{Y}_{iF} = \left[Y_i \hat{B}_{di} - Y_{Ti}\right] \\ &\theta_{iF} = vec([\theta_i \;\; T_i]) \\ &\hat{B}_{di} = I_{m_i+n_i+1} \otimes \hat{B}_i \end{aligned} \tag{37}$$

makes the error system (21) to converge to a residual set as (35). Adaptation of the matrix T_i will be addressed in the following section.

4 Simulation results

In order to demonstrate the effectiveness of the proposed method, some simulations were carried out. In these simulations, the proposed algorithm described in section 3, was applied to the power system described in section 2 for load-frequency control.

The first step was the design of the adaptive observer (6). The matrices A_i's and G_{0i}'s were selected to be:

$$\begin{aligned} A_1 &= \begin{bmatrix} -2.0496 & -0.0078 & 0 & 0 \\ -9.9566 & 9.5496 & 5 & 0 \\ 24.7232 & -24.6802 & -5 & 5 \\ -38.8996 & 37.3504 & 0 & -12.500 \end{bmatrix}, \\ G_{01} &= \begin{bmatrix} -11.8862 & 9.5351 \\ 19.2960 & -81.6645 \\ 79.6732 & -193.2059 \\ 102.8572 & -188.5061 \end{bmatrix} \\ A_2 &= \begin{bmatrix} -2.0002 & -0.0004 & 0 & 0 \\ -0.6507 & -8.6668 & 5.9880 & 0 \\ 1.2666 & -18.5977 & -4.1667 & 4.1667 \\ -2.5651 & 37.2200 & 0 & -12.5000 \end{bmatrix}, \\ G_{02} &= \begin{bmatrix} -23.8302 & 35.7628 \\ 29.9808 & -111.1410 \\ 160.3170 & -357.8501 \\ 173.9856 & -367.1521 \end{bmatrix} \\ A_3 &= \begin{bmatrix} -2.1005 & -0.0087 & 0 & 0 \\ -16.2549 & -8.6005 & 6.6667 & 0 \\ -27.9323 & -16.5684 & -4.0000 & 4.0000 \\ 59.7642 & 35.2097 & 0 & -12.5000 \end{bmatrix}, \\ G_{03} &= \begin{bmatrix} -4.7478 & 4.7555 \\ 2.5575 & -68.8186 \\ 34.4818 & -141.3355 \\ 19.8337 & -119.5056 \end{bmatrix} \end{aligned} \tag{38}$$

Use of the adaptive law of (14) resulted in very good parameter and state estimations. They converged in 3 s and no steady state errors were observed. These simulation results are not shown here for the sake of brevity (there were too many graphs in the simulation results, showing them all would make the paper very lengthy).

It should be noted that in the case of regulation problems (which was considered in this example), the reference input is identical to zero. However, in the adaptation period, we can consider a nonzero reference input to increase the gain and parameter convergence. A measurement noise was also added to each output.

The next step was to design the decentralized robust adaptive controller. The following reference model was considered for each area (it was selected by trial and error).

$$A_{mi} = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ -6 & -0.1 & 6 & 0 \\ 0 & 0 & -5 & 5 \\ -1448.9 & -73.2 & -53.2 & -27.9 \end{bmatrix},$$

$$B_{mi} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The adaptive control law of (36) was then applied. Many simulations with different disturbances and operating conditions were carried out. All simulation results showed the very good performance of the proposed controller. Here are just a few of them:

Figure 3 shows the response of frequency and tie-line power deviations in each area following a 0.01 pu disturbance step change in area 2, without the load-frequency controller. As the figure shows, the plant becomes unstable after 3 s. Figure 4 shows the same system response but with the proposed controller. It is clear from the figure that frequency and tie-line power deviations converge rapidly without any steady state errors.

Figure 5 shows the response of frequency and tie-line power deviations and control signals in each area following a 100 ms three-phase short circuit, with the the proposed controller.

The final control test was the investigation of the performance of the proposed controller following parameter variations. Figure 6 shows the performance of the proposed controller following a 50% deviation for the parameter M_2 and a 0.01 pu load disturbance in area 2.

The simulation results of Figs. 5 and 6 without the proposed LFC showed instability in the plant (similar to Fig. 3).

Adaptation of T_i

The aim of adaptation of θ_i is to adjust the control gains such that the overall state error and the control gains remain uniformly bounded. However, the aim of adaptation of T_i is to satisfy the conditions of robust stability and the applicability of the proposed adaptive control. The simulation results showed that without T_i, it is impossible to track all outputs of the model reference except in some special cases. In fact, although the inclusion of T_i increases the number of unknown parameters, it provides the existence of a unique solution for θ_i.

The simulation results showed that the adaptation of T_i must be carried out at a much slower rate compared with the adaptation rate of θ_i. Without adaptation of T_i, there are some steady state errors. The simulation results showed that it is sufficient that the adaptation of T_i is carried out during the steady state intervals. By doing this, the steady state error for all outputs was decreased considerably, and T_i was brought to a final value T_i^*. After this stage, the adaptation of T_i can be traced very slowly.

In the power system of the above example, the final value of transformation matrix T_i was obtained to be:

$$T_1 = T_2 = T_3 \begin{bmatrix} 0.9737 & 0 & 0 & 0.0532 \\ 0 & 0.9737 & 0 & 0.0000 \\ 0 & 0 & 0.9737 & -0.0001 \\ 0 & 0 & 0 & 0.9723 \end{bmatrix}$$

5 Conclusion

In this paper a new model reference-decentralized adaptive-output feedback controller for large-scale power plants with unknown parameters has been developed such that the error between the outputs of the system and outputs of the reference model, and the control gains

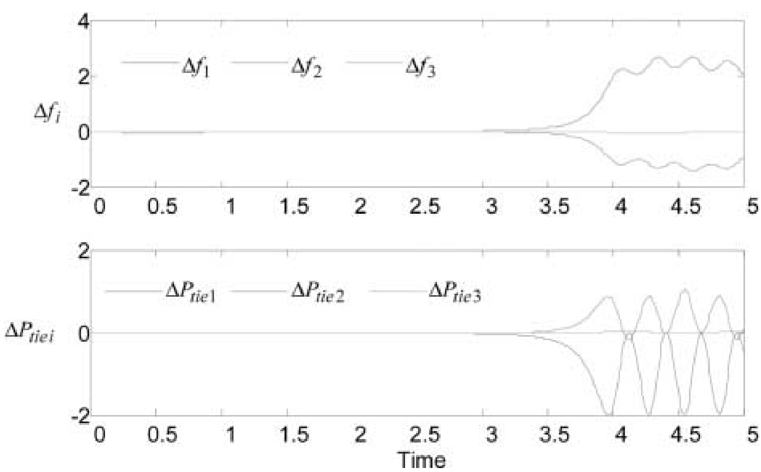

Fig. 3. The response of the frequency and tie-line power deviations in each area, following a 0.01 pu disturbance step change in area 2, without the load-frequency controller

remain uniformly bounded. In the proposed method, first for each area, an adaptive observer was designed to estimate the state variables only using local input–output data. Then, a linear combination of these estimates and model reference states with adjustable gain were fed back as a stable robust adaptive control such that the outputs of the plant tracked the outputs of the desired reference model asymptotically.

A suitable transformation matrix was introduced which transformed the initial reference model to an equivalent reference model such that the convergence of the output errors was guaranteed. In fact, this transformation matrix made an equivalent reference model such that some convergence conditions were satisfied. An appropriate adaptive law was derived for adjusting this transformation matrix.

The simulation results demonstrated the effectiveness of the proposed method for decentralized load-frequency control in a multi-area power system. Presenting all the simulation results was not possible in the paper. All simulation results showed a very good performance for the proposed controller.

Bearing in mind that the proposed controller is decentralized and need no prior information about the system parameters, and the fact that it is robust against system parameter uncertainties and needs only the output of the subsystems which are readily available, this makes the proposed algorithm a very good candidate for load-

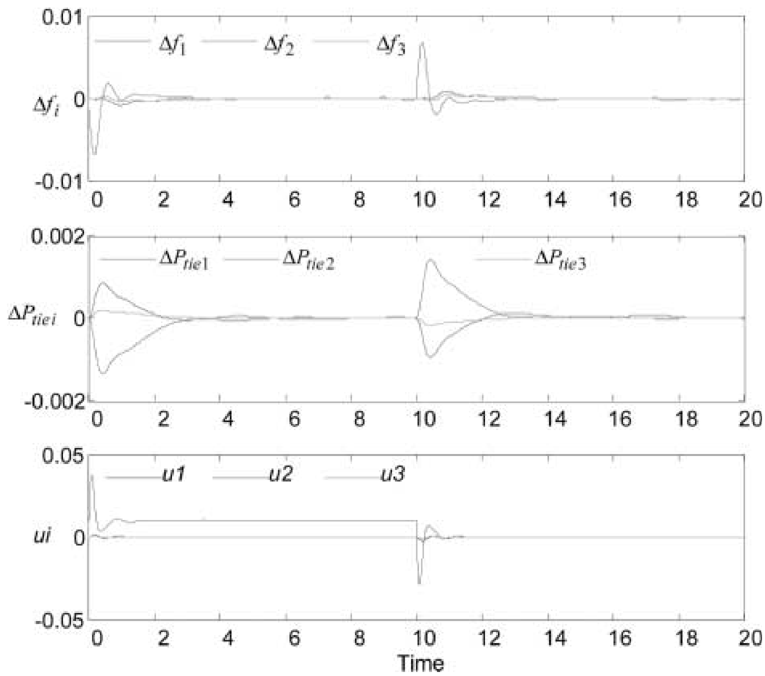

Fig. 4. The response of the frequency and tie-line power deviations in each area, following a 0.01 pu disturbance step change in area 2, with the load-frequency controller

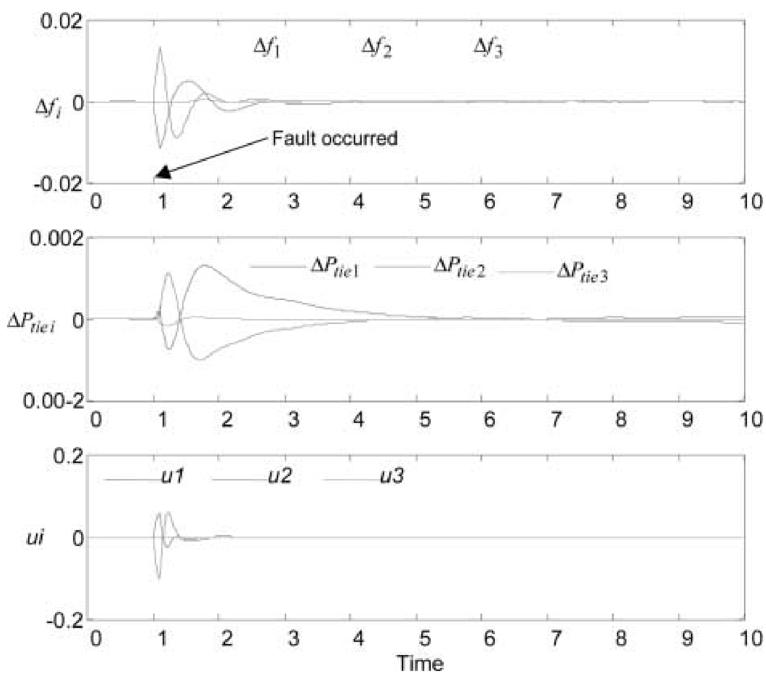

Fig. 5. The response of the frequency and tie-line power deviations in each area, following a 100 ms short circuit, with the load-frequency controller

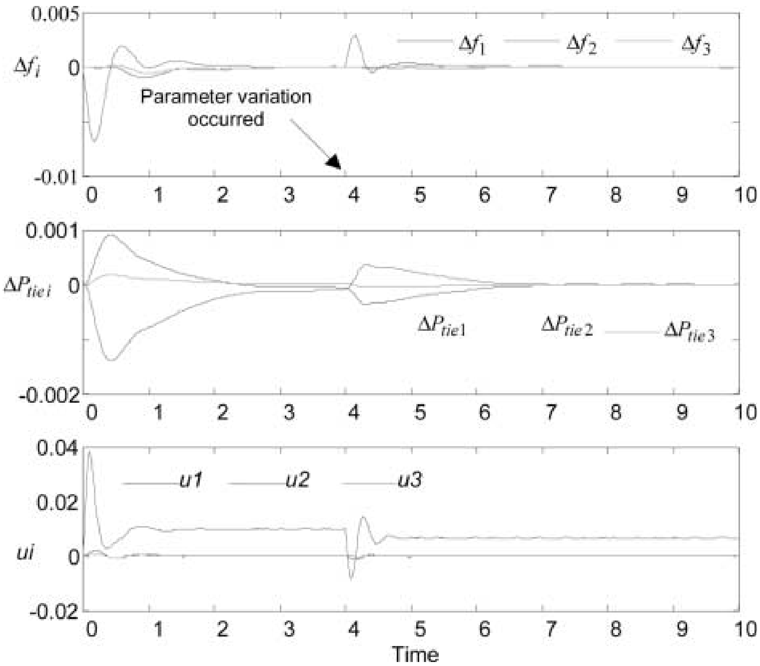

Fig. 6. The response of the frequency and tie-line power deviations in each area, following a 50% parameter variation, with the load-frequency controller

frequency control of power systems. In fact it seems to be "The Ultimate Control Strategy for LFC".

Proofs of the theorems

Proof of Theorem 1

The following lemma is first stated before the proof of the Theorem 1.

Lemma 1 [26] Let $Z(s)$ be a matrix of rational functions such that $Z(\infty)$ and $Z(s)$ has poles only in Re[s]-μ, (μ>0). Let (C, A, B) be a minimal realization of $Z(s)$. Then, $Z(s)$ is Strictly Positive Real (SPR) if and only if there exist a symmetric positive-definite matrix P and a matrix L such that:

$$A^T P + PA = -LL^T - 2\mu P = -Q$$

$$PB = C \tag{A1}$$

Now, we return to the proof of Theorem 1. Equation (9) can be rewritten as:

$$\dot{e} = Ke + \sum_{i=1}^{m} (\phi_i z_i + v_i) \tag{A2}$$

where z_i's are elements of z and ϕ_i are column vectors of matrix ϕ.

Defining $v_i = X_i \dot{\phi}_i$ and substituting z_i from (12) in (A2), results:

$$\dot{e} = Ke + \sum_{i=1}^{m} \left\{ \left(\dot{X}_i - KX_i\right)\phi_i + X_i\dot{\phi}_i \right\}$$

$$= Ke + \sum_{i=1}^{m} \left\{ \frac{\mathrm{d}}{\mathrm{d}t}(X_i\phi_i) - K(X_i\phi_i) \right\} \tag{A3}$$

Since (C, K) is observable, a matrix D exists such that K+DC is asymptotically stable. By choosing matrix D such that the system (C, K, D) is SPR, and defining

$$\eta = \sum_{i=1}^{m} \left\{ \frac{\mathrm{d}}{\mathrm{d}t}(X_i\phi_i) - (K + DC)(X_i\phi) \right\}, \quad \ell = Vec(\phi) \tag{A4}$$

we have:

$$\dot{e} = Ke + DCX\ell + \eta \tag{A5}$$

Now, we define the Lyapunov function V=$\xi_i^T P \xi_i$ for the system

$$\dot{\xi}_i = (K + DC)\xi_i + \eta_i \tag{A6}$$

with, ξ_i=$X_i\phi_i$.

Evaluating the time derivative of V along the trajectory of the system (A6), and using Lemma 1, we obtain:

$$\dot{V} = 0 - \xi_i^T LL^T \xi_i + 2\left(\xi_i^T P\eta_i - \mu\xi_i^T P\xi_i + \xi_i^T C^T C\xi_i\right) \tag{A7}$$

Now if we choose μ sufficiently large such that:

$$\left(\xi_i^T P\eta_i - \mu\xi_i^T P\xi_i + \xi_i^T C^T C\xi_i\right) < 0 \tag{A8}$$

Then $\dot{V}$ is negative, which implies the boundedness of η_i or η.

Subtracting (10) from (A5) implies:

$$\dot{\gamma} = K\gamma + \eta\,, \quad \gamma = e - \varepsilon \tag{A9}$$

Since K is a stable matrix, according to the input–output stability theorems [26], boundedness of η results in the convergence of $C\gamma$=$C(e$–$\varepsilon)$ to zero asymptotically. In other words, e_1 tends to ε_1 as t⟶∞.

QED

Proof of Theorem 3

Considering the Lyapunov function:

$$V = \varepsilon_i^T P_i \varepsilon_i + \tilde{\theta}_{iF}^T \tilde{\theta}_{iF} \tag{A10}$$

and evaluating $\dot{V}$ along the error system (32) and the adaptive gain (33) and using the result of Lemma 1, it follows that

$$\begin{aligned}\dot{V} = &-\varepsilon_i^T L_i L_i^T \varepsilon_i - 2\mu\varepsilon_i^T P_i \varepsilon_i + 2\tilde{\theta}_{iF}^T Y_{iF}^T C_i^T D_i^T P_i \varepsilon_i \\ &- 2\tilde{\theta}_{iF}^T \hat{Y}_{iF}^T C_i^T C_i \varepsilon_i - 2\tilde{\theta}_{iF}^T \theta_{iF}\sigma_{iF} + 2(\hat{x}_i - x_i)^T K_{ei}^{*T} B_i^T P_i \varepsilon_i \\ &+ 2x_i^T \Delta A_i^T P_i \varepsilon_i + 2\varepsilon_i^T P_i Y_{iF} \hat{Y}_{iF}^T C_i^T C_i \varepsilon_i \\ &+ 2\varepsilon_i^T P_i Y_{iF} \theta_{iF} \sigma_{iF} \end{aligned} \quad \text{(A11)}$$

It can be shown that

$$\begin{aligned}&- 2\mu\varepsilon_i^T P_i \varepsilon_i + 2\varepsilon_i^T P_i Y_{iF} \hat{Y}_{iF}^T C_i^T C_i \varepsilon_i + 2\varepsilon_i^T P_i Y_{iF} \theta_{iF}\sigma_{iF} \\ &\le -2\big[\mu\lambda_{\min}(P_i) - \|P_i Y_{iF} \hat{Y}_{iF}^T C_i^T C_i\| \\ &- \|P_i Y_{iF} \theta_{iF}\| \cdot \|C_i\|\sigma_{0i}\big] \cdot \|\varepsilon_i\|^2 \end{aligned} \quad \text{(A12)}$$

Choosing μ sufficiently large, such that the second side of the inequality (A12) becomes negative, the following inequality can be derived from (A11):

$$\begin{aligned}\dot{V} \le &-\varepsilon_i^T L_i L_i^T \varepsilon_i + 2\tilde{\theta}_{iF}^T \big(Y_{iF} - \hat{Y}_{iF}\big)^T C_i^T C_i \varepsilon_i - 2\tilde{\theta}_{iF}^T \theta_{iF}\sigma_{iF} \\ &+ 2(\hat{x}_i - x_i)^T K_{ei}^{*T} B_i^T P_i \varepsilon_i + 2x_i^T \Delta A_i^T P_i \varepsilon_i \end{aligned} \quad \text{(A13)}$$

Now if using the fact:

$$-2\sigma_{iF}\tilde{\theta}_{iF}^T \theta_{iF} \le -\sigma_{iF}\tilde{\theta}_{iF}^T \tilde{\theta}_{iF} + \sigma_{iF}\theta_{iF}^{*T}\theta_{iF}^* \quad \text{(A14)}$$

we have:

$$\begin{aligned}\dot{V} \le &-\varepsilon_i^T L_i L_i^T \varepsilon_i + 2\tilde{\theta}_{iF}^T \big(Y_{iF} - \hat{Y}_{iF}\big)^T C_i^T C_i \varepsilon_i \\ &+ 2(\hat{x}_i - x_i)^T K_{ei}^{*T} B_i^T P_i \varepsilon_i - \sigma_{iF}\tilde{\theta}_{iF}^T \tilde{\theta}_{iF} \\ &+ \sigma_{iF}\theta_{iF}^{*T}\theta_{iF}^* \end{aligned} \quad \text{(A15)}$$

This implies that the error system (32) with the adaptive gain (33) converge to the residual set $\Re_{\sigma i_F}$.

QED

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