The Hierarchical Hub Maximal Covering Problem with Determinate Cover Radiuses

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Abstract - **Hierarchical facility location problems with three levels deal with location of facilities in any level and assignment traffic to given routing. Hub covering problems covered the demand nodes, if they are within a particular radius of a facility that can supply their demand. This paper study the single allocation hierarchical hub maximal covering problem over complete network linking in the first level, that is consists of hub facilities known as central hubs and in the second and third levels are star networks linking the hubs to central hubs and the demand places to hubs or central hubs. The problem assigns every non-hub nodes and hubs to their top level facilities with predetermined cover radiuses, and minimizing the total cost. The cover radiuses computed with a heuristic method finally present a mixed integer programming model for this problem and test the performance of the problem on the CAB data.**

Keywords - **Hierarchical, hub covering, hub location.**

I. INTRODUCTION

The hub location problem scope is took care the choice of the location of hub facilities and the assignment of the demand nodes to them, and their goal is obtained minimum total cost for establish network. There are two style assignment for links in hub networks, single assignment and multi assignment. In single assignment any demand node allocated to exactly on hub. But in multi assignment, any nonhub can connected to more than one hub.

The single assignment hierarchical hub location problem with three level is deal with locating hub facilities in any level of hierarchy and allocating demand nodes to exactly one of hub and one of central hub so that constitute the route of traffic demand among origin–destination pairs. Frequently, the routing cost between hubs and central hubs and two central hubs, is discounted at a rate of in order take benefit of the economies of scale. Usually, in classical hub networks, hubs facilities in top level are connected with complete network and in the second and third level links are forms star networks. There are many work scopes for these problems, for example in transportation (airline, cargo delivery) and telecommunication systems.

At the first time was presented hub location problem with single allocation and minimizing total routing cost in a quadratic model formulation by O'Kelly [1]. Then O'Kelly illustrated a $\alpha \in [0,1]$ with topic constant discount factor, So as to reflect the economies of scale in hubbing links. There are many research on the hub location problem, but mainly executed on the linearization of the quadratic model introduce by O'Kelly [1], for example, Campbell [2], Ernst and Krishnamoorthy [3] and Skorin-Kapov et al. [4].

Hub covering location problems apply to finding the location of hub facilities and allocating the demand nodes to nearest located hub facilities. In continues researches on hub location problems existed combining model, hub covering is one of them. The hub covering presented for the first time in paper's Campbell [5]. Kara and Tansel [6] studied the single assignment hub set-covering problem and showed that it is NP-hard, also presented a new linear model of Campbell's quadratic model. Wagner [8] proposed new formulations for hub covering problems in two situations of single and multiple assignment, these formulations are needed less number of variables and constraints as compared to Kara and Tansel [6] formulation. Ernst et al. [7] presented new formulations for the single and multiple assignment hub set covering problem via considered β as the cover radius. These formulations are better than Kara and Tansel [6] formulation because is required less Processing time of CPU.

For first time presented result of a investigation on a major cargo company in Turkey, it showed that a cargo delivery company has three levels structure with two central hubs located in Ankara and Istanbul (see Elmastas, 2006). Elmastas make model for the problem and considered in airplanes and trucks with minimizing fixed charge cost objective and solved it [8].

Yaman [9] suggested other style of Elmastas's problem with two differences. These differences relate to the objective function and the network of links on each level (for details see Yaman [9]).

In this paper presented single allocation hierarchical hub maximal covering location model with a given delivery time bound (SA-TH-HMC), whereas in literature, there is no previous study on SA-TH-HMC. To the best of our knowledge, amounts of radiuses defined as decision variables and bring in to delivery time constraints, but where denote values of radiuses with heuristic method. So the demand and hub nodes assign to exactly one of hub and one of central hub, if they are in scope of given cover radiuses, respectively. The traffic from an origin to a destination can meet four or lesser than four hubs on its route. The central hubs are connecting to together on the complete network, so there are the direct links between facilities of first level. There is a transportation network with two levels of hub facilities in Fig. 1, in order that it shows the problem with best resolution. This network has 35 nodes and denoted nonhubs, hubs and central hubs with shapes of circle, square and hexagon, respectively. The cover radiuses of

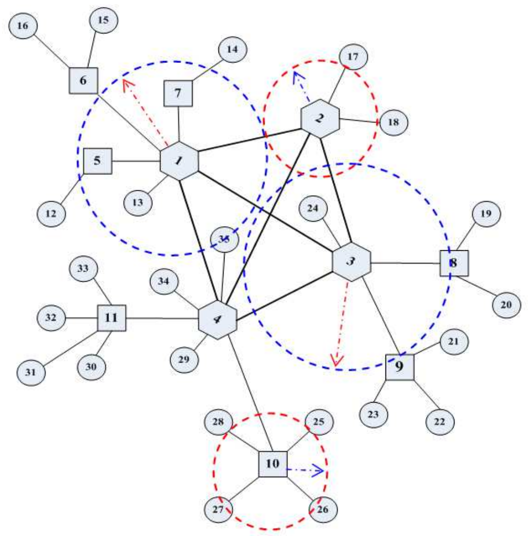

Fig.1. A three level network on 35 nodes with 11 hubs and 4 central hubs. Adapted from: Yaman [9].

hubs are drawing with not complete circles. In this paper, our aim is to find the location of hubs and central hubs, the allocation of demand nodes to the nearest located hub nodes and connecting hub to central hub in a unify cover situation. Also our model is required less Processing time of CPU with the heuristic method for specify the values of radiuses. The main contributions of this paper, define new hub location problem and solve it in best time by heuristic identify of cover constraints. Computational results obtained for the CAB data are emphasis on that the propose model solve in a good time unlike pervious studies.

This paper will be performing as follows: we present a mixed integer programming formulation in the second section. In the third section, we explain our heuristic method for finding amounts of radiuses. The fourth section is dedicated to the computational analysis. The end section is devoted to conclusions.

II. A MIXED INTEGER PROGRAMMING FORMULATION FOR SA-TH-HMC

In this section, we define the notations, parameters and decision variables for our model and then present the mathematical formulation for SA-TH-HMC problem. We assume that I be the set of nodes, $H \subseteq I$ be a potential hub set and $C \subseteq H$ be a set of possible locations for central hubs.

The parameters of the model are as follows. This model designed for maximal covering so must be fix number of hubs and central hubs to be opened; we denote the number of hubs and central hubs with P_H and P_C, respectively. In hub location problems use from discount Rate of α for establish link cost (or travel time) between hubs and $\alpha \in [0,1]$. We represent the decrease coefficients of routing cost and travel time among hubs and central hubs with α_H, $\hat{\alpha}_H$ and between central hubs with α_C, $\hat{\alpha}_C$ respectively. We assume that these parameters take values on found $\alpha_H \geq \alpha_C$ and $\hat{\alpha}_H \geq \hat{\alpha}_C$. The parameter f_{ij} is amount of traffic demand to be routed from node $i \in I$ to node $j \in I$, C_{ij} is the routing cost of a unit flow demand among two nodes of i and j, t_{ij} is the travel time from node i to node j and d_{ij} is the distance between node i to node j. r_H is amount of cover radius for all of hubs and r_C is amount of cover radius for all of central hubs.

Logically, there are f_{ii}, C_{ii}, t_{ii} and d_{ii} with attention to later description equal zero for all of $i \in I$. we symbolize the upper bound of delivery time by β that similar to literature, it is the maximum time that any flow demand should be arrived its destination.

We present and classify the decision variables of the model as follows:

X_{ijl} = 1 if node i is assigned to a hub at node j and a central hub at node l; 0 other wise.

Y^i_{jl}= The total demand flow is crossing among hub at node j and central hub at node l by node $i \in I$ as origin or destination.

Z^i_{kl} = The total demand flow is crossing from central hub at node k to central hub at node l by node $i \in I$ as origin or destination.

$\hat{D}_l$= The time which all of traffic from demand nodes and hubs assigned to central hub l comes to l.

D_l = The time which all the demand flow traverse from origin to destination central hub.

Consistent with the literature, if the variable $X_{jjl} = 1$ for some $j \in H$ that node j is assigned to a central hub at node l, it means that node j is a hub node and if $X_{lll} = 1$ for some $l \in C$, it means that node l is a central hub node.

The objective function of our mathematical model is to minimize the total costs of routing demand flow between nonhubs nodes and their hubs, among the hubs and their central hubs and central hubs together. The objective function is described in detail with the previously defined decision variables and parameters as follows:

$$\min \; z_1 = \sum_{i\in I}\sum_{r\in I}(f_{ir}+f_{ri})\sum_{j\in H} C_{ij}\sum_{l\in C} x_{ijl} + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C, l\neq j}\alpha_H C_{jl}\, y^i_{jl} + \sum_{i\in I}\sum_{k\in C}\sum_{l\in C, l\neq k}\alpha_C C_{kl}\, z^i_{kl} \qquad (1)$$

We classify and explain the constraints of our mathematical model for *SA-TH-HMC* as follows:

Single assignment hierarchical hub constraints:

$$\sum_{j\in H}\sum_{l\in C} x_{ijl} = 1 \quad \forall i \in I \tag{2}$$

$$x_{ijl} \le x_{jjl} \quad \forall i \in I, j \in H : j \ne i, l \in C \tag{3}$$

$$\sum_{m\in H} x_{jml} \le x_{lll} \quad \forall j \in H, l \in C : l \ne j \tag{4}$$

$$x_{ijl} = 0 \quad \forall j \in H, l \in C : l \ne j \tag{5}$$

$$x_{ijl} \in \{0,1\} \quad \forall i \in I, j \in H, l \in C \tag{6}$$

Because of we design our model with the single assignment attribute of the problem, so each nonhub node and every hub should allocated to exactly one of hub facility and one of central hub facility, respectively. Constraints (2), (4) and (6) guarantee the single assignment attribute for our model. Beside constraint (4) ensures that every hub cannot be linked to other node unless that node is a central hub node. Constraint (3) states that if a demand node be assigned to other node then that node should be a hub node. Constraints (18) are unnecessary however they make stronger the LP relaxation.

Number of Hubs and central hubs constraints:

$$\sum_{j\in H}\sum_{l\in C} x_{jjl} = p_H \tag{7}$$

$$\sum_{l\in C} x_{lll} = p_C \tag{8}$$

Our model found on P-hub median and maximal covering problems, thus we denote number of Hubs and central hubs with (7) and (8) constraints, respectively.

Flow balance constraints:

$$y^i_{jl} \ge \sum_{r\in I: r\ne j} (f_{ir}+f_{ri})(x_{ijl}-x_{rjl}) \quad \forall i \in I, j \in H, l \in C : l \ne j \tag{9}$$

$$\sum_{k\in C: k\ne l} z^i_{lk} - \sum_{k\in C: k\ne l} z^i_{kl} = \sum_{r\in I} f_{ir} \sum_{j\in H}(x_{ijl}-x_{rjl}) \quad \forall i \in I, l \in C \tag{10}$$

$$y^i_{jl} \ge 0 \quad \forall i \in I, j \in H, l \in C \tag{11}$$

$$z^i_{kl} \ge 0 \quad \forall i \in I, k \in C, l \in C : l \ne k \tag{12}$$

We define flow balance constraints, similarly to in Yaman [9]. For each link from a hub to its central hub, amount of traffic demand will be compute by constraints (9) and (11). This constraint let sum of traffic crossing from among a hub and its central hub as lower bound for y^i_{jl} variable, thus objective is minimizing type then estimation least of y^i_{jl}. Respondent of rules of flow network is equal sum of input flow with sum of output flow to a same node. Beside the central hubs are end nodes of their hierarchy, so this situation is ensured by flow balance constraints (10) and constraints (12).

Cover constraints:

$$\sum_{j\in H}\sum_{l\in C: j\ne l} d_{ij} x_{ijl} \le r_H \quad \forall i \in I \tag{13}$$

$$\sum_{l\in C} d_{il} x_{ill} \le r_H \quad \forall i \in I \tag{14}$$

$$d_{jl} x_{jll} \le r_C \quad \forall j \in H, l \in C \tag{15}$$

Briefly, all of assignments terminated, by two fix amounts which compute with a heuristic method which come at next section. Considering above description, we design three constraints (13), (14) and (15). Constraints (13) ensure that assignments demand nodes to hubs for obtain low service occur in specific closeness criterion of r_H. On the other hand a central hubs provide the services of a hub. So it is possible the nonhubs assign to a central hub for achieve to low Service. Then Constraints (14) guarantee that assignments demand nodes to central hubs for obtain low service happen in specific closeness criterion of r_H. Beside the assignments hubs nodes to central hubs for obtain top service occurs in specific closeness criterion of r_C, too. Constraints (15) are ensuring them.

Time bound constraints:

$$\hat{D}_l \ge \sum_{j\in H} (t_{ij} + \alpha_H t_{jl}) x_{ijl} \quad \forall i \in I, l \in C \tag{16}$$

$$D_l \ge \hat{D}_k + \alpha_C t_{kl} x_{kkk} \quad \forall l \in C, k \in C \tag{17}$$

$$D_l + \sum_{j\in H} (\alpha_H t_{lj} + t_{ji}) x_{ijl} \le \beta \quad \forall i \in I, l \in C \tag{18}$$

$$\hat{D}_l \ge 0 \quad \forall l \in C \tag{19}$$

We define the time bound constraints with using the ideas developed in Yaman [9]. The travel time every traffic demand until initial central hub computed with constraints (16) and insert travel time each route in its $\hat{D}_l$. Constraints (17) calculated travel time from each origin until destination central hub and let in its D_l. Constraints (18) ensure that the traffic of every origin arrives at destination the ultimate at time β or under it.

We explain output of our mathematical model comprehensively with an example that exhibited in Figs. 2 and 3. The node set in Fig. 2 is used for input model and ultimate presented the resulting network in Fig. 3. According to this solution, $x_{225} = x_{555} = x_{666} = x_{776} = x_{999} = x_{11\,11\,9} = 1$ so the nodes of 2, 5, 6, 7, 9 and 11 chosen as hub nodes and $x_{555} = x_{666} = x_{999} = 1$ nodes of 5, 6, and 9 selected as Central hub node. The below notations are time bound and cover for route between node 1 and node 8. The time bound constraints are $(t_{12}+\alpha_H t_{25})\ x_{125} \le \hat{D}_5$, $\hat{D}_5 + \alpha_C t_{56}\ x_{555} \le D_6$ and $D_6 + (\alpha_H t_{67} + t_{78})\ x_{876} \le \beta$. The cover constraints are $d_{12}\ x_{125} \le r_H$, $d_{25}\ x_{225} \le r_C$, $d_{76}\ x_{776} \le r_C$ and $d_{87}\ x_{876} \le r_H$. The other hand flow balance constrain (9) for nodes i=1, j=2 and l=5 is the notion of $(f_{12}+f_{21})(x_{125} - x_{225}) + (f_{12}+f_{21})\ x_{125} + (f_{13}+f_{31})\ x_{125} + (f_{14}+f_{41})\ x_{125} + \ldots + (f_{112}+f_{121})\ x_{125} \le y^1_{25}$. The constraint (10) is another flow balance, so for example it we spot the nodes of i=1 and l=5. the constraint is equal the $(z^{l}_{56}+ z^{l}_{95}) - (z^{l}_{65}+ z^{l}_{95}) = (f_{12}+ f_{13}+ f_{14}+ \ldots + f_{112})x_{125}- f_{12}\, x_{255}- f_{13}\, x_{355}- f_{15}\, x_{555}$. We introduce a heuristic method for computing r_H and r_C in the next section.

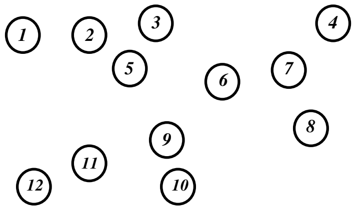

Fig. 2. The node set *I*.

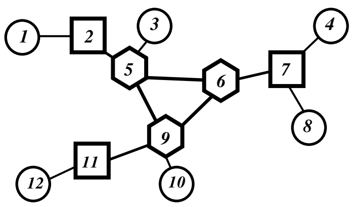

Fig. 3. The resulting network of a solution.

III. HEURISTIC METHOD FOR FINDING r_H AND r_C

Mainly optimality solve of the NP-complete problems is difficult for realistically sized cases. For the problem at hand, finding global solution is challenging. We saw that the problem solve in better time with time bound constraint. Then we understand that may exist other constraints, so as to reach global solution at least time. Hence, we classify all of solution space and modify assignment procedure for every nonhub node and every hub node to next facility on their top level. We specify that each assignment between two nodes when it must be implement that they are in specific closeness than together. The specific closeness criterion is equal the least distance among two nodes. The specific closeness criterions considered with r_H and r_C for assignments of the second level and the first level, respectively. So less number of nonoptimal solutions review in the slove, Hence soon reach the optimum solution. We used from distance table for finding the radiuses. Every column of distance table shows all of distances a city from other cities. Initially, we obtain the minimum number in any columns (without zero), these values denoted the nearest cities. Also we chose maximum number of between the minimum numbers of columns for r_H. This is best value for cover radius of nodes of second level. Because, if the choice was number less than it, the model was infeasible and if the choice was number further than it, the model was far from ideal. Logically, the cover radius of first level is bigger than r_H, so we need a dominant cover radius that is not very large and not very small. Hence we preferred minimum number of between the maximum numbers of columns for r_C. The maximum numbers of columns denoted two cities in greater distance from together, and minimum number of between them gives a dominant cover radius. We computed the r_H and r_C amounts for the model. As a result, we take r_H = 695 and r_C = 1506 for the CAB data. The next section is dedicated to the computational analysis.

IV. COMPUTATIONAL ANALYSIS

In our computational Analysis, we apply the CAB data set. The CAB data set introduced by O'Kelly [1]. The global solutions are scaled by dividing with the total flow. We get all of nodes as positional nodes for hubs and central hubs, i.e., I = H = C.

IV. I. EVALUATION OF THE TOTAL COST

In first stage, we investigated about effect of time bound constrains and the number of central hubs on the routing costs. We solved the problem with n = 25, P_H = 5, $\alpha_H = 0.9$, $\alpha_C = 0.8$, $\hat{\alpha}_H = 0.9$, $\hat{\alpha}_C = 0.8$ and $P_C \in \{1,\ldots, P_H\}$. For each data set similarly to the one used in Yaman (2009), we procured the smallest β amounts for which the problem is feasible when $\alpha_H = \hat{\alpha}_H = 0.9$ and $\alpha_C = \hat{\alpha}_C = 0.8$. We calculated averages of these smallest amounts for various values of P_C. In order to have a strong time bound, we present β amounts which are 120 and 240 more than the averages values. Therefore, we take $\beta \in \{2880, 3000, \infty\}$. We do not report for infeasible. The results showed with becoming great P_C, the total cost has decreasing procedure in all of cases. For see procedure of cost with more resolution, so we assume basic state of cost with $\beta = r_H = r_C = \infty$. Then for each state, we compute percent of increasing cost with well-known formula of

$$\%\text{increasing cost} = \frac{\text{New cost} - \text{Basic cost}}{\text{Basic cost}} * 100$$

. Our computational cost is coming in Table I and Fig. 4. The total cost is increase averagely 6.27% whereas cover radiuses impose to model. The problem is infeasible whereas P_C = 1 and impose the time bound constraints to model. when P_C = 2, the problem becomes feasible with the time bound constraints, however the total routing cost increases with growth rate of 13.29%. Generally, the total cost increase with the cover and the time bound constraints and increase percentage of the total cost is equal averagely 3.69% from $\beta = \infty$ to β = 2880. But, the Fig. 4 shows that rate growth of the total cost is very petty against perform covering situation.

TABLE I
TOTAL ROUTING COST AND %INCREASING COST FOR THE CAB DATA WITH n = 25, P_H = 5.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	PC	$(r_H, r_C) = \infty$	(695 , 1506)		
		$\beta = \infty$	∞	**3000**	**2880**
(0.9, 0.8, 0.9, 0.8)	1	1200.13	10.67	-	-
	2	1146.79	6.04	13.29	13.29
	3	1108.35	5.23	9.92	9.92
	4	1065.06	5.54	7.53	8.58
	5	1034.1	3.87	4.73	8.69

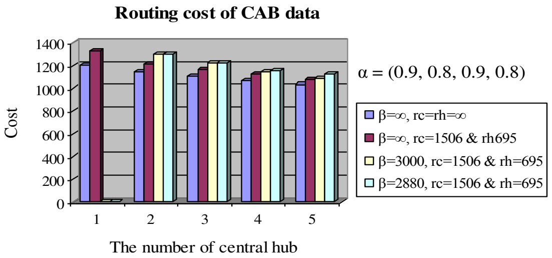

Fig. 4. A chart of total routing cost for the CAB data with n = 25, P_H = 5.

Finally, we infer that with impose the cover radiuses and delivery time bound, the total cost increase in both of data sets. The distances between hubs and central hubs are in range of r_C and the range of distances among nonhubs and hubs is r_H by cover constraints. So if two demand nodes and their hubs assigned to different hubs and central hub, the distances between origin–destination pairs is included two r_H and one r_C, but with move from star network to complete network for hubs (i.e., with increase central hubs), the r_C do not come in the distances between every origin–destination pairs.

IV. II. EVALUATION OF COMPUTATION TIME

In this section, we study effect of parameters of the problem on the computation times. We are solved the problem by using GAMS 21.7 and CPLEX 11.0.0 on a intel(R) Core^{TM} 2 Duo T8100 processor (2.10 GHz) with 3 GB of RAM operating under the system windows 7 Ultimate.

TABLE II

TOTAL ROUTING COST AND %INCREASING COST FOR THE CAB DATA WITH n = 25, P_H = 5.

$(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)$	P_c	$(r_H,r_C) = \infty$	(695 , 1506)		
		$\beta = \infty$	∞	3000	2880
(0.9, 0.8, 0.9, 0.8)	1	635.74	16.11	-	-
	2	61561.77	621.98	1845.51	32.64
	3	51699.33	3026.62	3665.95	505.02
	4	19831.40	3022.81	2627.58	373.06
	5	141.30	24.42	46.18	21.25

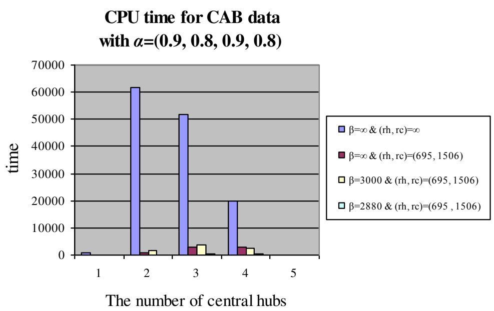

Fig. 5. A chart of CPU time for the CAB data with n = 25, P_H = 5.

The results come to Table II and Fig. 5. The Table II showed that the different cases of the problem are solved averagely 91.62% faster with cover constraints toward state of without covering. Whereas the computation times of the problem are increased in all of cases except state of P_C=4 with adding the time bound constraints with β = 3000. All of cases with P_H=P_C are the easiest instances. The Fig. 5 display that process time has extremely decreasing procedure. This procedure is very good against increasing procedure of poor of cost that show in Fig. 4.

V. CONCLUSION

In this paper, we studied the single assignment hierarchical hub maximal covering problem over complete network in first level and star networks in second and third levels. We presented a mixed integer programming formulation of the problem and in order to rapidly solve the model on data sets, we proposed a heuristic method for calculating amounts of the cover radiuses. To the best of our knowledge, our model and heuristic method are the first time in the literature they are proposed for the hierarchical hub problem. We experienced our model on the CAB data set and compared the productivity of our model than basic model. We understand that it obtained efficient solutions with less CPU time and did not see considerable effect on the total routing cost.

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