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Analytical solution of scattering by a DB/D′B′ cylinder coated with multilayer anisotropic media for cloaking applications

Nasser Montaseri, Gholamreza Dadashzadeh

Department of Electrical and Electronic Engineering, Faculty of Engineering, Shahed University, Tehran, Iran
E-mail: n.montaseri@shahed.ac.ir

Abstract: In this study, analytical solution of plane wave scattering is presented by a circular DB/D′B′ cylinder coated with multilayer anisotropic media. The relative permittivity and permeability tensors of each layer, when referring to principle axes (ρ, φ, z), are biaxial and diagonal; so, the radial eigenfunctions are complex ordered Bessel's functions. In order to compare different boundary conditions and cloaks, scattering width is calculated in some examples. Theory and simulation results show that scattering width of the circular DB (or D′B′) cylinder coated with Cloak I (or Cloak II) has the same value in both TM_z and TE_z polarisations. Furthermore, it is demonstrated that the DB boundary condition and Cloak I have good performance in the invisibility of cylindrical structures, which may provide a potential way to design a dual polarised cloak.

1 Introduction

Recently, Pendry *et al.* [1] investigated an interesting idea based on coordinate transformation to design transparency and low scattering phenomena. An optical conformal mapping method was used to create perfect invisibility in [2]. Independently, many groups have also begun to pursue other methods of realising cloaks. The proposed method by Alu and Engheta [3–5] was the scattering cancellation principle, relying on layered isotropic plasmonic or metamaterials to reduce total scattering cross-section. Furthermore, the cloaking effects associated with regions of anomalous localised resonance were illustrated based on 'scattering cancelling cloak' in [6]. Also, Silveirinha *et al.* [7] analysed the cylindrical structures coated with metamaterial that was suitably designed at microwave frequencies. The analysed metamaterials consisted of simple parallel-plate conducting implants in a host dielectric. Later, the same authors demonstrated the possibility of designing realistic metamaterials that obtained an effective plasmonic response at infrared and optical frequencies [8]. In [9], design of electromagnetic cylindrical cloaks for both TE_z and TM_z polarisations employing both magnetic inclusions and parallel plate medium already used by Silveirinha *et al.* [7] was illustrated. Inspired by the above works, many further investigations have been performed on invisible cloaking. Ruan *et al.* [10] studied scattering for an ideal invisibility cloak on the two-dimensional (2D) cylindrical cloak, Chen *et al.* [11] analytically analysed spherical cloaks in the Mie scattering model, Cai *et al.* [12] presented two novel designs for optical cloaking based on high-order transformations for TM_z and TE_z polarisations, Ivsic *et al.* [13] investigated the analysis of uniaxial multilayer cylinders used for invisible cloak realisation and Bojanjac and Sipus [14] analysed electromagnetic scattering by anisotropic multilayer cylindrical structures for oblique incident waves using a semi-analytical approach. In addition to the cloaking of the cylindrical structure, components of permeability and permittivity tensors of ideal cloak are [1]

$$\varepsilon_\rho = \mu_\rho = \frac{\rho - a}{\rho}, \quad \varepsilon_\varphi = \mu_\varphi = \frac{\rho}{\rho - a}, \quad \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{\rho - a}{\rho} \tag{1}$$

Equation (1) shows that all components of the tensors are a function of radius, which implies a very complicated design. Therefore, to realise the practical cloak, Schurig *et al.* [15] proposed metamaterial cloaks for TM_z polarisation (transverse magnetic) of the incident wave at microwave frequencies. Then, realisation of the TE_z (transverse electric) cloak was proposed in [16]. Only radial component of the permeability and permittivity tensors has gradients as a function of radius in [15, 16], respectively.

Concepts of DB and D′B′ boundaries that have been recently introduced by Lindell and Sihvola are described by the following relations [17, 18]

$$\text{DB boundary}: n \cdot D = 0, \quad n \cdot B = 0 \tag{2}$$

$$\text{D}'\text{B}'\text{boundary}: \nabla \cdot (nn \cdot D) = 0, \quad \nabla \cdot (nn \cdot B) = 0 \tag{3}$$

where n is the unit vector normal to the boundary. In the DB and/or D′B′ boundary conditions, normal components and/or

Table 1 Comparing DB/D′B′ with PEC and PMC [18]

	TM polarisation	TE polarisation
DB boundary	PMC	PEC
D′B′ boundary	PEC	PMC

normal derivatives of the fields vanish. DB boundary is the generalisation of both PMC (perfect magnetic conductor) and PEC (perfect electric conductor) in TM and TE polarisations, respectively. Performance of the D′B′ boundary is completely dual to the DB boundary. In Table 1, DB and D′B′ boundary conditions are compared for TM and TE polarised incident fields with respect to the normal of the plane. In this work, the considered polarisations in cylindrical coordinate are TM_z and TE_z; but, the unit vector normal to the boundary is $\hat{\rho}$ (i.e. is not $\hat{z}$). In Section 2, it will be demonstrated that the DB/D′B′ boundaries have different performance compared with Table 1 for normal incidence wave.

It should be noted that realisation of DB and D′B′ boundaries has been investigated in several studies. In [17], it was shown that the DB boundary could be realised by an interface of uniaxially anisotropic metamaterial half-space with zero axial medium parameters. Furthermore, Lindell *et al.* illustrated that the planar D′B′ boundary can be realised by a layer of wave-guiding medium, called a quarter-wave transformer, with strongly uniaxial permittivity and permeability dyadic upon the DB boundary condition [19]. Inspired by this work, realisation of spherical D′B′ boundary by a layer of wave-guiding medium was presented in [20].

Also, DB boundary has been recently analysed in some cloaking structures. In [21], the inner surface of the spherical cloaks was represented to behave like a DB boundary when the incident source was placed in the concealed region. In [22], electromagnetic scattering from spherical and cylindrical structures was illustrated. The analysis revealed that the inner surface of the circular cylindrical cloak, illuminated from inside of the inner cylinder, did not behave like DB boundary. Furthermore, the general formulation of the cloaking structures at the inner surface of the cloak layer was analysed in [23]. In [24], boundary conditions for point transformed electromagnetic cloaking structures in anisotropic media were described. The obtained results from [21–24] demonstrated that normal components of D and B fields at the inner surface of the cloak were zero when electromagnetic source was located outside the structures.

Although the boundary condition at the inner surface of the cylindrical structure has been presented by previous works (such as [21–24]), the interaction between DB (or D′B′) boundary and the proposed anisotropic cloaks by [15, 16] has not been studied yet. Thus, in this work, the analytical solution of plane wave scattering is investigated by an infinite DB or D′B′ cylinder coated with a multilayer anisotropic cloak. The main target of using these boundary conditions is to apply duality principle to obtain the same scattered fields in both polarisations. This article is divided into four sections. In Section 2, total fields are determined based on analytical approach. The anisotropic inhomogeneous cloak is discretised into N layers of anisotropic homogeneous media. To verify the analytical solution, some numerical examples are provided in Section 3. The presented analysis is validated by comparing the result of the analytical method with those in the published literature. Finally, conclusions are presented in Section 4.

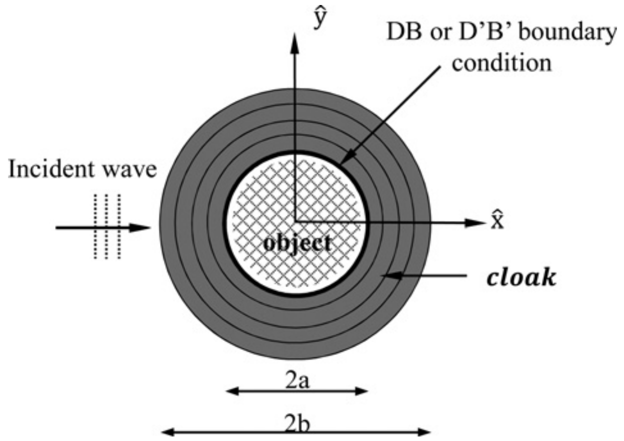

Fig. 1 *Cross-section of the analysed structure*

2 Theory

Cross-section of a circular-cylindrical cloak oriented with the axis in the z-direction is shown in Fig. 1. The main object is a DB or D′B′ boundary with radius a while surrounded by a cloaking shell of outer radius b. Since $\rho \geq b$ is free space, the wave number is $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. In the expressions of the fields, the time dependence $e^{+j\omega t}$ is assumed. The incident plane wave for both TM_z and TE_z polarisations can be expressed as

$$TM_z\text{mode}: E_z^{inc}(\rho,\phi) = \sum_{n=-\infty}^{+\infty} j^{+n} J_n(k_0\rho) e^{jn(\phi-\phi_0)} \quad (4)$$

$$TE_z\text{mode}: H_z^{inc}(\rho,\phi) = \sum_{n=-\infty}^{+\infty} j^{+n} J_n(k_0\rho) e^{jn(\phi-\phi_0)} \quad (5)$$

The scattered fields in $\rho \geq b$, can be expressed as

$$TM_z\text{mode}: E_z^{scat}(\rho,\phi) = \sum_{n=-\infty}^{+\infty} j^{+n} a_n H_n^{(2)}(k_0\rho) e^{jn(\phi-\phi_0)} \quad (6)$$

$$TE_z\text{mode}: H_z^{scat}(\rho,\phi) = \sum_{n=-\infty}^{+\infty} j^{+n} b_n H_n^{(2)}(k_0\rho) e^{jn(\phi-\phi_0)} \quad (7)$$

where $J_n(k_0\rho)$ and $H_n^{(2)}(k_0\rho)$ represent the nth order Bessel function of the first kind and Hankel function of the second kind, respectively.

It is proved that electromagnetic scattered fields by a circular cylinder in TM_z are dual with TE_z polarisation when the boundary condition is DB (or D′B′). The main idea in this paper is to apply the duality principle for both

Table 2 Permittivity and permeability tensors referring to the principle axes of the proposed cloaks

Cloak I	Cloak II
$\varepsilon_\rho = \mu_\rho = [(\rho - a)/\rho]^2$	$\varepsilon_\rho = \mu_\rho = [(b/(b-a)).((\rho - a)/\rho)]^2$
$\varepsilon_\varphi = \mu_\varphi = 1$	$\varepsilon_\varphi = \mu_\varphi = [b/(b-a)]^2$
$\varepsilon_z = \mu_z = [b/(b-a)]^2$	$\varepsilon_z = \mu_z = 1$

polarisations using these boundary conditions and specified cloak shell. Values of permittivity and permeability tensors of the suggestion cloaks (Cloaks I and II) are shown in Table 2. It should be noted that the considered cloaks in [15, 16] have been applied only for one polarisation; but, Cloaks I and II are intended to work for both TM_z and TE_z polarisations. In this paper, the scattered fields from an infinite DB (or D′B′) cylinder coated with Cloak I (or Cloak II) in TM_z is shown to be dual with TE_z polarisation.

To simplify physical implementation, the cloak shell is discretised into N layers of anisotropic media. Relative permittivity and permeability tensors of each anisotropic layer referred to the cylindrical coordinate system are considered as follows

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_\rho & 0 & 0 \\ 0 & \varepsilon_\varphi & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}_{\rho\phi z} ; \quad \bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu_\rho & 0 & 0 \\ 0 & \mu_\varphi & 0 \\ 0 & 0 & \mu_z \end{bmatrix}_{\rho\phi z} \tag{8}$$

Also, each anisotropic layer can be realised by double-layered isotropic dielectrics [25]. Total electric and magnetic fields inside the lth layer of the shell can be written as [26] (see (9) and (10))

where c_n, d_n, e_n and f_n are unknown coefficients for each layer, $m_{\text{TM}} = n\sqrt{\mu_\varphi/\mu_\rho}$, $m_{\text{TE}} = n\sqrt{\varepsilon_\varphi/\varepsilon_\rho}$, $u_{\text{TM}} = k_0\rho\sqrt{\mu_\varphi\varepsilon_z}$, $u_{\text{TE}} = k_0\rho\sqrt{\varepsilon_\varphi\mu_z}$ and $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. By applying the DB or D′B′ boundary at $\rho = a$ and continuity of tangential electric and magnetic fields across the boundary of each layer, the linear matrix equation about the unknown coefficients is obtained. For example the DB boundary (2) requires that

$$D_\rho(a, \varphi) = 0 \Rightarrow E_\rho(a, \varphi) = 0 \tag{11}$$

$$B_\rho(a, \varphi) = 0 \Rightarrow H_\rho(a, \varphi) = 0 \tag{12}$$

For TM_z modes, (11) is automatically satisfied. Using (12) and (9), the following equation is found

$$E_z(a, \varphi) = 0 \tag{13}$$

As the field H_φ does not vanish at $\rho = a$, in conclusion, the DB boundary in a cylindrical structure equals the PEC condition at TM_z polarisation. As another property for the DB boundary, it is similar to PMC condition at TE_z polarised wave. Similarly, the D′B′ boundary appears as a PEC for a TE_z and as a PMC for a TM_z. As the normal unit vector normal to the boundary is $\hat{\rho}$ (i.e. is not $\hat{z}$), these boundaries have inverse performance in comparison with the results of Table 1 for normal incidence wave. In order to describe the effect of dual polarised cloak by DB/D′B′ boundary, the scattering width (also known as the radar cross-section (RCS)) per unit length is calculated that is taken in the following forms

$$\text{TM}_z\text{mode}: \sigma^{\text{TM}_z} = \lim_{\rho\to\infty}\left[2\pi\rho\frac{|E_z^{\text{scat}}|^2}{|E_z^{\text{inc}}|^2}\right] \tag{14}$$

$$\text{TE}_z\text{ mode}: \sigma^{\text{TE}_z} = \lim_{\rho\to\infty}\left[2\pi\rho\frac{|H_z^{\text{scat}}|^2}{|H_z^{\text{inc}}|^2}\right] \tag{15}$$

where ρ is the distance from the structure. At a large distance from the structure and using the approximation of Hankel's function, the normalised scattering width for both TM_z and TE_z polarisations is obtained as

$$\text{TM}_z\text{mode}: \sigma^{\text{TM}_z}/\lambda_0 = \frac{2}{\pi}\left|\sum_{n=-\infty}^{+\infty} a_n e^{jn(\phi-\phi_0)}\right|^2 \tag{16}$$

$$\text{TE}_z\text{mode}: \sigma^{\text{TE}_z}/\lambda_0 = \frac{2}{\pi}\left|\sum_{n=-\infty}^{+\infty} b_n e^{jn(\phi-\phi_0)}\right|^2 \tag{17}$$

where λ_0 is the wavelength in free space.

3 Numerical results and validation

As the first step, the accuracy of the analytical formulations is tested using the published literature [13]. For validating the presented formulations, total scattering width of the cloaked cylinder (σ_T) is considered. The incident plane wave is propagated in the positive x-axis ($\varphi_0 = \pi$) with a frequency of 8.5 GHz and the cloak inner and outer radii are $a = 27.1$ mm and $b = 58.9$ mm, respectively. The total scattering width of the bare PEC cylinder for both TM_z and TE_z polarisations is obtained 12.7 and 8.9 cm, as demonstrated in Figs. 2a and b, respectively. In order to compare different boundary conditions (DB or D′B′) in central

$$\text{TM}_z\text{mode}: \begin{cases} H_\rho^l = \dfrac{-1}{j\omega\mu_0\mu_\rho\rho}\dfrac{\partial E_z^l}{\partial\varphi} \\ H_\varphi^l = \dfrac{1}{j\omega\mu_0\mu_\varphi}\dfrac{\partial E_z^l}{\partial\rho} \\ E_z^l = \displaystyle\sum_{n=-\infty}^{+\infty} j^{+n}[c_n J_{m_{\text{TM}}}(u_{\text{TM}}) + d_n H_{m_{\text{TM}}}^{(2)}(u_{\text{TM}})]e^{jn(\varphi-\varphi_0)} \end{cases} \tag{9}$$

$$\text{TE}_z\text{mode}: \begin{cases} E_\rho^l = \dfrac{1}{j\omega\varepsilon_0\varepsilon_\rho\rho}\dfrac{\partial H_z^l}{\partial\varphi} \\ E_\varphi^l = \dfrac{-1}{j\omega\varepsilon_0\varepsilon_\varphi}\dfrac{\partial H_z^l}{\partial\rho} \\ H_z^l = \displaystyle\sum_{n=-\infty}^{+\infty} j^{+n}[e_n J_{m_{\text{TE}}}(u_{\text{TE}}) + f_n H_{m_{\text{TE}}}^{(2)}(u_{\text{TE}})]e^{jn(\varphi-\varphi_0)} \end{cases} \tag{10}$$

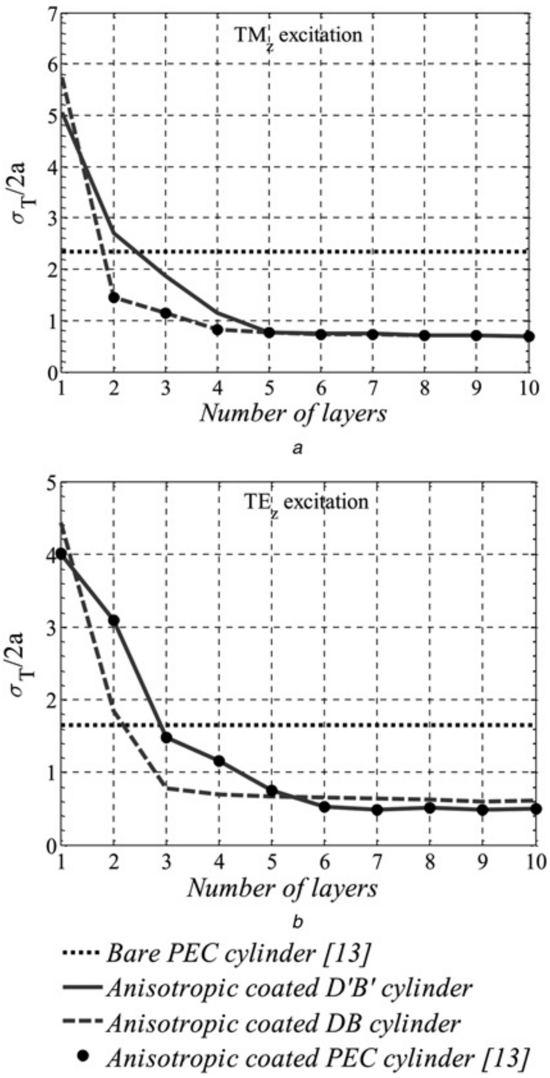

Fig. 2 *Normalised total scattering width against number of layers for*

a Cloak I
b Cloak II

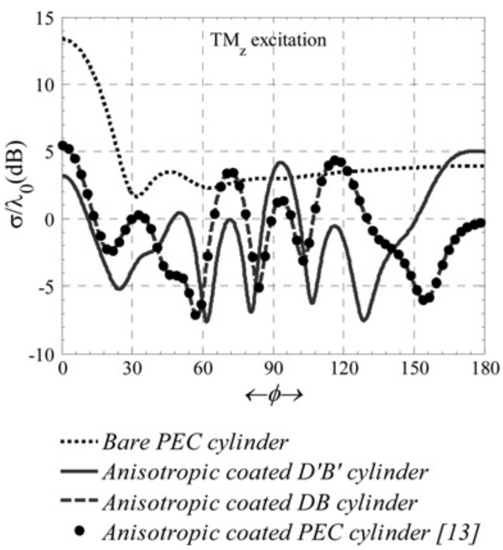

Fig. 3 *Normalised bi-static scattering width of 10-layers of Cloak I (a = 2.71 cm, b = 5.89 cm and Freq = 8.5 GHz)*

cylinder, Cloak I (Fig. 2*a*) and Cloak II (Fig. 2*b*) is selected for TM_z and TE_z excitation, respectively. In Fig. 2*a*, it is obvious that the total scattering width for multilayer anisotropic coated DB boundary has the same behaviour as PEC boundary obtained from [13]. Also, the total scattering width of the D′B′ boundary condition is given in Fig. 2*b* compared with the results that were calculated by Ivsic *et al* [13] for TE_z excitation. This comparison shows an excellent agreement and validates the presented relations and results.

In Fig. 3, normalised bi-static scattering width of the bare PEC cylinder and DB, D′B′ and PEC cylinders coated with multilayer anisotropic media is demonstrated when the cloak shell is discretised into ten layers of Cloak I. In Cloak I, only μ_ρ, μ_φ and ε_z components are relevant for the normal incidence of TM_z polarisation. Second validation of the presented relation is provided by comparing the result of normalised bi-static scattering width of DB and PEC cylinder coated with Cloak I in TM_z excitation [13]. This result can prove the validity of the presented formulations. Fig. 3 shows that the level of the forward ($\varphi = 0$) and the backward ($\varphi = 180$) scattering width can be decreased using the DB cylinder coated with the multilayer anisotropic media. Using the DB (or D′B′) boundary and Cloak I, normalised bi-static scattering width has the same behaviour in both TM_z and TE_z cases. It is interesting to notice that

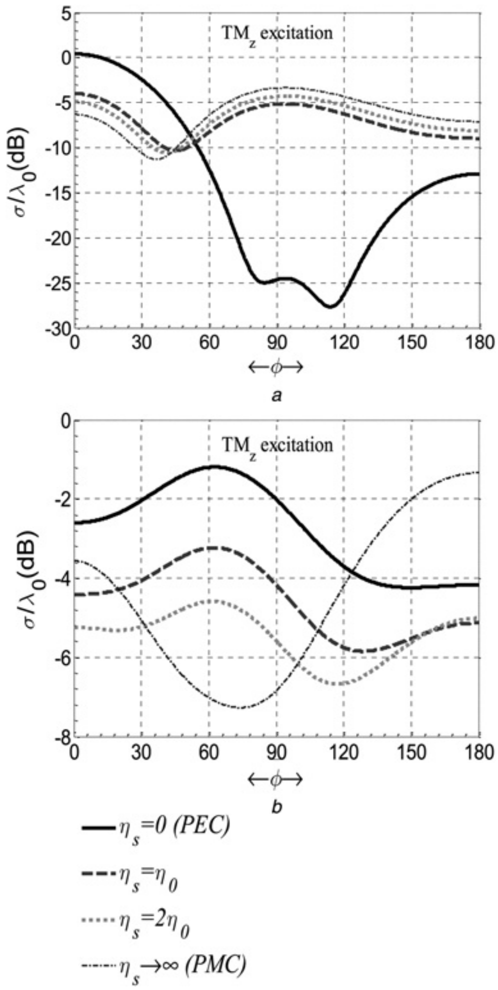

Fig. 4 *Normalised bi-static scattering width of impedance surface cylinder coated with 10-layers of*

a Cloak I
b Cloak II, *(a = 1/k*$_0$ *and b = 2/k*$_0$*)*

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the result of scattering width obtained from D′B′ boundary is not reduced in $\varphi = 180$ (backward scattering) and this boundary is not suitable for this structure.

In the second example, we presented the behaviour of the bi-static scattering width with 10-layers of Cloaks I and II. Radii of core cylinder and cylindrical shell are considered $a = 1/k_0$ and $b = 2/k_0$, respectively. First, to consider the effect of inner boundary condition, the surface impedance boundary condition is assumed inside the cloak ($\rho = a$). Surface impedance is the generalisation of both PEC and PMC. Furthermore, field components of impedance surface are described by $E_z^1(\rho = a) = \eta_s H_\varphi^1(\rho = a)$ for TM_z polarisation, where η_s denotes surface impedance [27]. As incidence of TM_z polarised plane wave, normalised bi-static scattering width of the impedance surface cylinder coated with ten layers of Cloaks I and II is presented in Figs. 4*a* and *b*, respectively. Fig. 4 shows that the scattered fields are clearly more sensitive to the boundary condition at $\rho = a$. In addition, TM_z [15], TE_z [16] and the proposed cloaks (Cloaks I and II) have sensitivity to the inner boundary condition of the multilayer cylindrical cloaks. Complementary to TM_z, for TE_z excitation, scattered fields can be obtained by a duality principle.

In Section 2, it is proved that the bi-static scattering width of the bare PEC cylinder is similar to that of the DB and D′B′ cylinders for TM_z and TE_z polarisations, respectively. Furthermore, using Cloak I (or Cloak II), the bi-static

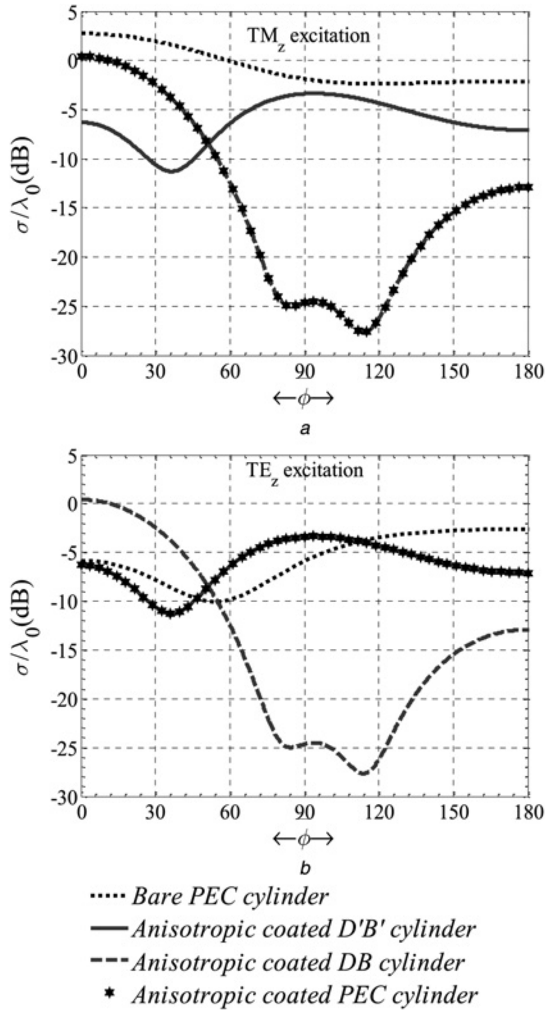

Fig. 5 *Normalised bi-static scattering width of 10-layers of Cloak I ($a = 1/k_0$ and $b = 2/k_0$)*

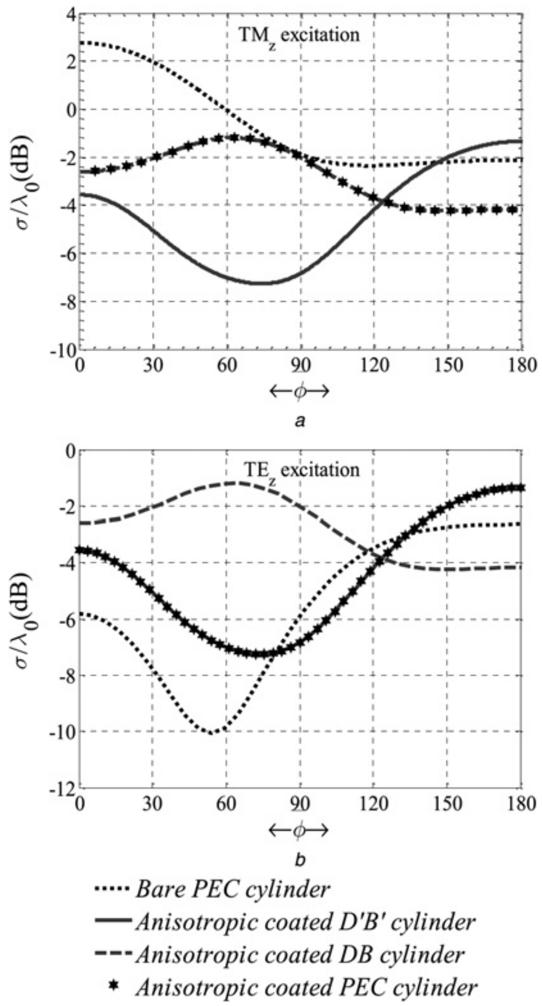

Fig. 6 *Normalised bi-static scattering width of 10-layers of Cloak II ($a = 1/k_0$ and $b = 2/k_0$)*

scattering width of the structure for DB or D′B′ boundary has the same value in both TM_z and TE_z polarisations. Using the DB (or D′B′) boundary and Cloak I (or Cloak II), the bi-static scattering width is given in TM_z polarisation as completely dual to TE_z polarisation. In other words, the structure has dual performance in TM_z and TE_z polarisations, which can be seen in Figs. 5 and 6. On the other hand, Fig. 5 shows that the level of backward scattering width for both polarisations is reduced when Cloak I and DB boundary are selected. The same result can be seen in Figs. 6*a* and *b* for Cloak II. Comparison of the obtained results in Figs. 3, 5 and 6 shows that the DB boundary and Cloak I are suitable to reduce the backward scattering width.

4 Conclusions

In this paper, solution of scattering from multilayer anisotropic cylinders used for dual polarised cloaks is analysed. First, it is demonstrated that the scattering width of the multilayer cylindrical cloaks has sensitivity on the variation of inner boundary condition. Furthermore, it is demonstrated that the ten layers of the cloak can be successfully approximated for realised inhomogeneous cloaks. The DB (D′B′ or surface impedance) boundary and Cloak I (or Cloak II) are utilised in the design process. For DB and D′B′ boundary conditions, scattering width of the structure behaved like PEC boundary condition in TM_z and TE_z polarised incident fields, respectively. In fact, for applying the duality principle in both polarisations, the boundary condition and cloak shell are chosen, while they have dual performances in TM_z and TE_z polarisations. Finally, to provide better cloaking, the numerical examples show Cloak I and DB boundary as excellent selections for reducing the backward scattering width for cylindrical structures.

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