Equivalent Circuit Model for Square Ring Slot Frequency Selective Surface

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***Abstract*- An equivalent circuit model for predicting the frequency response of a square ring slot frequency selective surface (SRS-FSS) for normal incidence is described. The proposed FSS consists of an array of square patches centered within a wire grid. The presented circuit model is formed by the impedance of the wire grid that is parallel with the impedance of the patch array, also the mutual coupling between the wire grid and the patch array is taken into account in the patch impedance. Through using the provided relations, the values of circuit elements can be calculated. The proposed relations are applied to many SRS-FSSs with different dimensions and the results are compared to those of the simulations obtained from two well-known software packages, Ansoft HFSS and CST Microwave Studio. The results show that the proposed model well approximates the frequency behavior of the SRS-FSS.**

***Index Terms*-** Circuit model, square ring slot frequency selective surface (SRS-FSS), lumped element, frequency response, mutual coupling.

I. INTRODUCTION

Among the methods for evaluating frequency selective surfaces (FSSs), the circuit approach is of great importance that has recently received much attention due to its simplicity. Equivalent circuits with appropriately chosen lumped elements can directly represent reactances and mutual coupling in simple structures and also give insight into the working principles of the structures observed in [1]-[13]. This approach can model these surfaces as a circuit including inductive and capacitive elements whose values can be calculated either by Method of Moment (MoM) as in [3]-[6] or by analytical formulas as in [2], [7] and [9]. Unlike MoM, analytical formulas can evaluate the impedance behavior of these surfaces as a function of their physical parameters; hence many researchers have tried to derive theoretical formulas that can predict the frequency behavior of these structures. The worthwhile work of Marcuvitz in analyzing an array of metallic strips [15] and also the works of Langley *et al*. in deriving relations for more complex FSSs [16]-[18] are good examples. These formulas are of great importance because they can be used to analyze more complex FSSs such as those in [2] and [7]. Among all known types of FSSs, the square loops received much attention in recent years because of their high bandwidth and small dimensions (almost one quarter wavelength at

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the resonant frequency) as shown in [6], [7] and [9]. Efficient formulas were presented for predicting the frequency response of this type of FSS [17]. Through these relations, more complex FSSs based on this type of FSS have been analyzed [7], [9]. The square ring slot FSS, which is the complementary structure of the square loop FSS, is composed of an array of square ring slots and has properties similar to that of the square loop FSS. In the recent decade, the square ring slot FSS has been used to reduce the radar cross section (RCS) of antennas as shown in [10], also it has been used as a selective surface in [8] and [9] or as a ground plane of a slot antenna to suppress its higher order modes in [14]. The square ring slot FSS has been analyzed using square loop FSS relations and the duality theorem as illustrated in [9]. However, for complex structures such as analog absorbers or cascading two or more surfaces, the duality theorem cannot be employed. In this paper, the relations for an array of metallic strips derived by Marcuvitz are extended to obtain the frequency response of the proposed FSS. The proposed theoretical formulas are applied to many square ring slot FSSs with different dimensions and the results are compared with the simulation results obtained from two different software packages, Ansoft HFSS and CST Microwave Studio. It is shown that the proposed relations give a good approximation of the frequency behavior of the proposed FSS. It can be noted that the proposed model is for FSS in free standing case. A practical structure is composed of a dielectric substrate on which the FSS is printed. The effect of dielectric substrate can also be considered to the model based on the previous works [2]-[10].

II. EQUIVALENT CIRCUIT MODEL FOR THE SQUARE RING SLOT FSS

The structure of the square ring slot FSS (SRS-FSS) is depicted in Fig. 1. As shown in this figure, the proposed FSS consists of an array of square patches centered within a wire grid. To obtain the equivalent circuit model, the impedances of the wire grid and the patch array are separately calculated. The circuit model is realized through considering parallel impedances of the two structures and taking into account the mutual coupling between the square patches and the wire grid. Fig. 2 shows the equivalent circuit model of the proposed SRS-FSS, in which L_1 is the inductance of the wire grid and L_2, C_2 represent the impedance of square patches and the mutual coupling. The transmission lines with a characteristic impedance of Z_0 represent the free space at both sides of the structure.

Parker in [17] showed that a square loop array has a frequency response similar to a series LC circuit. As discussed in this paper, the strip width determines the value of inductance while the gap between two adjacent loops determines the capacitance value of the array. The value of capacitance is increased as the gap decreased. Now, if the gap between the loops becomes negligible, the capacitive impedance will be very small and the resulting structure takes on the wire grid form shown in Fig. 1, which can be modeled by an inductor.

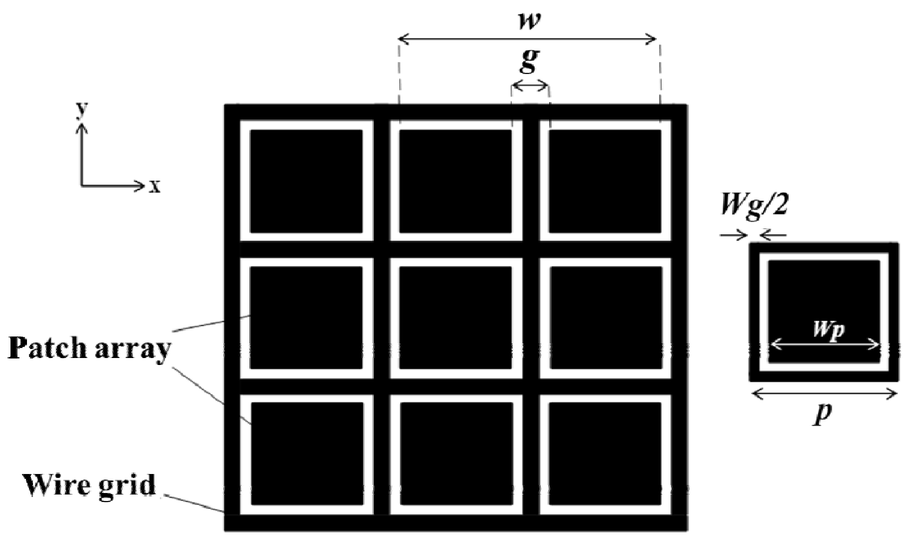


Fig. 1. Geometry of the proposed SRS-FSS and its constituting unit cell.

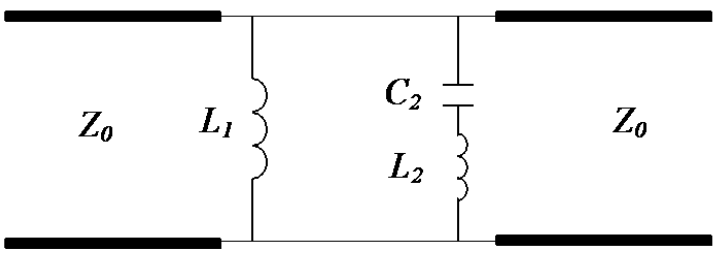


Fig. 2. Equivalent circuit model for the proposed SRS-FSS.

It can be mentioned that in this case, the loop width is equal to the loop period, so the normalized reactance of the inductive grid, X_{nL1}, can be determined using the following equation:

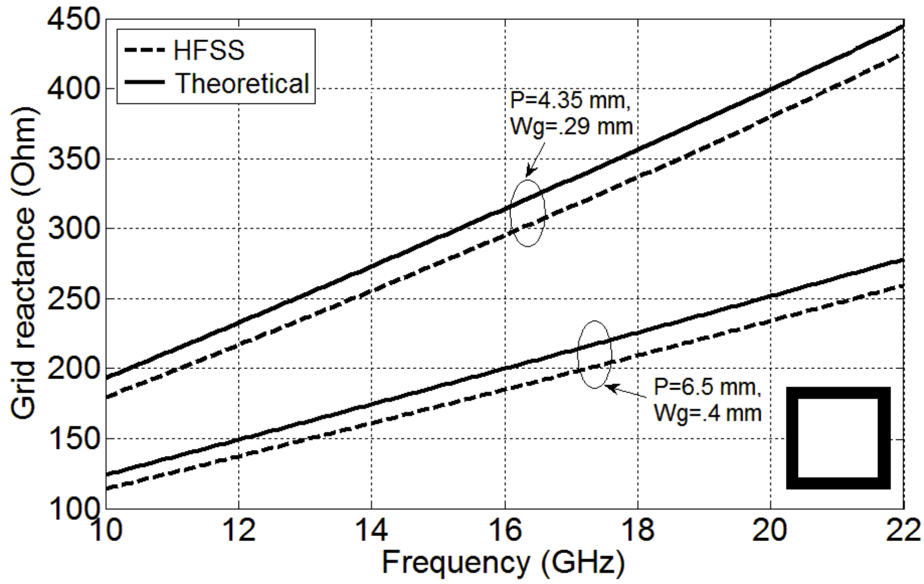
$$X_{nL1} = F\,(P,\ Wg,\ \lambda) \tag{1}$$

where Wg is the grid width, λ is the wavelength, P is the period and the function F has the same definition as presented in [17]. For normal incidence, F is

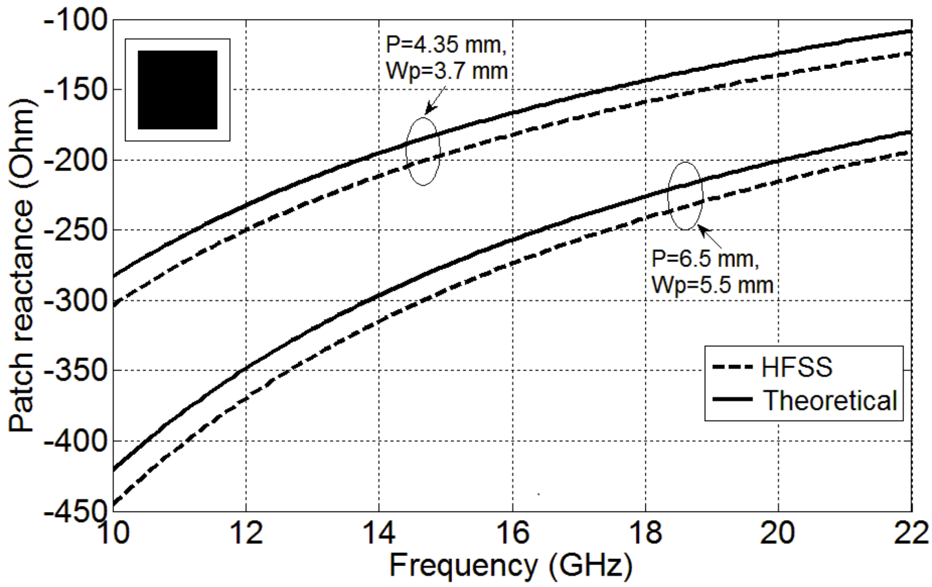
$$F\,(P,\ Wg,\ \lambda) = \frac{P}{\lambda}[\ln(\operatorname{cosec}\frac{\pi Wg}{2P}) + \frac{1}{2}\frac{(1-\beta^2)^2[2A(1-\frac{\beta^2}{4})+4\beta^2 A^2]}{(1-\frac{\beta^2}{4})+2A\beta^2(1+\frac{\beta^2}{2}-\frac{\beta^4}{8})+2\beta^6 A^2}]$$

where $A = \frac{1}{\sqrt{[1-(\frac{P}{\lambda})^2]}} - 1$ and $\beta = \frac{\sin(\pi Wg)}{2P}$

Fig. 3(a) shows the reactance of two inductive grids with different dimensions over the 10-22 GHz frequency range, which are calculated and simulated using (1) and HFSS software, respectively.



(a)



(b)

Fig. 3. Reactance of (a) the wire grid and (b) the patch array with respect to frequency.

As can be seen in Fig. 3, the theoretical and simulation results are in good agreement, indicating the proper performance of the inductive model.

To investigate the frequency behavior of the square patch array, it is assumed that the structure is normally illuminated by an x-polarized incident plane wave (see Fig. 4). In this case, every two adjacent y-directed sides of the square patches act as the plates of capacitors, whereas those along the x-axis act as inductors. It is clear that these capacitors and inductors are in series in the equivalent

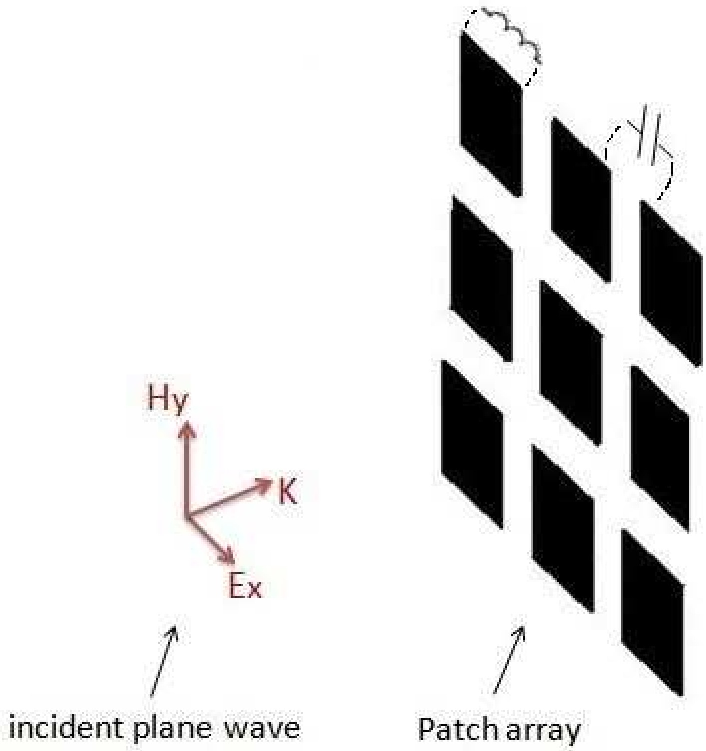


Fig. 4. Describing model for calculating the reactance of patch array.

circuit of the patches. The normalized inductive reactance, X_{nL_patch}, and capacitive susceptance, B_{nC_patch}, of the patches are calculated as follows:

$$X_{nL\_patch} = R \times F\,(P,\ Wp,\ \lambda) \tag{2}$$

$$B_{nC\_patch} = 4R \times F\,(P,\ P-Wp,\ \lambda) \tag{3}$$

where Wp is the patch width, the coefficient R can have a value from zero to unity and represents the reduction in the inductance or capacitance of the patch array compared with those of continuous strips. Our purpose is to calculate this coefficient as a function of the physical parameters. It should be noted that the inductive reactance of the patches is much smaller than their capacitive reactance, however to have a better approximation of the frequency behavior of the SRS-FSS, the inductive reactance is considered in the equivalent circuit.

Two planar strip conductors with a total width of w separated by a narrow gap, g, form a coplanar strip (CPS) transmission line whose characteristic impedance, $Z_{0,CPS}$, is determined in [19] as follows (see Fig. 5):

$$Z_{0,\,CPS} = 120\pi \frac{K(k')}{K(k)} \tag{4}$$

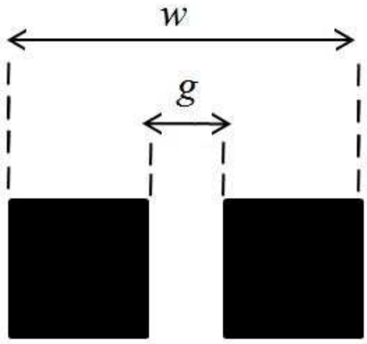


Fig. 5. Geometry of a CPS transmission line comprising two planar strip conductors.

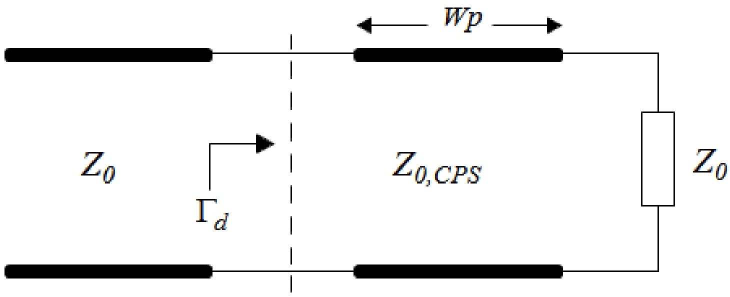


Fig. 6. The proposed model for calculating the coefficient *R*.

where K is the complete elliptical integral of the first kind, k and k' are dependent on the geometry of the CPS given in [18] as:

$$k=\sqrt{1-(\frac{g}{w})^2} \quad \text{and} \quad k'=\sqrt{1-k^2} \tag{5}$$

Now, it can be said that every two adjacent *y*-directed patches act as the previously mentioned CPS, keeping in mind that $g = (P - Wp)$ and $w = (P + Wp)$. The model for calculation of the coefficient R is shown in Fig. 6 in which the CPS transmission line embeds within a transmission line with characteristic impedance of 120π. In this case, The reflection coefficient, Γ_d, indicates the value of R. Assuming w to be a constant value, if the gap becomes negligible then the value of $Z_{0,CPS}$ becomes negligible as well. This case is equivalent to continuous conductive strips where the coefficient R equals unity. On the other hand if $K(k') = K(k)$, $Z_{0,\,CPS}$ is equal to 120π. This happens when $g = \sqrt{2}\,w$. In the latter case, the capacitive susceptance or inductive reactance is very small (due to high fragmentation of strips) and the value of R is considered to be zero (Γ_d=0). According to the previous statements, the minimum and maximum values of R are achieved in the following situations:

- The maximum value of R is achieved when $g = 0$.
- The minimum value of R is achieved when $g = \sqrt{2}\,w$.

Fig. 3(b) shows reactance of two patch arrays with different values of P and Wp using the proposed theory and HFSS software simulations. As can be seen in this figure, the theoretical formulas provide a good estimation of the impedance of the patch array.

In the previous discussions, estimating the impedances of the wire grid and the patch array has been described. As mentioned at the beginning of this section, the frequency response of the SRS-FSS is determined by considering the coupling between the patch array and the wire grid in the model. This is made possible by making two changes in the circuit model of the patch array. The first consideration is associated with the factor R and the second one is related to the gap between the patches. Since we model every two y-directed side of patches as a CPS, in (5), g and w should be replaced by $g- Wg$ and $w+ Wg$, respectively. This change is due to the presence of the wire grid, which results in a decrease in the distance between the y-directed sides of square patches (see Fig.1). The second consideration is that putting the patches within the grid causes the distance between the x-directed sides of patches to decrease which is followed by the increase in the capacitive susceptance of the patches. Therefore, in (3), the distance between the patches, which is equal to (P-Wp), should decrease by a fraction of Wg. Generally, we suppose this fraction as a function of patch width and periodicity. The findings of several simulations show that this fraction is directly proportional to the distance between the patches and inversely proportional to the width of the patches. Thus, the mentioned fraction is considered to be equal to $(P-Wp)/Wp$. Finally, according to the mentioned facts, the normalized reactance, X_{nL2}, of the inductance L_2 and susceptance, B_{nC2}, of the capacitance C_2 in the equivalent circuit model of the SRS-FSS shown in Fig. 2, are obtained as follows:

$$X_{nL2} = R' \times F\,(P,\ Wp,\ \lambda) \tag{6}$$

$$B_{nC2} = 4R' \times F\,(P,\ (P\text{-}Wp) \times (1\text{-}\ Wg/Wp),\ \lambda) \tag{7}$$

where $R' = R$ when $g = (P - Wp - Wg)$ and $w = (P + Wp + Wg)$.

It should be mentioned that the inductive impedance of X_{nL1}in the circuit model is calculated without any changes according to (1).

III. EXAMPLES AND DISCUSSIONS

In order to examine the proposed circuit model, several SRS-FSSs with different dimensions are modeled in two software packages, Ansoft HFSS and CST Microwave Studio, and their frequency responses are investigated and compared with those of the proposed LC model shown in Fig. 2. In the simulated models, the metal layers of the SRS-FSSs are defined as perfect electric conductor (PEC) with a thickness of .02 mm and are illuminated by a normal incident plane wave with a horizontal polarization. It should be mentioned that due to symmetry, the SRS-FSSs are insensitive to the incident wave polarization. Consequently, similar results were obtained for incident waves with a vertical polarization. The calculated and simulated resonant frequencies of some investigated SRS-FSSs are listed in Table I. It is notable that number 1 to number 7 SRS-FSSs in Table I are in fact the

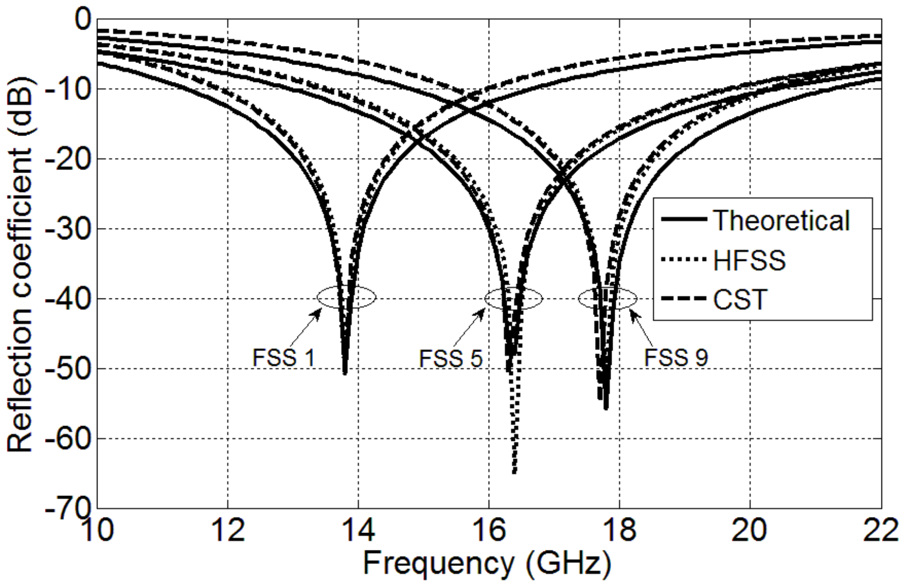


Fig. 7. Reflection coefficients of three different SRS-FSSs for normal incidence.

Table I. Theoretical and simulation results for resonant frequencies of some srs-fss

	Dimensions (mm)			**Resonant frequency (GHz)**		
SRS-FSS	**P**	**Wp**	**Wg**	**Equivalent circuit**	**Simulated by CST**	**Simulated by HFSS**
1	5.25	4.06	0.25	16.4	16.3	16.4
2	4.15	3.35	0.20	19.5	19.4	19.6
3	4.31	3.33	0.36	21.7	21.3	21.4
4	4.35	3.70	0.29	18.0	18.2	18.3
5	4.80	3.95	0.39	17.8	17.7	17.8
6	4.35	3.47	0.28	19.8	19.6	19.7
7	5.00	4.04	0.40	17.5	17.3	17.4
8	6.50	5.50	0.40	12.0	12.2	12.2
9	6.04	4.96	0.42	13.8	13.8	13.8
10	7.40	6.40	0.40	9.9	10.2	10.2

complementary patterns of the investigated arrays in [17]. As shown in Table I, the simulation results are in good agreement with the results obtained from the proposed circuit model. According to Table I, it is clear that the difference between the resonant frequencies determined by the proposed equivalent circuit model and those obtained from the simulations is about 3%. Calculated and simulated reflection coefficients for number 1, 5 and 9 SRS-FSSs in Table I are compared in Fig. 7. From Fig. 7, it can be found that the calculated -10 dB bandwidth of these FSSs are about 38%, 45% and 36% while the -10 dB bandwidth of the simulated models are about 30%, 37% and 28% for the number 1, 5 and 9 SRS-FSSs respectively, indicating good performance of the equivalent circuit model. Similar results were found for other SRS-FSSs.

IV. CONCLUSION

An equivalent circuit model is presented to predict the frequency response of square ring slot frequency selective surface (SRS-FSS) for normal incidence. The values of the circuit elements can be calculated through the presented relations. The proposed relations are applied to many SRS-FSSs with different dimensions and the results are compared to those of the simulations obtained from two software packages, Ansoft HFSS and CST Microwave Studio. Results show that the proposed model can predict the resonant frequency of SRS-FSS to within 3%. It should be noted that the presented formulas are based on Marcuvitz relations for the impedance of an array of metallic strips that can be useful for design work.

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