A New Hesitant Fuzzy Analytical Hierarchy Process Method for Decision-making Problems Under Uncertainty

S.M. Mousavi[1], H. Gitinavard [2], A. Siadat [3]

[1] Industrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran
[2] Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
[3] LCFC, Arts et Métier Paris Tech, Metz, France
(sm.mousavi@shahed.ac.ir)

Abstract – **Hesitant fuzzy set is a useful and powerful tool for dealing with uncertainty and hesitant situations. This concept helps experts or decision makers so that they specify their evaluations under a set. In this paper, we present the hesitant fuzzy sets for the analytical hierarchy process (AHP) method. For convenience, we call the proposed method as HF-AHP. In the process of proposed method, decision makers' evaluations for comparison matrices are expressed by linguistic variables and then DMs' judgments are aggregated the by utilizing the hesitant fuzzy geometric operator. Finally, the performance of the proposed HF-AHP method is illustrated for the selection of the most appropriate bridge construction problem from the recent literature. Computational results and the comparative analysis demonstrate that the presented method can be used for multi-level evaluation process to assist decision makers for better judgment in selecting the best alternative.**

Keywords - **Hesitant fuzzy set, Analytical hierarchical process, Multi-criteria decision making.**

I. INTRODUCTION

A large number of studies introduced and proposed the methodologies and theories for dealing with uncertainty situations. Among these theories, fuzzy set theory has widely used and utilized to cope with this situation in different type of problems. Since the fuzzy sets theory is introduced by Zadeh [1, 2], it is most important concept that researchers utilized for uncertain and vagueness situations. In this respect, the classical fuzzy set theory is developed and largely used in multi-criteria decision making (MCDM) problems [e.g. 3-6]. One of the useful extensions is intuitionistic fuzzy set (IFS) that is defined by three indexes as included membership degree, non-membership degree and hesitancy degree. In this regard, Qin & Liu [7] based on Choquet integral proposed a method for solving the interval intuitionistic fuzzy multiple attribute group decision making problems. Xu [8-10] has useful studies in this field and proposed a series of IFSs aggregation operators and applied them successfully for solving the MCDM problems.

One of the efficient extension of fuzzy set theory is hesitant fuzzy sets (HFSs) that have been first introduced by Torra [11] and Torra & Narukawa [12]. HFSs are utilized when membership degrees of an element should be expressed as a set. For this reason, decision makers (DMs) could manage the hesitant situation by assigning their opinions under a set. For widely used the HFS, Rodríguez et al. [13] presented an overview for preparing the viewpoint on different concepts, tools and the extension of the new fuzzy set.

The above-mentioned theories and other extension of fuzzy theory are applied through decision making tools for solving the MCDM problems. Yue [14] utilized the IFS to develop the TOPSIS method. Vahdani et al. [15] proposed a modified TOPSIS method based on the interval-valued fuzzy set. Hashemi [16] proposed a compromise ratio method based on the IFS to solve the MCDM problems. Also, some researchers focused on the HFSs to develop the decision making tools for solving the decision masking problems in different fields [17-19]. Based on the above-mentioned explanations, HFSs is a powerful tool to deal with vague situations. Thus, in this paper, the DMs' judgments are described by the HFSs and we plan to apply the HFSs within the AHP method for solving the MCDM problems under uncertainty.

The structure of this paper is represented as follows; in section II, we express the concept of the HFS and some operations in this area. In section III, the process of the proposed HF-AHP method is defined. In section IV, we present a numerical example and in section V, the proposed method is applied to numerical example to show the verification of the proposed method. Finally, in section VI, some conclusions are considered to end our paper.

II. PRELIMINARY

In this section, we express some definitions about HFSs concept and also describe some operations.

Definition 1 [20]: Let X be a universe set, then HFS is defined as E on X in terms of a function $h_E(x)$ that is applied to X returns under [0, 1]. Also, we explain the HFS by a mathematical symbol:

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$$E = \{< x, h_E(\mathrm{x}) >| \mathrm{x} \in \mathrm{X}\} \quad (1)$$

where $h_E(\mathrm{x})$ is expressing some possible membership degrees for an element, in [0,1].

Definition 2 [11, 12]: Some basic operators are defined as follows; let *h, h₁*, and *h₂* be HFSs:

$$h^-(x) = min\ h(x) \quad (2)$$

$$h^+(x) = max\ h(x) \quad (3)$$

$$h_\alpha^+(\mathrm{x}) = \{h \in h(\mathrm{x}) \mid \mathrm{h} \geq \alpha\} \quad (4)$$

$$h_\alpha^-(\mathrm{x}) = \{h \in h(\mathrm{x}) \mid \mathrm{h} \leq \alpha\} \quad (5)$$

$$h^c(\mathrm{x}) = \cup_{\gamma \in h(\mathrm{x})} \{1-\gamma\} \quad (6)$$

$$(\mathrm{h}_1 \cup \mathrm{h}_2)(\mathrm{x}) = \{h \in (\mathrm{h}_1(\mathrm{x}) \cup \mathrm{h}_2(\mathrm{x})) \mid \mathrm{h} \geq \max(\mathrm{h}_1^-, \mathrm{h}_2^-)\} \quad (7)$$

$$(\mathrm{h}_1 \cap \mathrm{h}_2)(\mathrm{x}) = \{h \in (\mathrm{h}_1(\mathrm{x}) \cap \mathrm{h}_2(\mathrm{x})) \mid \mathrm{h} \leq \min(\mathrm{h}_1^+, \mathrm{h}_2^+)\} \quad (8)$$

Definition 3 [11, 12]*:* All IFSs are HFSs. Let the IFS denoted by $\{< x, \mu_E(x), \upsilon_E(x) >\}$, then the HFS could be obtained by the following operations:

$$h(x) = [\mu_E(x), 1-\upsilon_E(x)] \ \ if \ \ \mu_E(x) \neq 1-\upsilon_E(x) \quad (9)$$

Definition 4 [20]*:* Some operators are defined under the HFSs by regarding the relationship between the HFE and IFV:

$$\tilde{h}_1 \oplus \tilde{h}_2 = \cup_{\gamma_1 \in \tilde{h}_1, \gamma_2 \in \tilde{h}_2} \{\gamma_1 + \gamma_2 - \gamma_1.\gamma_2\} \quad (10)$$

$$\tilde{h}_1 \otimes \tilde{h}_2 = \cup_{\gamma_1 \in \tilde{h}_1, \gamma_2 \in \tilde{h}_2} \{\gamma_1.\gamma_2\} \quad (11)$$

$$h^\lambda = \cup_{\gamma \in h} \{\gamma^\lambda\} \quad (12)$$

$$\lambda h = \cup_{\gamma \in h} \{1-(1-\gamma)^\lambda\} \quad (13)$$

Definition 5 [20]*:* Defining some aggregation operators based on the relationship between IFS and HFSs. In this regard, the hesitant fuzzy geometric (HFG) is expressed as follows:

$$HFG(h_1, h_2, ..., h_n) = \overset{n}{\underset{i=1}{\otimes}} h_i^{\frac{1}{n}} = \cup_{\gamma_1 \in \mathrm{h}_1, \gamma_2 \in \mathrm{h}_2, ..., \gamma_n \in \mathrm{h}_n} \left\{ \prod_{i=1}^n \gamma_i^{\frac{1}{n}} \right\} \quad (14)$$

where the $h_j\ (j = 1, 2, \ldots, n)$ is a collection of HFSs and the weight vector of them is denoted by $w = (1/n, 1/\mathrm{n}, ..., 1/n)^T$.

III. METHODOLOGY

Step 1. According to DMs' judgments, we construct the hesitant fuzzy comparison matrix for criteria, sub-criteria and indicators respectively. On each level, the sub-criteria of each criterion must be compared together. Also, we establish a table similar to Table I.

TABLE I
THE SAMPLE HESITANT FUZZY LINGUISTIC COMPARISON MATRIX FOR EACH PAIR OF CRITERIA

	C_1^K	C_2^K	…	C_n^K
C_1^K	1	{UI,LI,…,EI}	…	{VU,EI,…,MI}
C_2^K	{VI,MI,…,EI}	1	…	{EI,VU,…,LI}
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
C_n^K	{VI,EI,…,LI}	{EI,VI,…,MI}	…	1

Step 2. Construct the hesitant fuzzy comparison matrix for alternatives respecting to each criterion by regarding the opinions of the DMs. This matrix is shown in Table II.

TABLE II
THE SAMPLE HESITANT FUZZY LINGUISTIC COMPARISON MATRIX FOR EACH PAIR OF ALTERNATIVES REGARDING TO EACH CRITERION

	For criterion C_j			
	A_1	A_2	…	A_m
A_1	1	{UI,LI,…,EI}	…	{VU,EI,…,MI}
A_2	{VI,MI,…,EI}	1	…	{EI,VU,…,LI}
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
A_m	{VI,EI,…,LI}	{EI,VI,…,MI}	…	1

Step 3. Aggregate the DMs' opinions for establishing the aggregated hesitant fuzzy comparison matrix. In this respect, we utilize the HFG operator as mention on definition 5.

$$HFG(h_1, h_2, ..., h_n) = \overset{n}{\underset{i=1}{\otimes}} h_i^{\frac{1}{n}} = \cup_{\gamma_1 \in \mathrm{h}_1, \gamma_2 \in \mathrm{h}_2, ..., \gamma_n \in \mathrm{h}_n} \left\{ \prod_{i=1}^n \gamma_i^{\frac{1}{n}} \right\} \quad (15)$$

Step 4. Determine the weight of *i*th criterion, *j*th sub-criterion, *r*th indicator and *a*th alternative with respect to *k*th level by hesitant fuzzy geometric mean operation as below:

$$\omega_{C_i} = \sqrt[n]{\mu_{i1} \otimes \mu_{i2} \otimes ... \otimes \mu_{in}} \quad (16)$$

$$\omega_{SC_j} = \sqrt[m]{\mu_{j1} \otimes \mu_{j2} \otimes ... \otimes \mu_{jm}} \quad (17)$$

$$\omega_{In_r} = \sqrt[b]{\mu_{r1} \otimes \mu_{r2} \otimes ... \otimes \mu_{rb}} \quad (18)$$

$$\omega_{A_a} = \sqrt[h]{\mu_{a1} \otimes \mu_{a2} \otimes \ldots \otimes \mu_{ah}} \tag{19}$$

Step 5. Compute the final weight of each criterion if between each level we have a relation.

$$w_{C_i} = \sum_{a}^{h} \sum_{r}^{b} \sum_{j}^{m} \omega_{C_i}^{K} \omega_{SC_j}^{K} \omega_{In_r}^{K} \tag{20}$$

Step 6. Compute the score of each alternative with respect to each weight of criterion.

$$S_{A_a} = \sum_{i} w_{C_i} w_{A_a, C_i} \tag{21}$$

Step 7. Rank the possible alternatives by decreasing sorting of scores.

IV. APPLICATION EXAMPLE

In this section, a practical example is utilized to represent the efficiency of the proposed HF-AHP method. The numerical example is adopted from Pan [17] that is utilized in selecting the best bridge construction method. Also, a group decision making is formed (eight DMs). They assign their judgments about comparison matrices. For this problem, five criteria (C_j) and three alternatives (A_i) are considered. In addition, the hierarchy of the application example is indicated in Fig 1.

C_1: Quality;
C_2: Cost;
C_3: Safety;
C_4: Duration;
C_5: Shape.

and

A_1: Full-span pre-cast and launching method;
A_2: Advancing shoring method;
A_3: Balanced cantilever method.

In this paper, DMs' opinions are stated by linguistic variables and also their importance is expressed in Table III. Also, the risk preferences of the DMs have been considered in the process of the proposed HF-AHP method. As shown in Table I, if the risk preferences of the DM are pessimist, then the lower bound of hesitant interval-valued fuzzy element (HIVFE) is selected and if the DM is moderate the average between lower and upper bound of HIVFE is chosen. Also if the DM is optimist the upper bound of HIVFE is selected. The above-mentioned explanations are represented in Table III.

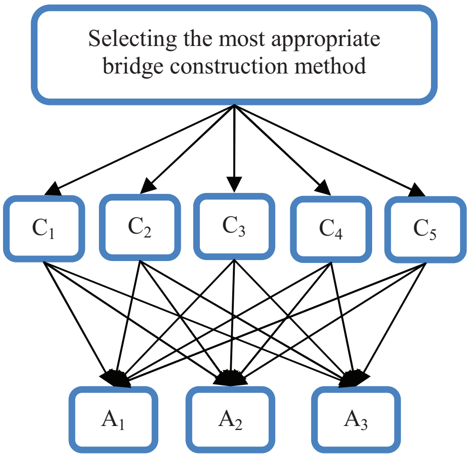

Fig. 1. Hierarchical structure of the problem.

TABLE III
HESITANT LINGUISTIC TERMS FOR RATING THE IMPORTANCE OF CRITERIA

Hesitant linguistic variable	Hesitant interval-valued fuzzy	DMs' risk preferences: Pessimist	Moderate	Optimist
Very important (VI)	[0.40,0.50]	0.40	0.45	0.50
More important (MI)	[0.30,0.40]	0.30	0.35	0.40
Equally important (EI)	[0.20,0.30]	0.20	0.25	0.30
Less important (LI)	[0.10,0.20]	0.10	0.15	0.20
Very unimportant (VU)	[0.10,0.10]	0.10	0.10	0.10

V. RESULTS

By utilizing steps 1 and 2, the DMs assign their opinions for choosing the appropriate construction method of the bridge. In this respect, the DMs' evaluations have been considered for establishing the hesitant fuzzy linguistic comparison matrix for each pair of criteria and also constructing the hesitant fuzzy linguistic comparison matrix for each pair of alternatives regarding to each criterion. The results are shown in Tables IV and V, respectively.

In addition by applying steps 3 to 5, the aggregated hesitant fuzzy comparison matrix is constructed for each pair of criterion and the final weight of criterion is computed and the results are represented in Table VI. Also, computing the final weight of hesitant fuzzy comparison matrix for each pair of alternatives respecting to each criteria is similar to the calculation process of the above-statement. Computational results are expressed in Table VII.

TABLE IV
HESITANT FUZZY LINGUISTIC COMPARISON MATRIX FOR EACH PAIR OF CRITERIA

Pair wise attributes	DMs' judgments							
	1st DM	2nd DM	3rd DM	4th DM	5th DM	6th DM	7th DM	8th DM
C_1 vs. C_2	VI	MI	VI	MI	VI	EI	EI	MI
C_1 vs. C_3	MI	EI	EI	LI	EI	EI	EI	EI
C_1 vs. C_4	VI	MI	VI	MI	EI	EI	MI	EI
C_1 vs. C_5	VI	MI	MI	VI	MI	MI	MI	EI
C_2 vs. C_3	LI	LI	LI	VU	EI	EI	EI	LI
C_2 vs. C_4	MI	EI	EI	EI	EI	EI	EI	EI
C_2 vs. C_5	LI	MI	MI	EI	MI	MI	EI	LI
C_3 vs. C_4	VI	MI	MI	VI	EI	EI	MI	MI
C_3 vs. C_5	VI	MI	VI	VI	MI	VI	VI	EI
C_4 vs. C_5	LI	EI	EI	EI	MI	MI	MI	EI

TABLE V
HESITANT FUZZY LINGUISTIC COMPARISON MATRIX FOR EACH PAIR OF ALTERNATIVES REGARDING TO EACH CRITERION

Pair wise attributes	DMs' judgments							
	1st DM	2nd DM	3rd DM	4th DM	5th DM	6th DM	7th DM	8th DM
C_1*1	LI	LI	EI	LI	EI	VU	EI	EI
C_1*2	MI	EI	LI	MI	EI	VI	EI	EI
C_1*3	MI	MI	EI	MI	EI	MI	EI	EI
C_2*1	VU	EI	LI	LI	EI	LI	LI	LI
C_2*2	LI	LI	EI	MI	EI	LI	EI	LI
C_2*3	MI	MI	MI	MI	EI	MI	VI	VI
C_3*1	LI	EI	EI	EI	EI	EI	MI	MI
C_3*2	MI	EI	LI	EI	EI	EI	MI	MI
C_3*3	VI	EI	MI	EI	EI	MI	EI	MI
C_4*1	EI	LI	EI	EI	LI	LI	EI	LI
C_4*2	EI	MI	EI	EI	EI	EI	EI	EI
C_4*3	EI	MI	EI	EI	EI	MI	MI	EI
C_5*1	LI	MI	EI	LI	EI	EI	MI	EI
C_5*2	VU	EI	LI	VU	EI	EI	EI	LI
C_5*3	VU	EI	LI	LI	EI	EI	EI	LI

Note: *1 denotes the relative importance of A_1, when it compared to A_2 regarding to each criteria.
*2 denotes the relative importance of A_1, when it compared to A_3 regarding to each criteria.
*3 denotes the relative importance of A_2, when it compared to A_3 regarding to each criteria.

TABLE VI
THE AGGREGATED DMS' JUDGMENTS FOR CONSTRUCTING THE HESITANT FUZZY COMPARISON MATRIX OF CRITERIA

	C_1	C_2	C_3	C_4	C_5	Final weight
C_1	1	0.34572	0.24298	0.332999	0.35838	0.39831
C_2	0.15882	1	0.16093	0.259002	0.25677	0.27931
C_3	0.23546	0.31978	1	0.343626	0.39282	0.39940
C_4	0.16931	0.22576	0.15298	1	0.25302	0.27166
C_5	0.14351	0.20133	0.12068	0.21955	1	0.23812

According to steps 6 and 7, the score of alternatives regarding to the criteria' final weights is computed and also the alternatives is sorted by decreasing value. In this paper, the proposed HF-AHP method has been compared with Pan [17]. The above-statement results are indicated in Table VIII.

TABLE VII
FINAL WEIGHT OF HESITANT FUZZY COMPARISON MATRIX FOR EACH PAIR OF ALTERNATIVES REGARDING TO EACH CRITERION

	C_1	C_2	C_3	C_4	C_5
A_1	0.354904	0.400045	0.395611	0.367342	0.339423
A_2	0.446858	0.48705	0.414142	0.432295	0.35162
A_3	0.32976	0.328146	0.342416	0.358455	0.458238

TABLE VIII
THE SCORE OF EACH ALTERNATIVES AND COMPARATIVE ANALYSIS

	Score of A_i	Ranking by the proposed method	Ranking by Pan [21] method
A_1	0.59172	2	2
A_2	0.68061	1	1
A_3	0.56626	3	3

As it is clear in Table VIII, the ranking of the proposed HF-AHP amethod is similar to the ranking of Pan [17] method. It shows that the proposed HF-AHP method can cope with hesitant situations properly. The capability of the proposed method is illustrated in complex situations, in which the DM for margin of error could assign their preferences by some membership degrees under a set for an element.

VI. CONCLUSIONS

This paper has presented a new AHP method under uncertainty that is based on the HFSs. Firstly a group of the DMs has been formed to evaluate the relative importance of each criterion compared to other one. Also, this process has been done for comparing each pair of alternatives regarding to each criterion. Then, the DMs' opinions have been aggregated by utilizing the hesitant fuzzy geometric operator and also the final weights have been obtained from the comparison matrices. Finally, the score of each alternative has been computed by regarding to the final weights. The comparison analysis has shown that the ranking of the proposed HF-AHP method is similar to the ranking of Pan [17] method. For this respect, we have concluded that the proposed HF-AHP has been verified and it could be applied in large-scale and in multi-level problems. For future work, we plan to consider the interval-valued hesitant fuzzy set (IVHFS) for extending the AHP method under uncertainty. In addition, the extension method could be provided by using the hesitant fuzzy sets along with stochastic approach.

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