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A Case Study on Monitoring Polynomial Profiles in the Automotive Industry

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In some statistical process control applications, the quality of a process or product can be characterized by a relationship between a response variable and one explanatory variable, which is referred to as profile. We give an example here of a profile that can be described using a polynomial model. This example comes from the automotive industry, where one of the most important quality characteristics of an automobile engine is the relationship between the torque produced by an engine and the engine speed in revolutions per minute. We find for this data set that a second-order polynomial works well. In addition, we show that there is autocorrelation within each profile, thus an ordinary least-square method that ignores the autocorrelation is inappropriate. We propose a linear mixed model method as an alternative approach. After the reduction of the data to a series of parameter estimates, we then conduct a step-by-step Phase I analysis of the polynomial profiles monitoring using a T^2-based procedure to check the stability of the process and whether or not there are outlying profiles. The remaining profiles are used to form the estimated mean vector and variance–covariance matrix to be used in Phase II studies. Finally, a brief discussion is presented to show how one can use these parameters in Phase II. Copyright © 2009 John Wiley & Sons, Ltd.

Keywords: automotive industry; linear mixed model; minimum volume ellipsoid (MVE); successive differences; T^2 statistics

1. Introduction

Sometimes, quality of a process or product is characterized by a relationship between a response variable and one or more explanatory variables, which is referred to as profile. We consider here profiles obtained from the automotive industry. One of the most important quality characteristics of an automobile engine is the relationship between the torque produced by an engine and the engine speed in revolutions per minute (RPM). For this particular engine type (TU3, assembled in the Peugeot automobile), the engine is run at different RPM values and the corresponding torque values are obtained. In other words, the torque produced by each engine is considered as a response variable and the correspondent speed values are considered as an explanatory variable. If the manufacturing process is in control, the profiles that describe the relationship between RPM and torque should be similar. An engine with mechanical defects or other issues will yield a profile that is different from the good engines. It is desirable to implement a procedure to monitor the quality level of the engines. Because there are multiple RPM values obtained for each engine it is natural to try to implement a multivariate quality control procedure to detect engines that are not acceptable. However, if the number of RPM values is large, it will be better to fit a parametric model to describe the relationship and monitor the estimated parameters in place of the actual torque values.

In Phase I of a control chart scheme, we have a historical data set at our disposal from which we are trying to determine if the profiles form a stable set of similar profiles with no outlying profiles. Once we are satisfied that the data set contains no outlying profiles, we can estimate the process parameters that are then fixed for Phase II of the control chart scheme. In Phase II, we are monitoring the process and are interested in detecting shifts as quickly as possible for real-time profiles that we continue to collect. Thus, our goal in Phase II is to know when the process has shifted from its historical, stable state so that we can quickly react to those changes and prevent problems.

A good introduction and literature review on profile monitoring can be found in Woodall *et al.*[1]. A more recent review can be found in Woodall[2]. A wide variety of literature has appeared in recent years indicating a growing popularity in the approach. Several applications of profiles monitoring have been reported by authors such as Kang and Albin[3], Mahmoud and Woodall[4], Woodall *et al.*[1], Wang and Tsung[5], Montgomery[6], Woodall[2], Zou *et al.*[7], and Williams *et al.*[8] Simple linear profiles for Phase I and

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Phase II applications were addressed by Mestek *et al.*[9], Stover and Brill[10], Kang and Albin[3], Kim *et al.*[11], Mahmoud and Woodall[4], Noorossana *et al.*[12], Wang and Tsung[5], Gupta *et al.*[13], Zou *et al.*[7,14], Mahmoud *et al.*[15], Zhang *et al.*[16], and Saghaei *et al.*[17].

There have been several recent papers dealing with extensions to profiles that can be modeled by a general linear model. Kazemzadeh *et al.*[18] transformed a polynomial regression model to an orthogonal polynomial regression model and proposed a method based on using EWMA control charts to monitor the parameters of orthogonal polynomial regression model in Phase II. Kazemzadeh *et al.*[19] proposed three Phase I methods for monitoring a kth-order polynomial profiles. They also compared the performance of their methods under both step and isolated shifts in the regression parameters. Mahmoud[20] proposed a Phase I methods for multiple linear profiles and compared the F-statistic method of Mahmoud and Woodall[4] with a T^2-based control chart in Phase I.

In all the previous references, it is assumed that the error terms in the linear model are independently and identically distributed normal random variables. However, in certain cases these assumptions can be violated. Noorossana *et al.*[21] investigated the effect of non-normality of the error terms on the performances of the EWMA/R method by Kang and Albin[3]. Jensen *et al.*[22] proposed a linear mixed model (LMM) to account for the autocorrelation within a linear profile in Phase I. Jensen and Birch[23] showed that the use of mixed models could have significant advantages when there is autocorrelation within nonlinear regression models. Soleimani *et al.*[24] proposed a transformation to eliminate the AR(1) structure between observations within a simple linear profiles and then used the traditional control procedures in the literature to monitor the profiles in Phase II. Noorossana *et al.*[25] and Kazemzadeh *et al.*[26] considered the linear and polynomial profiles over time and modeled autocorrelation between profiles as a first-order autoregressive process, respectively. In other words, it is assumed that the time is short between profiles, thus the corresponding observations in two successive profiles are autocorrelated. They proposed some methods based on a time-series approach and evaluated the performance of their methods.

In this case study we focus on a Phase I application of profile monitoring and show the step-by-step analysis we performed on the data. The structure of the paper is as follows: In the next section, it is shown that the relationship between the torque and the RPM of an engine can be modeled by a second-order polynomial profile. In addition, we discuss that there is an autocorrelation structure between residuals within a profile and as a result using a standard approach is inefficient. An alternative approach for modeling this relationship that can take into account the autocorrelation within a profile is discussed in Section 3. In Section 4, using the alternative approach, a Phase I study is conducted to check the stability of the process and determine if there are any outlying profiles. Then, the mean vector and variance–covariance matrix of regression parameters are estimated by using the observations from a stable process. Our concluding remarks are given in the final section.

2. Model and structure of the process

In the literature of profile monitoring in Phase I, it is assumed that there are m samples of size n_j in the form of $\{(x_{ij}, y_{ij}), i=1,2,\ldots,n_j, j=1,2,\ldots,m\}$. In other words, the subscript i represents the ith observation within each profile and subscript j shows the jth profile collected over time. One approach in the literature is referred to as a parametric approach, where one estimates the regression parameters after fitting an adequate parametric model and then uses the vector of regression parameters in a suitable statistic. This approach of fitting parametric regression models to the profiles is used throughout the paper.

For the engine data, we have a total of 26 engines at our disposal for Phase I analysis. In each engine, the speed values are set equal to 1500, 2000, 2500, 2660, 2800, 2940, 3500, 4000, 4500, 5000, 5225, 5500, 5775, and 6000 RPM and its corresponding torque values are measured. Hence, 14 points are gathered for each engine and form the profile of interest. The data set for 26 engines was collected and is shown in Appendix A. As with many situations, it is important to ensure that the fitted model is appropriate for the data at hand. A good modeler will try a variety of potential models, trying to keep the models as simple as possible while still explaining a significant percentage of the variability in the data. In our situation, a quadratic polynomial turns out to relatively simple model that explains the data well. That is not to say that an alternative model could not be fitted which would explain more of the variability of the data. Thus, for each profile we fit the following model as:

$$y_j = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \varepsilon_j \tag{1}$$

where y_j is the vector containing the torque values, x_j is the vector containing the RPM values, and ε_j is the vector containing the errors. Because the RPM values are the same for all the profiles we have $x_j = x$ for all values of j. The parameters β_0, β_1, and β_2 are estimated for each profile. After fitting this model, we obtain high values of the adjusted coefficient of determination for all engines. The scatterplot for one engine (Engine number 329) is illustrated in Figure 1 as an example. As is shown in Figure 1, the speed values are mean corrected to decrease the effect of multicollinearity. In other words, we replace x_j with $x_j - \bar{x}$ in the above equation. The adjusted coefficient of determination (R^2 adjusted) is equal to 96.3% and the model fits well. Similar results (not reported here) were obtained for the other 25 investigated engines.

The estimates of the regression parameters, the variance inflation factors (VIF), and analysis of variance table for engine number 329 are shown in Table I.

Based on the analysis of variance as well as separate individual tests in Table I, the significance of the regression parameters is confirmed. The VIFs show that there is no multicollinearity between explanatory variables. The other engines were evaluated using the above procedure and the adjusted coefficients of determination and estimates of regression parameters are summarized in Table II.

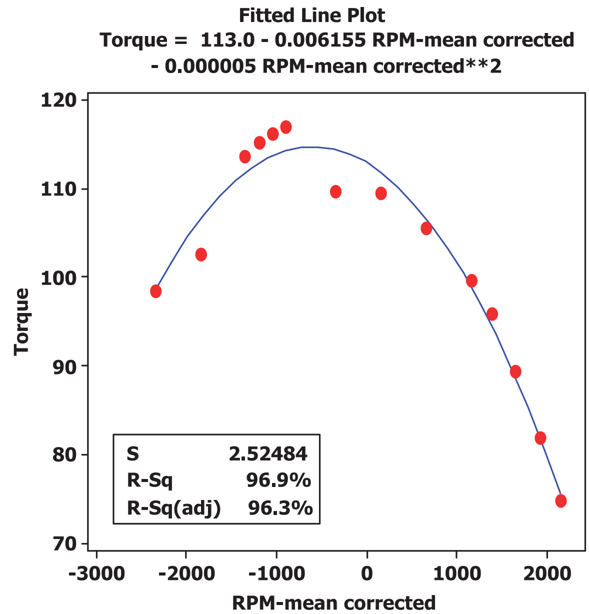

Figure 1. A second-order polynomial profile fit for engine number 329. This figure is available in colour online at www.interscience.wiley.com/journal/qre

Table I. Regression analysis results for Engine 329 for the model fit from Equation (1) with torque as the response and RPM as the explanatory variable

Predictor	Coef	T	P	VIF	
Constant	112.992	104.02	0.00		
RPM	−0.0061551	−13.11	0.00	1.0	
RPM^2	−0.00000522	−12.67	0.00	1.0	
Analysis of variance					
Source	DF	SS	MS	F	P
Regression	2	2156.2	1078.1	169.12	0.00
Residual	11	70.1	6.4		
Total	13	2226.3			

The independence of residuals over time for each engine is one of the assumptions of regression analysis when one uses ordinary least square (OLS) method. We used a run chart to check this assumption. Run charts for standardized residuals of four engines are depicted in Figure 2. The results show that the clustering and trend hypothesis tests are significant and as a result it can be concluded that the residuals are correlated. Similar results are obtained for the other engines (the results are not reported here).

Model adequacy checking showed that we are faced with a case in which there is autocorrelation between residuals and as a result between observations in each profile. The least-square method is not recommended to estimate the regression parameters in this case because it ignores the presence of autocorrelation. As a result we propose an alternative modeling approach to the standard approach to address the limitations of the OLS approach.

3. An alternative approach

The proposed approach that we consider here will account for the autocorrelation within profiles. Note that this approach is simply a method for reducing the dimensionality of the data set prior to using a control chart to determine outlying profiles. The approach is an LMM approach, as was proposed by Jensen *et al.*[22]. A good introduction to the LMM can be found in Verbeke and Molenberghs[27]. An LMM treats the profiles as a random sample of profiles and allows for the inclusion of autocorrelation within a profile. The LMM is given as $y_j = X\beta + Zb_j + \varepsilon_j$, where y_j is the response vector for the jth profile, X is the observation matrix, β is the vector of fixed parameters, Z is the matrix associated with the random effects, and b_j is the vector of the random effects, which needs to be obtained from the data. The random effects indicate the deviation in the fixed parameters (β) for a particular profile. We assume that the random effects are multivariate normally distributed, that is $b_j \sim \mathrm{MN}(0, D)$, where 0 is a zero vector and D is a positive-definite variance–covariance matrix with all off-diagonal elements equal to 0. In other words, the random effects are uncorrelated with each other. To allow for the correlated errors, we assume that $\varepsilon_j \sim \mathrm{MN}(0, \Sigma)$, where Σ is a

Table II. Results from the OLS analysis of the 26 engine profiles including adjusted coefficient of determination and estimates of regression parameters

Engine No.	R^2_{adj}%	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
329	96.3	112.99	−0.0061	−0.0000052
449	96.3	110.95	−0.0058	−0.0000050
529	97.8	112.13	−0.0052	−0.0000047
642	95.8	108.95	−0.0056	−0.0000045
724	96.8	107.82	−0.0060	−0.0000045
803	97.8	110.98	−0.0067	−0.0000052
930	97.5	111.14	−0.0061	−0.0000047
1148	96.7	112.45	−0.0061	−0.0000055
1171	96.3	111.09	−0.0058	−0.0000051
1516	97.8	111.66	−0.0067	−0.0000050
1791	98.5	111.97	−0.0071	−0.0000049
2600	96.8	113.28	−0.0061	−0.0000053
3100	98.0	110.96	−0.0061	−0.0000046
3720	97.5	109.92	−0.0051	−0.0000045
4025	98.2	110.27	−0.0064	−0.0000047
4068	97.2	112.60	−0.0070	−0.0000054
4926	96.4	114.70	−0.0061	−0.0000054
5155	98.4	111.96	−0.0058	−0.0000046
6143	94.8	110.04	−0.0053	−0.0000050
6844	97.1	111.46	−0.0057	−0.0000053
7811	97.6	109.75	−0.0060	−0.0000050
8007	96.3	109.87	−0.0061	−0.0000047
8623	96.6	110.89	−0.0057	−0.0000047
9388	97.6	113.31	−0.0057	−0.0000052
9404	95.3	109.43	−0.0056	−0.0000052
10430	97.2	112.16	−0.0057	−0.0000050

positive-definite matrix. In addition, we assume that the random effects and errors are independent of each other, that is Cov(b_j, ε_j)$=0$. It is common to allow Σ to have some structure (AR(1) and compound symmetry for example) that reduces the number of parameters that need to be estimated in the variance–covariance matrix. The choice of which structure to use will depend on the data and we will use Draftman's display as discussed by Jensen *et al.*[22] to obtain the appropriate structure for this data set.

To implement the LMM approach on the 26 profiles, we first determine which of the regression parameters need a random effect. From the estimated fixed effects shown in Table II from the OLS approach, we computed the sample standard deviation for the three regression parameters (intercept, linear, and quadratic term), which are shown in Table III.

The large standard deviation for the intercept indicates that just one random effect may be needed. However, initial attempts to fit an LMM on these data with just one random effect for the intercept were unsuccessful because of lack of convergence. This is because the intercept estimates are many orders of magnitude larger than the other terms. Thus, we had to scale the RPM values so that the estimates are of similar magnitude, thereby ensuring convergence of the LMM method. This is done by dividing the adjusted (by subtracting the mean) RPM values by 10 000. With this adjustment to ensure convergence, we are able to include random effects for the intercept, slope, and quadratic terms. To determine the autocorrelation structure within the profile, we use Draftman's display as discussed in Jensen *et al.*[22]. This display is a plot of the within-profile residuals (centered and scaled by torque) at RPM_j versus those at RPM_{j+1} for all levels of RPM. For this engine data, Draftman's display is shown in Figure 3. We see that the plots at the diagonal show a positive correlation between successive RPM levels. As we move away from the diagonal, the correlation decreases indicating that the RPM levels further apart are not as strongly correlated. Thus, an AR(1) structure for the Σ matrix is appropriate to model the autocorrelation of the errors within a profile. Note that the residuals used here are based on $(y_j-X\hat{\beta}-Z\hat{b}_j)$, computed under the assumption of uncorrelated errors, which means that the residuals represent the within-profile variability.

AS the variance–covariance matrices (D and Σ) are not known, we obtain the estimated variance–covariance matrices by restricted maximum likelihood estimation using PROC MIXED in SAS. To do so we assume that the AR(1) structure and then obtain the best estimates given that assumption. Given an estimate of D and Σ, we can compute, $\hat{\beta}$, the vector of fixed effects as

$$\hat{\beta}=(X'\hat{V}^{-1}X)^{-1}(X'\hat{V}^{-1}\bar{y}) \tag{2}$$

where $\hat{V}=Z\hat{D}Z'+\hat{\Sigma}$ is the overall estimated variance–covariance matrix and $\bar{y}$ is the average of the response vectors. The estimated random effects (also known as the eblups because they are estimated as best linear unbiased predictors) are then given by

$$\hat{b}_j=\hat{D}Z'\hat{V}^{-1}(y_j-X\hat{\beta}) \tag{3}$$

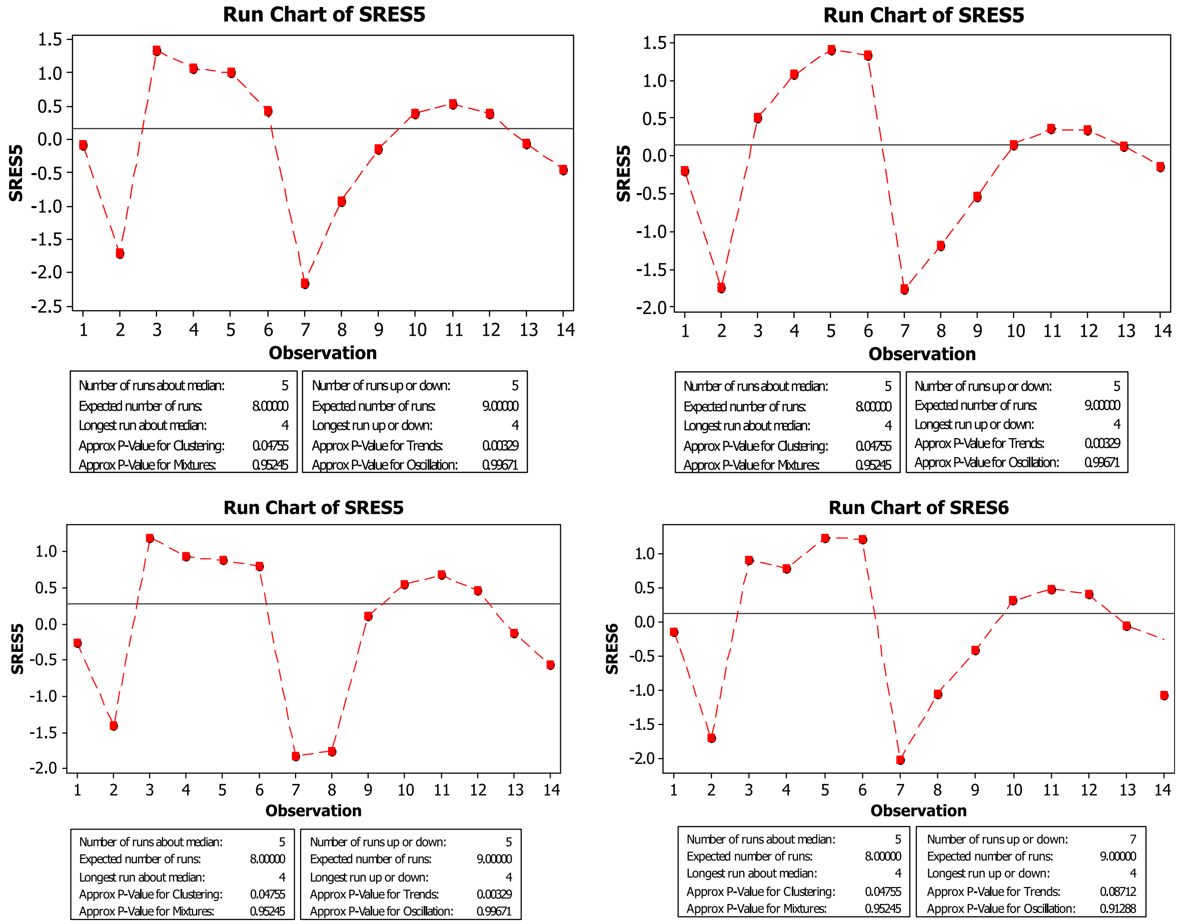

Figure 2. Run chart for standardized residuals of 4 engines. This figure is available in colour online at www.interscience.wiley.com/journal/qre

Table III. Sample standard deviation of the estimated parameters from the least-squares estimates from the 26 fitted profiles

Estimated parameters	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
Sample mean	111.2589	−0.005985	−4.958 e−6
Sample standard deviation	1.5299	0.0004961	3.101 e−6

Thus, fitting the LMM to our data results in a vector containing the estimates of fixed effects, $\hat{\beta}$, that is common for all profiles as well as a vector containing the random effects, $\hat{b}_j$, which is unique for each profile.

For the control chart procedure, Jensen *et al.*[22] showed that only the eblups ($\hat{b}_j$) are needed to determine the stability of the process. We obtained the eblups for the 26 engine profiles, which are given in Table IV. A profile plot showing the fitted profiles based on the eblups is shown in Figure 4.

As a check that the fitted LMM is appropriate, we show in Figure 5, a histogram of the residuals as an exploratory measure to determine if normality is a reasonable assumption. The residuals for the 26 profiles combined are shown here and are based on $(y_j - X\hat{\beta} - Z\hat{b}_j)$ as was done in Draftman's display from Figure 3, with the exception that we do not assume uncorrelated errors. Rather, we use the estimated variance–covariance matrix using the AR(1) structure to obtain the residuals. We do not do a formal test of normality which would be invalid for these data because the residuals are not independent of each other but rather show this histogram as a check that the normality assumption appears to be reasonable. While there is a slight amount of left skewness in the data, the residuals are centered around zero and we believe that the normality assumption is adequate.

4. Phase I studies

From the previous section, we have now shown a way that the profiles can be reduced to a smaller set of parameter estimates, which can be analyzed to determine if the Phase I data are in control. We will use a T^2-based control procedure to investigate

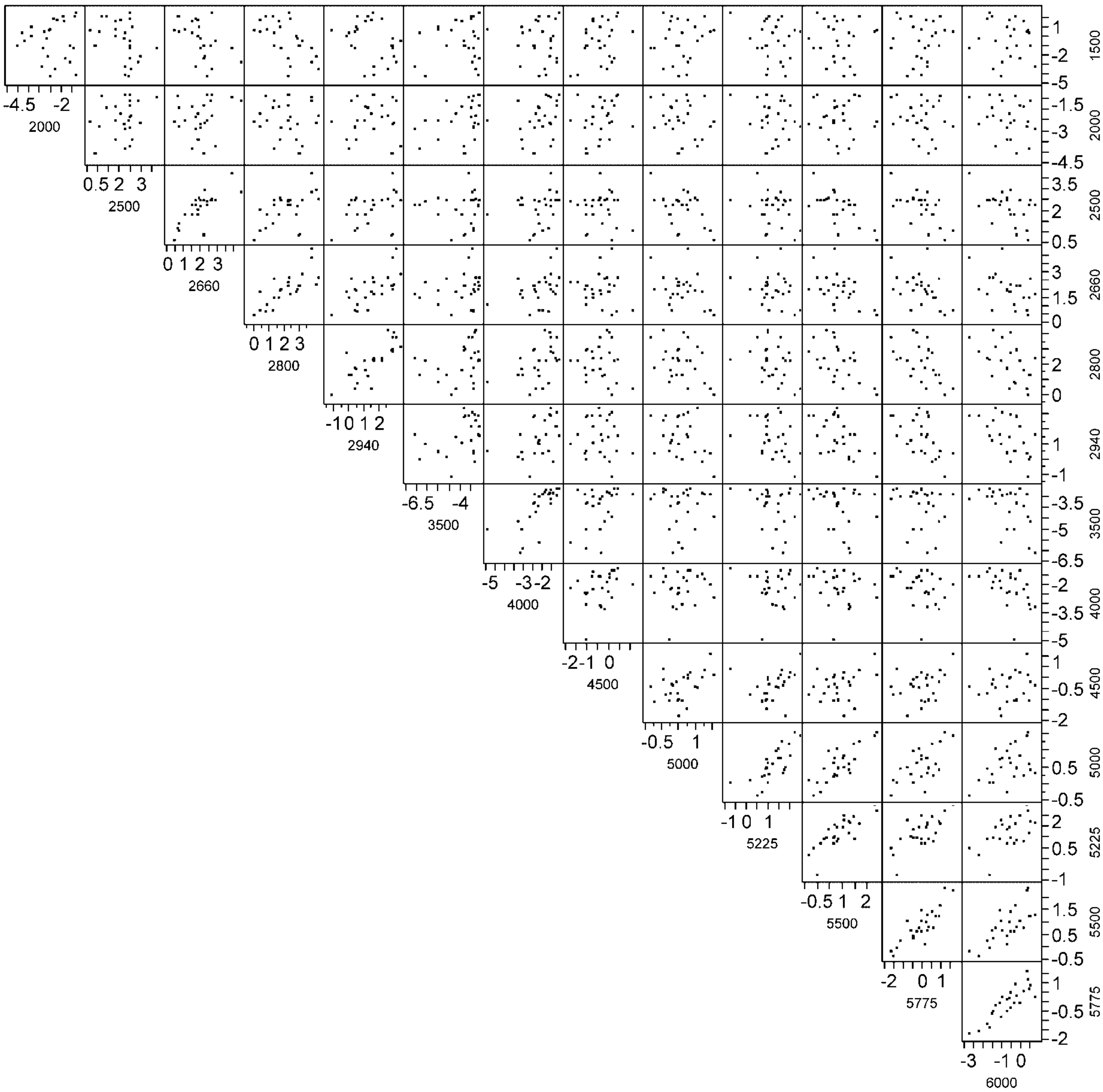

Figure 3. Draftman's display for the 26 profiles indicating an AR(1) error structure

the stability of the process as well as determine if there are any outlying profiles of the 26 profiles. Once we have removed any outlying profiles, we will be able to estimate the mean vector and covariance matrix of the process to be used to obtain control limits for Phase II of the control chart scheme.

As noted by Vargas[28] and Jensen *et al.*[29], there are multiple versions of the T^2 control chart that can be used for Phase I studies. Based on the recommendation of Figure 3 of Jensen *et al.*[29], we will use a robust T^2 control chart based on the minimum volume ellipsoid (MVE) as well as one based on the successive differences (SD) as shown in Vargas[28]. The MVE control chart is good for detecting outliers as well as small clusters of outliers. However, the SD control chart is good for detecting step changes in the profiles. The use of both of these two charts together will allow us to signal a wider variety of signals in the Phase I data.

The general form of T^2 statistic for the SD chart for profile j is

$$T_j^2 = (\hat{\beta}_j - \bar{\hat{\beta}})' S_{SD}^{-1} (\hat{\beta}_j - \bar{\hat{\beta}}) \tag{4}$$

where S_{SD} is calculated as follows:

$$S_{SD} = \frac{1}{2(m-1)} \sum_{j=1}^{m-1} (\hat{\beta}_{j+1} - \hat{\beta}_j)(\hat{\beta}_{j+1} - \hat{\beta}_j)' \tag{5}$$

In these equations, $\hat{\beta}_j$ represents the estimated vector containing the model parameters from the LMM approach and $\bar{\hat{\beta}}$ represents the mean of the estimated vectors for all the profiles. For the LMM approach, we replace the $\hat{\beta}_j$ in Equations (4) and (5) with the estimated vector of random effects, $\hat{b}_j$, from Equation (3). As discussed by Jensen *et al.*[22], the average of the

Table IV. The eblups from the LMM analysis of the 26 engine profiles

Engine No.	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$
329	0.953	−0.532	0.253
449	−0.381	0.297	−0.155
529	0.986	−0.019	0.047
642	−1.19	0.769	−0.36
724	−1.936	0.874	−0.421
803	−0.614	−0.093	0.01
930	0.373	−0.241	0.143
1148	0.102	−0.097	0.009
1171	−0.28	0.245	−0.129
1516	0.387	−0.574	0.281
1791	0.715	−0.926	0.458
2600	1.012	−0.502	0.233
3100	0.37	−0.265	0.161
3720	−0.412	0.65	−0.285
4025	−0.378	−0.054	0.034
4068	0.471	−0.763	0.341
4926	1.998	−1.002	0.485
5155	1.099	−0.441	0.259
6143	−0.992	0.823	−0.411
6844	−0.419	0.342	−0.197
7811	−1.209	0.567	−0.296
8007	−0.715	0.271	−0.127
8623	0.102	0.118	−0.035
9388	1.146	−0.379	0.184
9404	−1.853	1.07	−0.565
10430	0.667	−0.138	0.079

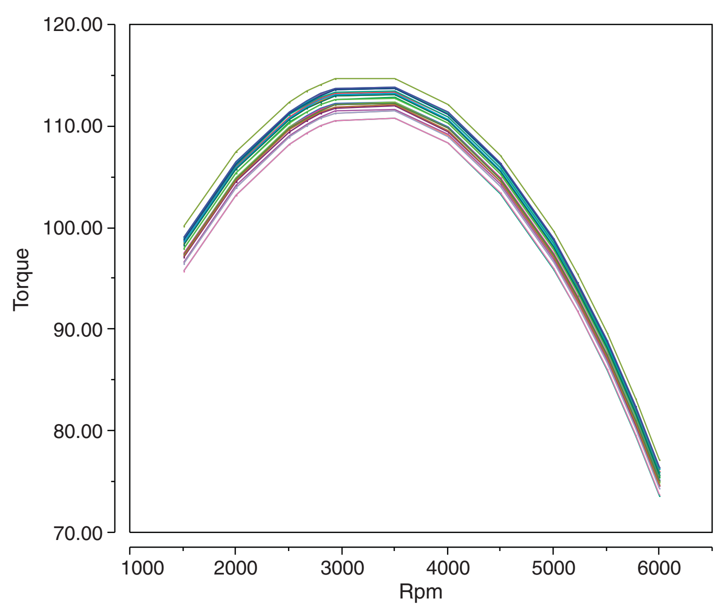

Figure 4. Profile plot showing the fitted LMM model to all 26 profiles. This figure is available in colour online at www.interscience.wiley.com/journal/qre

random effects vectors will be zero, thus we can rewrite Equation (4) as follows:

$$T_j^2 = (\hat{b}_j)' \left[\frac{1}{2(m-1)} \sum_{j=1}^{m-1} (\hat{b}_{j+1} - \hat{b}_j)(\hat{b}_{j+1} - \hat{b}_j)' \right]^{-1} (\hat{b}_j) \tag{6}$$

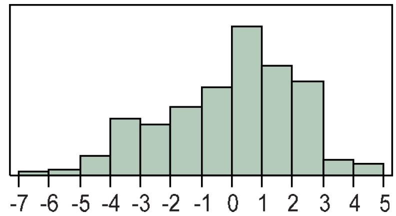

Figure 5. Histogram showing residuals from the fitted LMM model with and AR(1) structure. This figure is available in colour online at www.interscience.wiley.com/journal/qre

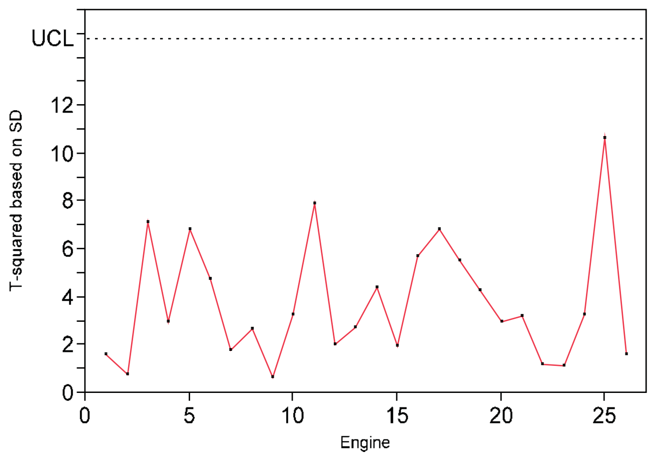

Figure 6. T^2 control chart based on successive differences. This figure is available in colour online at www.interscience.wiley.com/journal/qre

In multivariate control charts, If $m>p^2+3p$ (m is number of samples and p is number of quality characteristics), the T^2 statistic based on successive differences follows a chi-square distribution with p degrees of freedom, asymptotically (Williams *et al.*[30]). Hence, the upper control limit (UCL) for the statistic in Equation (6) is as follows:

$$UCL=\chi^2_{p,\alpha} \tag{7}$$

As the underlying model is a second-order polynomial profile, the number of regression parameters is equal to three and m for the investigated case is equal to 26. Hence, $m=26$ is greater than $p^2+3p=18$ in our case and the UCL in Equation (7) can be used here for the T^2 control chart based on successive differences. The UCL for T^2 statistics based on the SD considering significance level of $\alpha=0.00197$ and 3 degrees of freedom is calculated as follows:

$$UCL=\chi^2_{3,0.00197}=14.83 \tag{8}$$

For the T^2 chart based on the MVE, we use general form of T^2 statistic for the MVE chart for profile j, which is

$$T_j^2=(\hat{b}_j-\bar{\hat{b}}_{MVE})'S_{MVE}^{-1}(\hat{b}_j-\bar{\hat{b}}_{MVE}) \tag{9}$$

where $\bar{\hat{b}}_{MVE}$ and S_{MVE} are the center and variance–covariance matrix of the ellipsoid obtained from the data. The UCL for this chart must be obtained by simulation.

Because we are using two different control charts, one based on SD and one based on MVE, we need to control the overall significance level of the procedure. To obtain an overall significance level of $\alpha=0.1$, each control chart should have roughly the α equal to 0.05. This probability is calculated as follows:

$$\alpha=1-(1-\alpha_{overall})^{1/2}=1-(1-0.1)^{1/2}\cong 0.05 \tag{10}$$

Similarly, to obtain an overall significance level of $\alpha=0.05$ for each control chart, each statistic should have the following probability of Type I error:

$$\alpha=1-(1-\alpha_{overall})^{1/m}=1-(1-0.05)^{1/26}=0.00197 \tag{11}$$

The UCL for T^2 control chart based on MVE for m equal to 26, p equal to 3, and $\alpha=0.05$ was given as 31.76 in Jensen *et al.*[29], who obtained the value by simulation. The T^2 statistics based on the SD and MVE for LMM approach were computed and the corresponding T^2 control charts are shown in Figures 6 and 7.

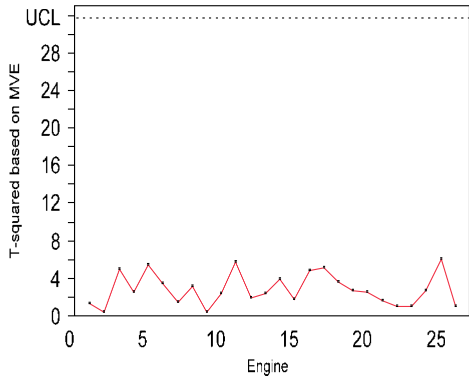

Figure 7. T^2 control chart based on MVE. This figure is available in colour online at www.interscience.wiley.com/journal/qre

The T^2 control charts based on both SD and MVE for the LMM approach show that the process is in statistical control and stable (Figures 6 and 7). If the process was not stable, the assignable cause should be detected and the data values related to that assignable cause should be omitted. It should be noted that if one statistic falls out of the UCL, one should first omit the corresponding observation, then recalculate the covariance matrix of observations and estimate the regression parameters for the remaining observations. This procedure is repeated until no assignable causes are detected.

As the process is stable, we can use the data set to estimate the parameters of the process including mean vector and variance–covariance matrix. These parameters can be used to construct the control charts for Phase II studies. The mean vector and variance–covariance matrix are estimated by using MVE estimator and the results are given as follows:

$$\bar{\hat{b}}_{MVE}=\begin{pmatrix}0\\0\\0\end{pmatrix},\quad S_{MVE}=\begin{pmatrix}0.94771 & -0.49118 & -0.24681\\ -0.49118 & 0.32360 & -0.15933\\ -0.24681 & -0.15933 & 0.07902\end{pmatrix} \tag{12}$$

The estimated parameters in Equation (12) can be used in Phase II to monitor the process in the future. We denote $\hat{b}_{m+1}$ as the estimate of a future observation, which is obtained by replacing y_j with y_{m+1} in Equation (3). Note that we use the adjusted RPM values as well as the original 26 in-control observations in the estimation of the vector of fixed effects and variance-covariance matrices in Equation (3) because there were no out-of-control observations in either of the control charts. The following T-squared statistic can be applied in this phase as

$$T^2_{m+1}=(\hat{b}_{m+1}-\mu)'\Sigma^{-1}(\hat{b}_{m+1}-\mu) \tag{13}$$

where μ and Σ are replaced by $\bar{\hat{b}}_{MVE}$ and S_{MVE} in Equation (12), respectively. For each future observation, $\hat{b}_j$, which is the estimator vector of the random effects, is computed by using Equation (3). Then, T-squared statistic is computed by using Equation (13) and is plotted on the T^2 control chart. In Phase II, the T-squared statistic in Equation (13) follows a chi-squared distribution with p degrees of freedom, which is equal to three in our case. Hence, to achieve the probability of Type I error equal to α, the UCL for the statistic in Equation (13) is equal to $\chi^2_{p,\alpha}$ Using the mentioned T^2 control chart, one can monitor the process of producing the engine in the future and decide whether or not process is in statistical control.

5. Conclusions

In this paper, the relationship between the torque produced by an automobile engine and its speed was investigated. Our studies showed that this relationship can be modeled by a second-order polynomial profile and there is autocorrelation structure within each profile. We proposed LMM approach to estimate polynomial regression parameters. Then, we used a T^2-based control procedure to conduct Phase I studies. The results showed that the process that produces the engine is stable. Furthermore, using the stable process, the mean vector and variance–covariance matrix of parameter estimators, which are required for monitoring the process in the future, were estimated. Finally, we discussed how one can use these estimators to form a T^2 based control chart to monitor polynomial profiles in Phase II.

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Appendix: A Historical Data Set of 26 Automobile Engines (Torque versus RPM)

RPM	Torque-Eng. No. 329	Torque-Eng. No. 449	Torque-Eng. No. 529	Torque-Eng. No. 642	Torque-Eng. No. 724	Torque-Eng. No. 803	Torque-Eng. No 930	Torque-Eng. No. 1148	Torque-Eng. No. 1171	Torque-Eng. No. 1516
1500	98.53	96.35	98.77	96.70	96.75	97.61	100.06	94.55	96.48	100.73
2000	102.65	100.74	103.03	100.05	100.87	102.46	103.60	103.22	100.87	103.82
2500	113.82	110.67	111.99	111.17	110.14	112.18	112.74	112.99	110.81	113.46
2660	115.26	113.06	112.78	111.51	110.48	112.99	113.56	114.18	113.20	113.69
2800	116.24	114.58	113.14	112.01	110.94	114.54	112.85	116.48	114.73	115.07
2940	117.06	114.98	113.73	111.23	111.17	115.00	114.49	115.33	115.13	115.32
3500	109.89	108.55	110.30	105.64	105.78	108.99	108.95	109.59	108.69	109.71

RPM	Torque-Eng. No. 329	Torque-Eng. No. 449	Torque-Eng. No. 529	Torque-Eng. No. 642	Torque-Eng. No. 724	Torque-Eng. No. 803	Torque-Eng. No 930	Torque-Eng. No. 1148	Torque-Eng. No. 1171	Torque-Eng. No. 1516
4000	109.65	107.41	109.35	106.02	103.37	107.95	108.24	108.47	107.55	109.25
4500	105.72	103.90	107.61	103.11	102.23	103.65	105.56	105.27	104.03	104.70
5000	99.74	97.99	100.64	97.40	96.06	96.94	98.92	97.90	98.12	98.29
5225	95.97	94.27	97.59	93.88	92.39	92.78	95.41	94.67	94.39	94.53
5500	89.47	88.45	91.68	88.17	86.54	86.41	89.19	88.23	88.56	88.02
5775	81.96	81.44	84.58	81.18	79.31	78.60	81.85	80.86	81.54	80.02
6000	74.90	75.00	77.48	75.03	73.13	71.97	75.09	73.93	75.09	73.39

RPM	Torque-Eng. No. 1791	Torque-Eng. No. 2600	Torque-Eng. No. 3100	Torque-Eng. No. 3720	Torque-Eng. No. 4025	Torque-Eng. No. 4068	Torque-Eng. No 4926	Torque-Eng. No. 5155	Torque-Eng. No. 6143	Torque-Eng. No. 6844
1500	101.98	96.83	100.07	97.44	100.00	97.98	97.29	100.97	93.13	93.11
2000	105.40	103.78	103.91	101.84	103.27	104.98	105.86	104.96	101.02	103.43
2500	113.94	114.30	112.52	110.14	111.46	114.90	115.25	112.47	111.25	112.02
2660	115.11	114.62	113.25	111.02	111.80	116.06	117.83	113.04	111.83	113.20
2800	115.45	117.19	114.10	111.37	113.02	116.65	117.97	113.91	113.27	113.77
2940	115.17	116.61	114.10	110.78	113.55	116.18	117.77	114.22	113.04	113.77
3500	110.41	110.43	109.21	107.87	108.87	109.65	111.31	110.46	105.60	109.15
4000	109.70	109.61	108.34	107.26	107.60	109.06	110.97	109.66	106.15	108.05
4500	106.05	106.32	104.87	105.27	104.44	105.01	107.37	106.79	104.12	103.46
5000	98.04	99.44	98.35	99.36	97.50	97.43	100.53	100.27	97.45	98.26
5225	92.96	95.62	94.76	95.97	94.04	94.04	97.17	96.48	94.68	94.26
5500	87.83	89.46	88.93	90.38	87.89	87.51	90.47	90.74	88.59	89.09
5775	79.84	82.00	82.19	82.77	80.04	79.36	83.51	83.50	81.08	81.06
6000	73.55	75.83	75.80	76.15	73.18	72.34	76.34	76.38	75.77	74.14

RPM	Torque-Eng. No. 7811	Torque-Eng. No. 8007	Torque-Eng. No. 8623	Torque-Eng. No. 9388	Torque-Eng. No. 9404	Torque-Eng. No. 10430
1500	95.38	98.28	96.79	96.45	91.53	98.37
2000	101.25	101.29	103.64	104.52	100.72	102.40
2500	111.53	112.20	112.73	113.78	110.71	112.67
2660	112.11	112.57	113.92	114.59	111.72	113.76
2800	112.60	113.06	113.35	115.40	112.29	115.41
2940	111.76	112.37	112.78	115.86	111.61	113.01
3500	108.12	107.03	108.20	110.78	105.21	110.08
4000	106.62	106.37	107.06	110.21	106.22	109.51
4500	102.92	104.10	105.27	106.75	101.73	106.09
5000	96.35	98.01	98.47	99.94	96.59	99.84
5225	93.14	94.21	95.67	96.94	93.78	96.46
5500	86.75	87.53	89.41	90.24	87.29	90.16
5775	80.27	80.08	82.57	82.65	78.97	82.74
6000	73.47	73.90	76.31	76.76	72.80	75.82

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