LOCATION ROUTING PROBLEM WITH SIMULTANEOUS PICKUP AND DELIVERY OF DEMANDS WITH LEAKING CHARACTERISTIC

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ABSTRACT

This paper addresses a location routing problem with simultaneous pickup and delivery of demands in which delivery products have leakage, this characteristic leads to decreasing products volume during transportation. The amount of leaking products depends on traveled distance between nodes, volume of loaded products on vehicle and property of traveled arc such as weather condition. The problem has a number of applications in real life while products have leakage during the delivery distribution. The goal of this study is to determine location, allocation and routing decisions to minimize the total network cost including of location, routing and operational cost such as the product leakage cost. We develop a mathematical model for this problem. Some numerical examples on well-known data sets are presented to evaluate efficiency of the proposed model. Moreover some sensitivity analysis is performed to confirm model validity.

Keywords: Location routing problem, Pickup and delivery, Leakage, Mixed integer nonlinear programming.

1 INTRODUCTION:

Two major components of distribution networks are location and routing decisions, solving these sub problems separately brings to highly suboptimal solutions and increases overall cost of supply chain system. Location routing problem (LRP) thus has to combine these two problems to take economic decisions. In classic form of LRP, customers have only delivery demands, but in various cases customers have two types of demands: pickup and delivery, and request of both demands should be served simultaneously which is called LRPSPD (location routing problem with simultaneous pickup and delivery) as an extension of LRP. Also, in practice some of products may have leakage (called leaking characteristic) and the volume of loaded products on vehicle decreases during transportation because of traveled arc property and weather condition. The leaking characteristic can be occurred in distribution of some products such as cement, fluid, fertilizer, and etc.

To clarify the problem, consider a distribution system that distributes products such as bulk cement or fluids. During transportation of these types of products, because of arc property such as road or weather conditions, some of loaded products on vehicle will be leaked and fall down or evaporate on passed road. The amount of leakage in an arc depends on the volume of loaded products on vehicle passing mentioned arc while leaking coefficient differs in different arcs.

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Finally the total volume of leakage will depend on the arc, volume of loaded product and travelled distance by the vehicle.

Neglecting this characteristic may face us to following issues. First issue is that customers' demands will not be satisfied completely because of product leakage during the transportation. To tackle with the first issue more volume of product rather than required can be loaded to the vehicle when starting the tour. It will be another issue because of increasing of total leakage while the tour plan is not optimal. In this paper a new model of location routing problem with simultaneous pickup and delivery with leaking characteristic called LRPSPD-L is presented. Figure 1 illustrates the main concept of the problem by comparing the result of classic LRPSPD (Figure 1.a) and our proposed problem as LRPSPD-L (Figure 1.b). Figure 1.a shows that for serving customer 2, loaded products on the vehicle when starting corresponding tour is 18. Products are leaked as much as 0.572, because of leaking characteristic. Consequently, when vehicle visits customer 2, loaded products on this vehicle is 17.428 while demand of customer 2 is 18, hence about 3% of customer 2's demand is not satisfied. For serving other customers, classic model and proposed one select different tours. By applying proposed model all customers demand will be satisfied, while by applying classic model, similar to mentioned pattern, products will have leakage and about 18.5% of last customer demand (5.7 % of total demands in respected tour) will not be satisfied. It is necessary to note that, in classic model, if products are loaded more than total request of assigned customers, the products leakage cost will be increased comparison with the proposed model and it is not economic, especially when products are more valuable.

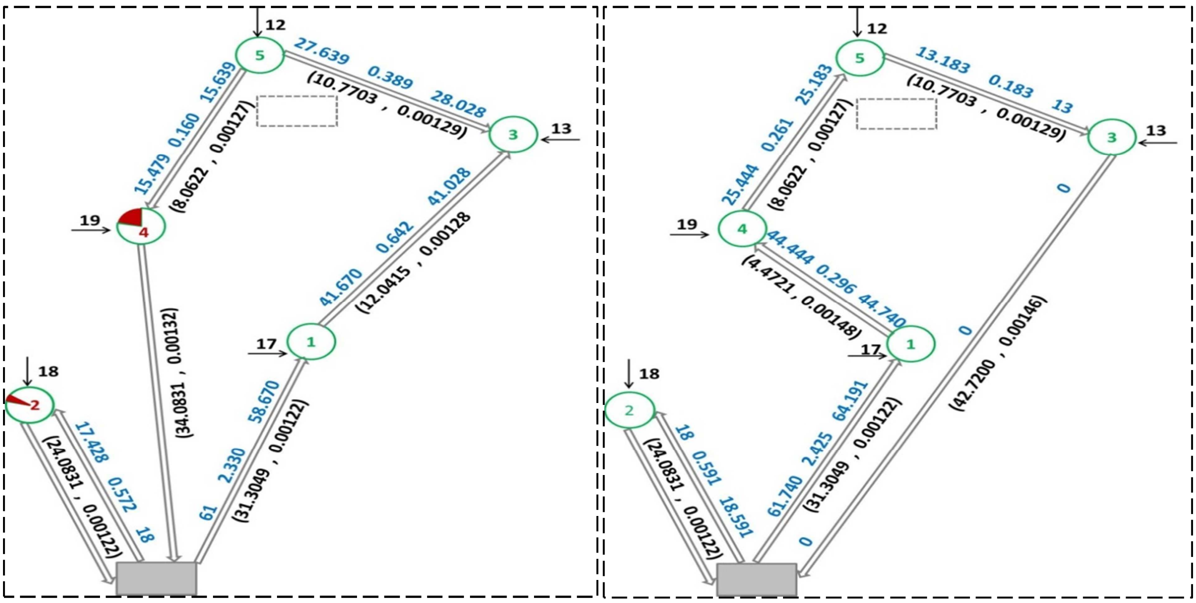
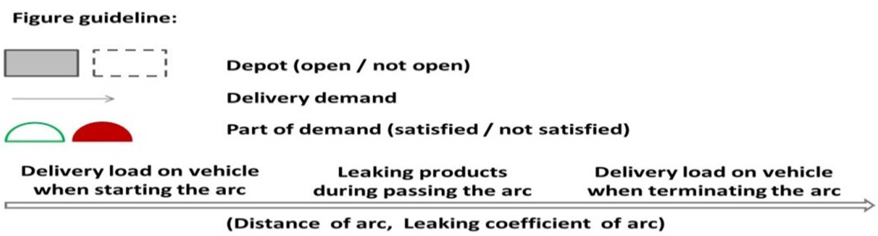


Figure 1.a: A schematic example for the performance of classic LRPSPD.

Figure 1.b: A schematic example for the performance of the LRPSPD-L.



To the best of authors' knowledge, there is no pervious study on the LRP in which products have leaking characteristic. After modeling of the problem, different numerical examples are solved by the Gams software.

The rest of this research is organized as follows: literature review is given in section 2, in section 3 after the problem definition, mathematical formulation is developed, section 4 reports computational results and sensitivity analysis to evaluate and illustrate performance of the proposed model and finally conclusion and direction for future researches are presented in the last section.

2 LITERATURE REVIEW

Location routing problem as a field of operation research have been studied for many years. First classification for LRP was provided by Balakrishnan et al. [1] and recently Prodhon and Prins [2] and also Drexl and Schneider [3] provided a comprehensive survey on variant, applications and solution methods of this problem.

Pickup and delivery location routing problem is an extension of LRP, in which goods are delivered to customers from one of open depots and also goods are picked up at customers and delivered to depot or other customers. Berbeglia et al. [4] and Parragh et al. [5] reviewed pickup and delivery problem and its different variants. If every customer has both pickup and delivery demand and both demands are served at the same time, the problem is called LRPSPD.

LRPSPD was introduced by Karaoglan et al. [6] at the first time. They proposed a flow base mixed integer linear programming for this problem and an exact branch and cut algorithm to solve the LRPSPD. Same authors [7] proposed another node base formulation for LRPSPD and then presented valid inequalities to strength mentioned formulations. They applied two initialization heuristics to generate an initial solution and a heuristic approach based on simulated annealing to solve the problem. Wang [8] developed this problem in urban-rural dual directions logistics network and extended a nonlinear mixed integrated programming model, then applied a two-phase heuristic approach based on Tabu search to solve it. Vincent and Lin [9] adopted multi start strategy and proposed multi start annealing heuristic for LRPSPD.

In all previous studies, loaded products on the vehicle don't have leakage characteristic. While in some industrial applications there are some products with leakage during distribution. In this paper we focus on formulating of mentioned characteristic in the LRPSPD.

3 PROBLEM DEFENITION AND MATHEMATICAL FORMULATION

The proposed mathematical model is an extended version of the proposed formulation by Karaoglan et al. [7] for the location routing problem with simultaneous pickup and delivery of demands with leaking characteristic. This model will decide the location of depots, allocation of customers to the opened depots, vehicles routing plan for delivery (or pickup) products to (from) customers and the amount of loaded products on vehicles in the beginning of tours to satisfy all customers' demands, when delivery products have leakage during transportation.

The formulation for LRPSPD with leaking characteristic is proposed with following notations:

Indices:

N: The set of nodes (depots and customers)

N_0: The set of potential depots

N_c: The set of customer nodes

Parameters:

d_i : Delivery demand of customer i ($i \in N_c$)

p_i : Pickup demand of customer i ($i \in N_c$)

α : Value of per unit leaking product

c_{ij} : Transportation cost of arc (i,j) ($i,j \in N$)

s_{ij} : Distance of arc (i,j) ($i,j \in N$)

r_{ij} : Leaking coefficient of arc ($i,j \in N$)

CD_k : Capacity of depot k ($k \in N_0$)

FD_k : Fixed cost of depot k ($k \in N_0$)

CV : Capacity of each vehicle

FV : Fixed operating cost of each vehicle

Decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if a vehicle travels directly from node } i \text{ to node } j \quad (\forall i, j \in N) \\ 0 & \text{o.w} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if depot } k \text{ is selected for opening} \quad (\forall k \in N_0) \\ 0 & \text{o.w} \end{cases}$$

$$z_{ik} = \begin{cases} 1 & \text{if customer } i \text{ is serviced by depot } k \quad (\forall i \in N_c,\ \forall k \in N_0) \\ 0 & \text{o.w} \end{cases}$$

ψ_i: *Delivery load on vehicle just after departing previous node of customer i considering leaking ($i \in N_c$). It includes delivery demands of remaining nodes and total leaking during remaining arcs in respected route.*

V_i: *Pickup load on vehicle just after having serviced customer i ($i \in N_c$)*

dd_i: *Amount of leaking products in direct arc to node i plus delivery demand of customer i ($i \in N_c$)*

Introduced decision variables dd_i and ψ_i, *and also parameters* r_{ij} *and* α are specifically related to leaking characteristics of products.

The model formulation is given by:

$$\min \sum_{k \in N_0} FD_k y_k + \sum_{k \in N_0} \sum_{i \in N_c} FV x_{ki} + \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} + \sum_{j \in N} \sum_{i \in N_c} \alpha \psi_i r_{ji} s_{ji} x_{ji} \tag{1}$$

$$\sum_{j \in N} x_{ij} = 1 \qquad \forall i \in N_c \tag{2}$$

$$\sum_{j \in N} x_{ji} - \sum_{j \in N} x_{ij} = 0 \qquad \forall i \in N \tag{3}$$

$$\sum_{k \in N_0} z_{ik} = 1 \qquad \forall i \in N_c \tag{4}$$

$$x_{ik} \le z_{ik} \qquad \forall i \in N_c,\ \forall k \in N_0 \tag{5}$$

$$x_{ki} \le z_{ik} \qquad \forall i \in N_c,\ \forall k \in N_0 \tag{6}$$

$$x_{ij} + z_{ik} + \sum_{m \in N_0, m \ne k} z_{jm} \le 2 \qquad \forall i, j \in N_c,\ i \ne j,\ \forall k \in N_0 \tag{7}$$

$$\sum_{i \in N_c} dd_i z_{ik} \leq CD_k y_k \qquad \forall k \in N_0 \tag{8}$$

$$\sum_{i \in N_c} p_i z_{ik} \leq CD_k y_k \qquad \forall k \in N_0 \tag{9}$$

$$\psi_j - \psi_i + CVx_{ij} + (CV - dd_i - dd_j)x_{ji} \leq CV - dd_i \qquad \forall i, j \in N_c,\ i \neq j \tag{10}$$

$$V_i - V_j + CVx_{ij} + (CV - p_i - p_j)x_{ji} \leq CV - p_j \qquad \forall i, j \in N_c,\ i \neq j \tag{11}$$

$$\psi_i + V_i - dd_i \leq CV \qquad \forall i \in N_c \tag{12}$$

$$\psi_i \geq dd_i + \sum_{j \in N_c, j \neq i} dd_j x_{ij} \qquad \forall i \in N_c \tag{13}$$

$$V_i \geq p_i + \sum_{j \in N_c, j \neq i} p_j x_{ji} \qquad \forall i \in N_c \tag{14}$$

$$\psi_i \leq CV - (CV - dd_i)\,(\sum_{k \in N_0} x_{ik}) \qquad \forall i \in N_c \tag{15}$$

$$V_i \leq CV - (CV - p_i)\,(\sum_{k \in N_0} x_{ik}) \qquad \forall i \in N_c \tag{16}$$

$$dd_i = d_i + \sum_{j \in N, j \neq i} \psi_i s_{ji} r_{ji} x_{ji} \qquad \forall i \in N_c \tag{17}$$

$$\sum_{j \in N_c, j \neq i} (\psi_i - \psi_j)x_{ij} + \sum_{k \in N_0} \psi_i x_{ik} = dd_i \qquad \forall i \in N_c \tag{18}$$

$$x_{ij} \in \{0,1\} \qquad \forall i, j \in N \tag{19}$$

$$z_{ik} \in \{0,1\} \qquad \forall i \in N_c,\ \forall k \in N_0 \tag{20}$$

$$y_k \in \{0,1\} \qquad \forall k \in N_0 \tag{21}$$

$$V_i \geq 0 \qquad \forall i \in N_c \tag{22}$$

$$dd_i \geq 0 \qquad \forall i \in N_c \tag{23}$$

$$\psi_i \geq 0 \qquad \forall i \in N_c \tag{24}$$

The objective function given in equation (1) minimizes total costs including fixed costs (depot and vehicle fixed costs) and variable costs (transportation and leaking products costs). In this paper it is considered that all terms of objective function have equal importance and relative weight of each term is one; but if the objective function seeks to minimize the hierarchical objective, equation (1) may need weighting coefficient corresponding to each term of objective function. Constraint (2) implies that each customer is served exactly once and split delivery is not allowed. Constraint (3) assures continuity of tour. Constraint (4) guarantees that each customer must be assigned to one depot. Constraints (5)-(7) ensure that each tour must be terminated at the same depot which started from and solutions don't contain illegal tours. Constraint (8) specifies that customers don't allow to be assigned to a closed depot and sum of products shipped from opened depot by considering leaking characteristic must be less than its capacity. Constraint (9) guarantees depot capacity for pickup loads. Constraint (10) describes flow inequalities for delivery demand with considering leaking characteristic and ensures that delivery demand of each customer is satisfied considering of leakage. Constraint (11) is similar to constraint (10) but concerns pickup demands; note that pickup demands don't have leaking characteristic. Constraints (10)-(11) also prevent sub tours. Vehicle capacity in each node is guaranteed by constraint (12), constraints (13)-(16) are bounding constraints for additional variables V_i and ψ_i. The amount of product leakage from previous node to customer i is calculated by constraints (17)-(18), also these constraints force that amount of leaking products on the last arc of each tour to be zero. Constraints (19)-(24) define variables types.

The introduced model is a mixed integer nonlinear programming. Objective function and constraints (8), (10), (13), (15), (17), (18) have nonlinear terms in which a continuous variable (dd_i or ψ_i) is multiplied by a binary variable.

4 COMPUTATIONAL RESULTS AND SENSITIVITY ANALYZE

In this section, computational experiments and sensitivity analysis are discussed to evaluate the efficiency of the proposed model. The model is coded by GAMS software version 23.2. Test instances are derived from data available in the LRPSPD literature generated by Karaoglan et al. [7]. For example the data set in which $|N_c|$=7 and $|N_0|$=2, have been generated by considering 7 first customers and 2 first depots, also leaking coefficient between nodes are generated from a uniform distribution of U[0.00100-0.00150]. Summary of data sets and obtained experimental results are reported in table 1. Table 1 confirms that when delivery products have leakage, for servicing all customer demands, vehicle should be loaded more than customers' demands starting its tour. By applying proposed model, the amount of over loaded products than customer demands will be equal to the product leakage during transportation.

Table 1: The results of the proposed model performance comparing with classic model

Test problem characteristics				Delivery demands results			Total cost
				Total demand	Loaded products		
NO	Original test problem	$\lvert N_c\rvert$	$\lvert N_0\rvert$		Classic model	Proposed model	
1	Prod_10_3_W_coord20-5-1	5	2	79	79	82.782	13194.528
2	Prod_10_3_Z_coord20-5-1b	6	3	93	93	98.110	8388.170
3	Prod_10_3_X_coord20-5-1	7	2	62	62	67.050	12259.814
4	Prod_10_3_Y_coord20-5-2b	8	3	67	67	71.535	7201.130
5	Prod_10_3_W_coord20-5-1b	9	2	143	143	151.488	14712.261
6	Prod_10_3_X_coord20-5-2b	10	3	73	73	74.640	7222.266

Moreover, to investigate the effect of important parameters such as demand and leaking coefficient, second instance of table 1 is selected and the problem is solved.

Figures 2 and 3 illustrate the effect of leaking coefficient and the effect of demand on the total cost, respectively. For different levels of these parameters, the selected problem is solved by the proposed model (scenario 1) and its result is compared with the scenario 2 in which the leaking characteristic is not included in the route planning. (In scenario 2, route decision is taken by solving of the classic LRPSPD model, and then these obtained tour plans are put in the proposed model). According to mentioned figures, by increasing of leaking coefficient and customers demand, the difference between mentioned scenarios will be decreased and it shows high importance of the proposed model in cases with sensitive leaking products.

In Figure 3 it is seen that in some cases the route derived by classic model as scenario 2 can be an infeasible solution for the proposed model because of capacity constraints. According to Figure 3 it is cleared that by increasing of demand as much as three times of the initial value, obtained solution is not reported, because using of the route obtained by solving the classical model vehicle capacity will be violated in the proposed model because this tour is not optimal solution for LRPSPD-L and the amount of required loaded products on vehicle for servicing customer, will be calculate over the optimum value (point 3*d* in Figure 3).

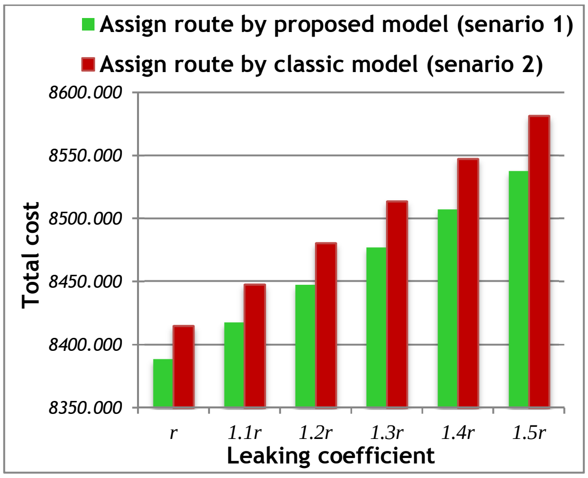


Figure 2: Comparison of total cost between scenario 1 and scenario 2. (Corresponding different levels of leaking coefficient)

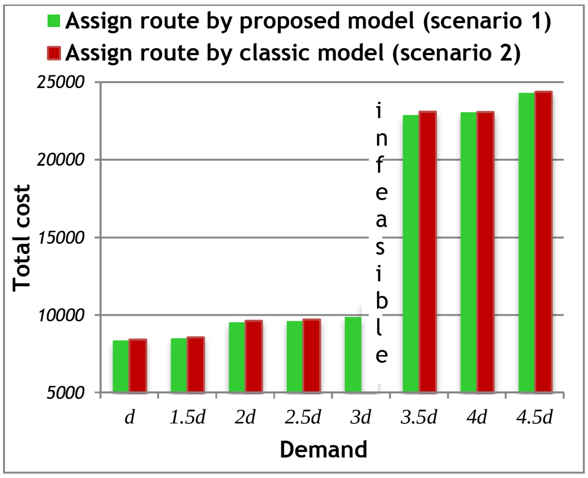


Figure 3: Comparison of total cost between scenario 1 and scenario 2. (Corresponding different levels of demand)

Figure 4 illustrates that if leaking characteristic is not considered in the routing planning (scenario 2) the amount of loaded products on vehicles in the beginning of tours will be over than the required for serving customers. Surplus products owing to leaking characteristic will be lost and will not be saved at the end of the tour. Numerical experiments confirm that mentioned extra cost will be increased by increasing of leaking coefficient.

In a real situation with leaking products, by using the classic model, we need to handle more products than customer demands, so in some cases product leakage will be increased. And in spite of our model, the leakage will be occurred in the last arc as well. So it confirms that the proposed model decision will be more economical for mentioned cases.

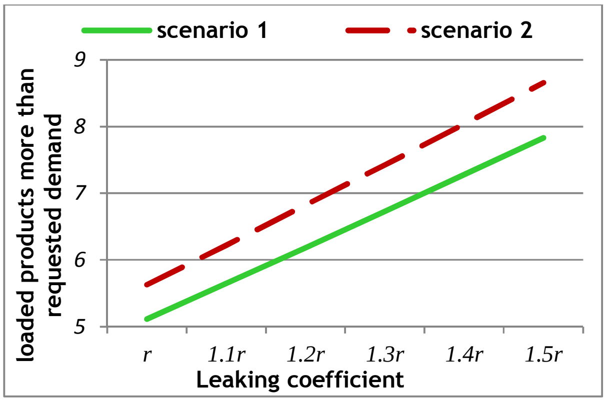


Figure 4: Effect of the leaking coefficient on the over loaded products for two scenarios

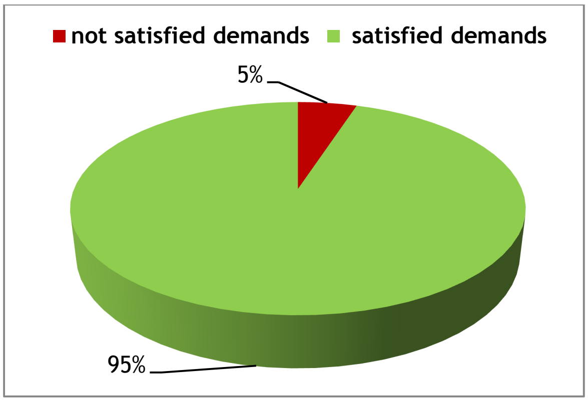


Figure 5: Unsatisfied demand percent by using of classic model decisions (average for different values of demands)

To consider necessity of the proposed model, we selected the decisions of the classic model and we realized that if the distributor does not consider the leakage, will face a shortage to satisfy all customer demands. We calculated the shortage for all different levels of demand. Our study confirms that we face an average shortage about 5% of demands for different values of demand which has been depicted in Figure 5.

In all of experiments, proposed model has better results on total cost and proportion of satisfying customers' requests. In fact when products have leakage, considering leaking characteristics leads to save cost and satisfy all customers' demands completely. The efficiency of the proposed model will be more tangible in a case with longer travel distances, larger number of customers, high value products, and uneven property of traveled arcs.

5 CONCLUSION

In this paper a new extension of LRPSPD was introduced, in where delivery products have leakage. This characteristic leads to decrease the volume of loaded products on vehicle during transportation. The leaking products volume on every traveled arc depends on some feature such as the amount of loaded products on vehicle in every arc, property of every traveled arc for example road property and distance of each arc. Distribution cement and fluid are examples of this context. A mathematical model was developed for mentioned problem and model has been solved by GAMS software on some instances extracted from the literature, moreover some sensitivity analyses were performed on selected problem from data sets to evaluate efficiency of the proposed model and also to investigate the effect of important parameters on the model. The results show that considering leaking characteristic, in situation that products have leakage, leads to save cost and let satisfy all customers' demands completely compared to cases without considering it. Future study may be focused on developing of split delivery model to service customers, when products have leakage characteristics. In this case the network tends to have split deliveries to decrease total amount of leakage.

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