Abstract

Green vehicle routing problem (GVRP) is one of recent variants of vehicle routing problem (VRP), dealing with environmental aspects of distribution systems. In the literature, economic aspect has been often used, while safety and social concerns have less proportion of studies. In this paper, we present two developed mixed integer-programming models for GVRP with social and safety concerns. Different numerical analyses have performed to evaluate suggested models and investigate the influence of several key factors in them. The results confirm that the proposed model has more social benefits such as balanced tours and less customer waiting time than the classic GVRP.

Keywords: Green Vehicle Routing Problem, Route Balancing, Mixed Integer Linear Programming

1. Introduction

Among harmful impacts that transportation has on the environment, air pollution is the most concerning [1]. One approach to deal with this problem is to switch vehicle fuels from fossil fuel to alternative ones. The short driving ranges of alternative fuel vehicles (AFVs), lack of infrastructures for alternative fueling stations (AFSs) and unevenly distribution of them, present obstacles to use a fleet of AFVs. The GVRP is introduced as a new variant of the VRP that takes into account these additional challenges associated with using AFSs [2]. On the social and safety aspects, being equity between employees and make a safer workplace for them increase job satisfaction among workers. Substantial differences among drivers’ working time could be considered as unfairness [3] and may increase the numbers of accidents caused by tiredness of drivers who work in lengthier tours. The Vehicle Routing Problem with Route Balancing (VRPRB) is introduced to deal with these problems. In this study, two models for GVRP with social and safety concerns are presented. In the first one an aggregated model is presented to study tradeoff between economic aspect (minimizing the total traveled distance and the refueling cost) and social aspect (minimizing the difference between tour lengths (duration). In other model, the risk cost, for probable accident that may occur during the tour length, are considered. The aim of model is to reduce the accident risk through reducing the time difference between tours. The result of different computational experiment is reported to assess models and different key factors on them. The structure of this paper is organized as follows: In the next section a brief review of recent related studies are presented. Then the mathematical models are presented in section 3. The numerical analysis is performed in section 4 with reporting of some sensitivity analysis results. Finally concluding remarks are provided in the last section.

2. Literature Review

In VRP literature, models consider fuel tank capacity limitations are rare. As described in introduction, Sevgi Erdogan and Elise Miler-Hooks [2], introduced GVRP for the first time and solved it with two modified heuristics. Michael Schneider et al [4] extended GVRP model. They considered capacity and time window restrictions and solved model with a hybrid metaheuristic. Jun Yang and Hao Sun [5] studied routing plan of a fleet of capacitated electric vehicles (EVs). They considered the...
strategic decision of determining the best location of AFSs and proposed two heuristics to solve the problem. None of these mentioned studies consider social or safety aspects in their models. One of other related problem to our study is VRPRB. In VRPRB models two intrinsically conflicting objectives are optimized. Some researchers formulate this problem as an aggregated single objective and some others consider it as a multi-objective optimization problem (MOP) and use multi-objective evolutionary algorithms (MOEA) to approximate the Pareto set. In VRPRB, different tour’s workload, such as the number of visited customers, the quantity of delivered goods, the tour lengths (distance or time), make different balancing objectives [6]. However there is no study considers safety aspects as a workload. A brief history of related works is shown in Table1.

### TABLE1. A brief review of previous studies on the route balancing

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Problem</th>
<th>Solution Methods</th>
<th>Workload</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sutcliffe and Board (1990)</td>
<td>VRP</td>
<td>LP</td>
<td>TT, CU</td>
<td>Min. TD, Max. EVTT, Max. CU</td>
</tr>
<tr>
<td>Lee and Ueng (1998)</td>
<td>VRP</td>
<td>A/H</td>
<td>TT</td>
<td>Min. TD, Min. DWT</td>
</tr>
<tr>
<td>Ribeiro and Lourenço (2001)</td>
<td>MPVRD</td>
<td>A, ILS</td>
<td>VT</td>
<td>Min. TD, MIN. DVT, Max. D/C R</td>
</tr>
<tr>
<td>Ramos et al. (2013)</td>
<td>MDPVRPI</td>
<td>MT</td>
<td>TT</td>
<td>Min. TD, Min. DWT</td>
</tr>
<tr>
<td>Oyola and Lakketangen (2014)</td>
<td>CVRP</td>
<td>H</td>
<td>TT</td>
<td>Min. TRC, Min. DWT</td>
</tr>
<tr>
<td>Lacomme et al. (2015)</td>
<td>VRP</td>
<td>MSSPR</td>
<td>TT</td>
<td>Min. TRC, Min. DWT</td>
</tr>
</tbody>
</table>

VRP: Vehicle Routing Problem; LP: Linear Programming; TT: Tour Time; TD: Total Distance; EVTT: Equalization of the Vehicle Travel Times; A: Aggregation; H: Heuristic; DWT: Difference between Working Time; MPVRD: Multi-Period Vehicle Routing Problem; ILS: Iterated Local Search; VT: Volume Transported; DVT: Difference between Volume Transported; D/C R: Driver/Customer Relationship; MOEA: Multi-Objective Evolutionary Algorithms; MDPVRPI: Multi-Depot Periodic Vehicle Routing Problem; MT: Metaheuristic; TRC: Total Routing Cost; CVRP: Capacitated Vehicle Routing Problem; MSSPR: Multi-Start Split based Path Relinking approach

### 3. Problem Definition

In this paper, we study a fleet of AFVs that deliver customer demand from a single depot. AFVs leave depot with full tank capacity and defeat their limited driving range by visiting a set of AFSs existed in the rout. Vehicles can visit a station many times and should complete their tours in a prespecified limited time \(T_{\text{MAX}}\). We use the original notation from Erdogan and Miller-Hooks [2] to describe our model. First we presented the non-linear model, then we use some technics to linearize it. Thereafter linearized model is presented.

Sets:

- \(v_0\) : Depot
- \(I\) : Set of customers
- \(I_0\) : Set of customers and depot, \(I_0 = \{v_0\} \cup I\)
- \(F\) : Set of AFSs
- \(F'\) : Set of stations and dummies (which is considered to permit several visits from each station)
- \(F_0\) : Set of AFSs and depot, \(F_0 = \{v_0\} \cup F'\)
- \(V\) : Set of real vertices, \(V = \{v_0\} \cup I \cup F\)
- \(V'\) : Set of vertices, including dummies vertices, \(V = \{v_0\} \cup I \cup F'\)
- \(K\) : Set of vehicles
Non-decision variables and parameters:

- $y_j$: Fuel level variable specifying the remaining tank fuel level upon arrival to vertex $j$.
- $\tau_j$: Time variable specifying the time of arrival of a vehicle at vertex $j$.
- $\ell_j$: Tour length for vehicle $k$.
- $\bar{\ell}$: Average of tours time.
- $l_k$: Difference between tour lengths and average of all tour lengths.
- $o_k$: Difference between tour lengths (which are longer than the average) and average of all tour lengths.
- $s_k$: Difference between tour lengths (which are shorter than the average) and average of all tour lengths.
- $p_i$: Service time (if $i \in I$, $P_i$ refer to the service time at the customer, if $i \in F$, $P_i$ means the refueling time at AFS).
- $r$: Vehicle fuel consumption rate (gallons per mile).
- $Q$: Vehicle fuel tank capacity.
- $T_{\text{MAX}}$: Maximum tour lengths.
- $d_{ij}$: Distance between vertex $i$ and $j$.
- $t_{ij}$: Traveling time between vertex $i$ and $j$.
- $W_1$: Weight Coefficient for the traveled distance.
- $W_2$: Weight Coefficient for the time part.
- $W_3$: Weight Coefficient for the refueling cost.
- $m$: Number of vehicles.

Decision variables:

- $x_{ijk}$: Binary variable equals to 1 if vehicle $k$ travels from vertex $i$ to $j$ and 0 otherwise.

\begin{align*}
\text{Min} \quad & W_1 \left( \sum_{k \in K} \sum_{i \in V, j \neq i} d_{ij} x_{ijk} \right) + W_2 \left( \sum_{k \in K} l_k \right) \tag{1} \\
\sum_{k \in K} \sum_{j \in V, j \neq i} x_{ijk} = 1 & \quad \forall i \in I \tag{2} \\
\sum_{j \in V, j \neq i} x_{ijk} \leq 1 & \quad \forall i \in F_0, \forall k \in K \tag{3} \\
\sum_{i \in V, j \neq i} x_{ijk} - \sum_{i \in V, j \neq i} x_{jik} = 0 & \quad \forall j \in V', \forall k \in K \tag{4} \\
\sum_{j \in V \setminus \{v_0\}} \sum_{k \in K} x_{0jk} = m & \tag{5} \\
\sum_{j \in V' \setminus \{v_0\}} x_{0jk} = 1 & \quad \forall k \in K \tag{6} \\
\tau_j \geq \tau_i + (t_{ij} + p_j)x_{ijk} - T_{\text{MAX}} (1 - x_{ijk}) & \quad \forall i \in V', j \in V' \setminus \{v_0\}, k \in K \text{ and } i \neq j \tag{7} \\
\sum_{i \in V', j \neq i} (t_{ij} + p_j)x_{ijk} = \ell_j & \quad \forall k \in K \tag{8} \\
tv_k \leq T_{\text{MAX}} & \quad \forall k \in K \tag{9}
\end{align*}
\[
\tilde{t} = \left( \sum_{k \in K} n_k \right) / m
\]

\[
|n_k - \tilde{t}| = l_k \quad \forall k \in K
\]  

(11)

\[
0 \leq y_{ij} \leq (r.d_{ij})x_{ijk} - Q(1 - x_{ijk}) \quad \forall j \in I, i \in V', k \in K \text{ and } i \neq j
\]  

(12)

\[
y_{ij} \geq (r.d_{ij})x_{ijk} \quad \forall j \in V', i \in V', k \in K \text{ and } i \neq j
\]  

(13)

\[
y_{ij} = 0 \quad \forall j \in F_0
\]  

(14)

\[
x_{ijk} \in \{0,1\} \quad \forall i, j \in V', \forall k \in K
\]  

(15)

\[
l_k \geq 0 \quad \forall k \in K
\]  

(16)

The objective function (1) minimizes four criteria simultaneously, the total traveled distance and the refueling cost and difference between each tours time with average of all ones. The value of variable \( l_k \) determine with respect to constraints (11). Constraints (2) ensure that each customer’s demand is definitely satisfied by a vehicle. Constraints (3) ensure that each AFS (and associated dummy vertices) is visited one time or not at all and will have one successor (a customer, AFS or depot vertex) if any vehicle visits it. Constraints (4) guarantee that at each vertex the numbers of arrivals are equal to numbers of departures. Constraint (5) denotes that exactly \( m \) vehicles leave depot. Constraints (6) make certain that each vehicle is assigned to only one trip. Constraints (7) track time at each vertex visited based on vertex sequence and also eliminate the possibility of subtour formation. Constraints (8) calculate tour length for each vehicle. Constraints (9) make sure that each trip is no longer than \( T_{MAX} \). Constraint (10) calculates the average of all tour lengths. Constraints (11) record tour length deviation from average of all tour lengths. Vehicles’ fuel levels based on customer sequence are tracked by Constraints (12). Constraints (13) guarantee that vehicles can pass a route if they have enough fuel for passing it. Constraints (14) reset tank fuel level to \( Q \) when vehicles leave the depot or AFSs. Finally, the decision variables’ binary and positive natures are stated by constraints (15) and (16). For linearizing Constraints (11), new two non-decision variables are presented.

<table>
<thead>
<tr>
<th>Non-decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( o_k )</td>
</tr>
<tr>
<td>( s_k )</td>
</tr>
</tbody>
</table>

The objective function, Constraints (11) and (16) are changed to:

\[
\text{Min } W_1 \left( \sum_{k \in K} \sum_{i \in I, j \in V, i \neq j} d_{ij} x_{ijk} \right) + W_2 \left( \sum_{k \in K} (o_k + s_k) \right) + W_3 \left( \sum_{k \in K} \sum_{i \in I, j \in V, i \neq j} x_{ijk} \right)
\]

\[
n_k - \tilde{t} = (o_k - s_k) \quad \forall k \in K
\]  

(17)

(18)

\[
o_k, s_k = 0 \quad \forall k \in K
\]  

(19)

\[
o_k, s_k \geq 0 \quad \forall k \in K
\]  

(20)

Constraints (19) are nonlinear, but they are satisfied automatically and can be omitted from the model; because of existing of two new positive non-decision variables in the objective function, one of these two variables is always zero, in order to minimize the objective function.

Given to importance of safety aspect, we also present a model that aims to reduce the risk of accident. By increasing the drivers’ working time, the risk of accident increases too. So the model considers, two different risk cost per hour for two levels of tour length while the second one has higher value because of its importance (the risk of the accident increase by tiredness of the
The presented model intends to reduce the risk of the accident through reducing risk cost in level 2 of the whole tours. For presentation of models we use this extra notation:

<table>
<thead>
<tr>
<th>Non-decision variables and parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tc_k$</td>
</tr>
<tr>
<td>$c_1$</td>
</tr>
<tr>
<td>$c_2$</td>
</tr>
<tr>
<td>$th$</td>
</tr>
<tr>
<td>$u_k$</td>
</tr>
</tbody>
</table>

The new GVRP with safety aspect is presented in the following:

$$\min \sum_{k \in K} tc_k$$  \hfill (21)$$

we use all constrains in model 1 except (5), (10), (11), and (20). New constraints are presented as below:

$$\sum_{k \in K} \sum_{j \in V \setminus \{v_0\}} x_{0,jk} \leq m$$  \hfill (22)$$

$$tv_k - u_k \leq th, \quad \forall k \in K$$  \hfill (23)$$

$$tc_k = c_1(tv_k) + (c_2 - c_1)u_k, \quad \forall k \in K$$  \hfill (24)$$

$$u_k \geq 0, \quad \forall k \in K$$  \hfill (25)$$

The objective function (21) minimizes the total risk cost that is computed with constraint (24). Constraint (22) denotes that up to $m$ vehicles can leave depot. Constraints (23) denote that if time length passes the predefined threshold, the risk cost for level 2 is calculated. Constraints (24) calculate the total risk cost for each vehicle. The $u_k$ positive nature is stated by Constraint (25).

4. Computational experiments

In the following sections, the effect of different parameters is investigated to show the model performance as well as checking its validity. Furthermore for investigating the effect of presented model in medium and large size, a genetic algorithm is developed. Different variants of genetic algorithm (GAs) have been used successfully for solving wide variants of NP-hard problems, such as the quadratic assignment problem and several variant of the TSP and the VRP problems[16] The main success of GAs are mainly due to their simplicity and great flexibility in dealing with complex functions[17]. These are the major reasons for selecting GA as an optimization method in this paper. The details of the algorithm aren’t presented because of space limitation. The model is coded by Gams software version 22 for small sized and the algorithm is coded by Matlab software. The data that is used are available at http://neo.lcc.uma.es/vrp/vrp-instances/.

4.1. Effect of presented GVRPRB model

Augerat et al. [18] introduced three sets of instances, of which part A, A-N32-K8 was used for solving small size problems (1-25 customer) in table 2. Instead of using all customers in the instance, each instance only contains the first $(n+s)$ nodes. For example, “A-n11-4s” uses the first 15 nodes: the first 11 nodes as customers and remained 4 nodes (from 12 to 15) as stations. The driving range is set to $Q = 2d_{max}$, where $d_{max}$ is the maximal Euclidean distance between any two points in the network. For samples 1 to 7, the amount of $W_1, W_2$ and $W_3$ for the first model are set to 1, 2 and 100 respectively and for remained samples the amount of $W_2$ changes to 5. Based on
the results of several experiments, the parameter setting is determined as follows: (1) The maximal number of iterations is set as 100, 300 and 800 for first, second and other remained instances. (2) The number of populations for two first samples is set to 10 and 20 for others. The amounts of $T^{MAX}$ for different instances are presented in the last column of table 2. In table 2, "**" represents feasible solutions found by Gams within 3 hours (hrs). "#" denotes that Gams failed to obtain a feasible solution in 3 hrs. The data in columns 5-7 is obtained by averaging data from 5 times run of the genetic algorithm. The gaps in column 8 is defined as (corresponding average objective value - objective value obtained by Gams)/objective value obtained by Gams.

### TABLE2. Results of comparison between Gams and genetic algorithm for the generated instances

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Sample</th>
<th>Gams</th>
<th>GAs</th>
<th>$T^{MAX}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Result</td>
<td>Time(s)</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>A-n5-2s</td>
<td>576</td>
<td>6.8</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>A-n6-2s</td>
<td>595</td>
<td>10.71</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>A-n8-1s</td>
<td>626</td>
<td>2132.9</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>A-n11-4s</td>
<td>698*</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>A-n15-4s</td>
<td>894*</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>A-n20-4s</td>
<td>768*</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>A-n25-4s</td>
<td>#</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>A-n30-5s</td>
<td>#</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>A-n40-5s</td>
<td>#</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>A-n50-4s</td>
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</tr>
<tr>
<td>11</td>
<td>A-n60-4s</td>
<td>#</td>
<td>10800</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>A-n75-4s</td>
<td>#</td>
<td>10800</td>
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</tr>
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<td>#</td>
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</tr>
<tr>
<td>14</td>
<td>Tia150a-n145-4s</td>
<td>#</td>
<td>10800</td>
<td>2</td>
</tr>
</tbody>
</table>

$|k|$: minimum number of used vehicles

Figure 1 shows the difference between maximum and minimum tours length in classic GVRP and GVRPRB for each sample of table 2. The data is obtained by reporting the difference between maximum and minimum tours length in the best answer. (If the best answer can’t be found by Gams in 3 hrs, the best answer is reported which is found by algorithm in 5 times run). The length difference between routes in GVRP which doesn’t consider social aspect, is substantial greater than GVRPRB. The results show that by using the proposed model, the network will tend to have balanced routes and will cause social benefits.

![Figure 1. Effect of green tour balancing model](image)
4.2. Effect of tour balancing on customer’s waiting time

The outcomes of considering social aspect in classical VRP may be not limit to equity between employees. When firm’s fleet work in parallel, the good distribution can be completed as soon as possible so the customers’ waiting time decreases and freshness of goods increases in several cases in GVRPRB in comparison to GVRP. This result is valuable for firm’s reputations.

![Figure 2. Effect of route balancing on the strategic points customers’ waiting time](image)

4.3. Effect of increasing the risk cost of accident in second level

As demonstrated in Figure 3, by increasing of the risk cost in level 2 from $c_2 = 10$ to $c_2 = 110$, the difference between tour lengths will be decreased to reduce the amount of total risk cost. (the amount of risk in level one is fixed and equal to 10). It leads to a reduction in the potential of the accident caused by tiredness of drivers who work in lengthier tours.

![Figure 3. Effect of increasing the risk cost of accident in second level](image)

5. Conclusion

The necessity of attention to environmental, social and safety aspects beside economic aspect in design of distribution networks, leads to increase the number of researches in this area. In this paper, GVRPRB is introduced and two models that take into account social and safety aspects for a fleet of AFVS are presented. The models aim to minimize the differences between tour lengths that lead to maximization of social fairness and minimization of the accident risk related to tiredness of drivers. Analysis of results to assess the effect of key parameters of the model in small, medium and large size instances confirms the validity of the proposed model and also highlights the social aspects effects in such networks.

6. References