Modeling of Periodic Location Routing Problem with Time Window and Satisfaction Dependent Demands

N. Nasherahkami[1], M. Bashiri[2], J. Bagherinejad[3]
[1]Department of Industrial Engineering, Payam noor University, Tehran, Iran
[2]Faculty of engineering Department of Industrial Engineering, Shahed University, Tehran, Iran
[3]Department of Industrial Engineering, Alzahra University, Tehran, Iran
(Bashiri@shahed.ac.ir)

Keywords - **In most of real cases such as distribution of perishable food with short life time or medicine products, delivery in shortest time or in specific time window is important, so ignoring these requirements may affect customer's satisfaction. In some cases, customers have different specific time windows in different periods and are eager to receive their demands in mentioned time windows. Therefore the predefined demand of customers in each period may be decreased due to the violation of time windows in previous periods. Since classical LRPs are unable to handle these kinds of assumptions simultaneously, so a new model of periodic location problem is proposed which minimizes the lost demand costs over the multiperiods. To show the verification of model and comparison with classic PLRP, a numerical example is used. The results show the efficiency of the proposed model.**

Keywords - **Customer satisfaction, Lost demands, Periodic location routing problem, Time window**

I. INTRODUCTION

Facility Location Problem (FLP) is defined as determining the optimal locations for a set of facilities such as factories/warehouses/service centers and etc. The Vehicle Routing Problem (VRP) is an integer programming seeking to find distribution routes between depots and customers. Location Routing Problem (LRP) which is combinatorial optimization problem and part of distribution management considers both of VRP and FLP decisions. LRP is surveyed and classified by [1] and [2]. One of the applied assumptions in LRP is time window. According to this assumption, delivery should be occurred in the interval [a, b], in which a and b are the earliest and the latest allowable times that the service should be taken place.

Another extension of routing problem is periodic vehicle routing (PVRP). In this case, customers can be visited in more than one period over the planning horizon. In classic Periodic Location Routing Problem (PLRP), it is not important for a customer to be visited in specific time window and customer's demand is fixed during the horizon. In this paper, we present a novel variant of PLRP with assumption of soft time window (PLRPSTW) where it's important to service to customer during its time window. The main contribution of this paper is as follows: Servicing a customer out of time window in each period leads to customer's dissatisfaction and as a result, a partial of customer's demand in next periods will be lost. This losing occurs cumulative and gradually over the planning horizon. Based on authors' knowledge, there is no previous study on PLRP which has been focused on these assumptions.

There are some real situations consider these assumptions, for example distribution of dairy products to retailers, distribution of medicine products to treatment centers or pharmacies are some applications of this problem. In these situations and other similar situations, delivery of demand in desired times helps to have customers loyalty.

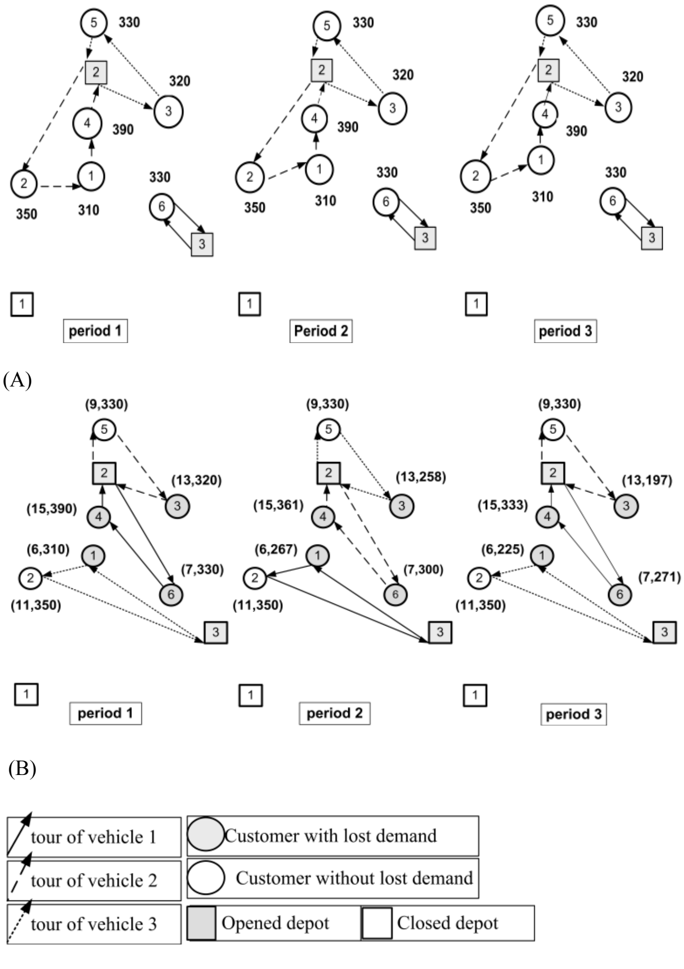

Fig. 1. The structural difference between classic PLRP (A) and proposed problem (B).

Fig. 1 depicts structural differences between our model and classic PLRP as contribution of this research. As shown in Fig. 1(A), in classic PLRP customer's demand is fixed over the horizon which is shown at the top of each circle. According to Fig. 1(B), for each customer in each period two components (a,b) have been defined, a refers to arrival time of vehicle to the customer and b refers to customer demand in the same period. Since customers 1,3,4,6 have been visited out of their time windows in all periods, then their demands have been decreased during the horizon. The rest of this paper is organized as follows. Section II includes a literature review. Definition of problem and formulation of model are presented in Section III and IV respectively. Numerical example is introduced in section V. Section VI shows the results of sensitivity analysis on model. The result of comparison between proposed model and classic PLRP is shown in section VII. Conclusion is discussed in section VIII.

II. LITERATURE REVIEW

In recent years, several studies have been conducted on VRPTW, but assumption of customer's satisfaction and penalty due to the violation of time window has newer history. Reference [3] modeled a perishable food delivery problem with two kinds of soft and hard time windows. In this problem delivery time and speed of vehicle are dependent on traffic conditions and they are assumed to be stochastic. One of the components of its objective is minimizing penalty cost due to violation of time window. Different service levels as customer satisfaction levels based on fuzzy membership function were shown in [4]. Reference [5] solved shortest path and scheduling problem with resource constraints and late arrival penalties. Assumption of time window for problems with pickup and delivery demands was proposed in [6]. In this problem, customer satisfaction is proportional to waiting time before time window. The competition between distributors with considering time window was shown in [7] which each customer has two types of demands: time dependent and time independent. For the first type, the partial of demand will be lost if vehicle arrives to customer after its rival. Arrival time of vehicle is stochastic parameter with uniform distribution function and desirability of arriving time is decreased from lower bound to upper bound of the time windows.

In reviewing of the PVRP studies, the visit frequency usually considered as a predefined parameter, but [8] defined it as a decision variable as a service choice.

Routing problem under assumption of two types customers such as inflexible (mandatory) and flexible, introduced in [9]. The mandatory requests must be served strictly within the current period of service and the flexible requests must be served within a certain number of subsequent periods. In some variants of PVRP, customers, orders and feasible service periods may be revealed uncertain (dynamic or stochastic) over the time. One of the first studies showed uncertainty associated with demand in PVRP is related to [10]. In their problem a set of customers can be served either in time period t or in time period t+1. Decision is that which subset of the customers should be served in current period and which customers are postponed to the next period in such a way that the sum of the costs over periods is minimized. Reference [11] proposed a multiperiod TSP with stochastic urgent and regular demands which appear stochastically at customer nodes. Urgent demands have to be satisfied immediately while regular demands can be satisfied either immediately or next day. The assumption of dynamic periods and demands was shown in [12]. The PVRP with time window is introduced in [13] , [14] ,[15]. In [14], the vehicles are not allowed to wait before time window. Reference [15] introduced a novel case of problem where each customer has time window comprising several periods. In each period there are two types of customers. For the first type, belonging the next period to time window is deterministic but for second one it's probabilistic. A model with assumption of limitation on visit quota in order to minimize the number of required vehicles for each customer was proposed in [16]. Assumption of time window in LRP was shown in [17]. Travel times in this research have been considered as fuzzy numbers.

The PLRP has newer history in comparison with another subject in LRP researches. According to our findings, most researches about this issue are related to [18], [19] and [20]. In this research for the first time Prodhun combined the PVRP and LRP into an even more realistic problem as PLRP covering all decision levels. She proposed different heuristic and metaheuristic approaches such as Memetic Algorithm, ELSxPath Relinking and evolutionary local search for generate initial solutions and improve them. Reference [21] introduced a new model about PLRP which captures the difference in the scope of the location and routing decisions by considering different scales within the time horizon. In their problem the tactical routing decisions are made at each time period $t \in T$, whereas the strategic location decisions are only made at a subset of time periods of the planning horizon. Reference [22] proposed a sequential and parallel large neighborhood search algorithms for solving PLRP. Table 1 describes main differences about some features of the proposed model and other related models.

III. PROBLEM DEFINITION

In a nonexclusive environment, satisfying the customers is very important, because it leads to gain more share of market compared with rivals and increases the profit. In this problem violation of time window decreases demands over the multiperiod. Then in commercial relationship, reduction of expected demands in the long term makes considerable cost for these companies. This dissatisfaction may be caused to customers gradually intend to cut common business communications. In proposed problem, each customer has a visit frequency. According to this frequency, a specific combination of day as a service pattern is assigned to each customer. The main assumptions considered in this problem are discussed as following: Each route must begin and end at the same depot within the same day and its total load must not

TABLE 1

Different aspects in proposed model comparing to previous studies

Feature	Proposed model	Another related model
Waiting time before time window	Delivery immediately after vehicle arrival	Considering waiting time for vehicles [17]
Service before time window	Allowed	Not allowed [14]
Lateness penalty in each period	No penalty in the same period (penalty in next periods)	Lost sale cost in the same period as penalty [7]
Earliness penalty in each period	No penalty in the same period (It affects on next periods demand)	Waiting time cost in the same period as penalty [23]
Customer's demand in each period	Satisfaction dependent demand over the horizon	Static demand over the horizon [20]
Visit frequency	Predefined parameter	Decision variable [8]
Objective function	Lost demand cost in addition of location cost, routing cost and vehicle cost	Location cost, routing cost and vehicle cost [20],[24]

exceed the vehicle capacity. Each node is visited only once by a single vehicle in each period. The fleet of vehicles is heterogeneous. As soon as the vehicle arrives to customer, delivery will begin immediately. In proposed model, unlike the classical models, violation of time window in each period doesn't cause to penalty in the same period. Customer's demand may be decreased over the planning horizon, but it is not allowed to be finished.

IV. MODEL FORMULATION

The problem is defined on a horizon composed of P periods (days) which O,I are set of nodes refer to potential depots and customers. In proposed model N is the set of customer's visit sequences related to the vehicle belongs to set K. Used parameters are discussed as following:

F_i : Visit frequency of customer i

H_{ip} : If customer i is visited in period p (Binary parameter)

t_{ij} : Travel time between customer i and customer j

tt_{oi} : Travel time between depot o and customer i

(el_i , ll_i) : Lower and upper bounds of time window at customer i

$Qveh$ & $Qdep$: Capacity of vehicle and depot respectively

DFC & VFC : Depot cost and vehicle cost

LDR & RR : Costs per unit of lost demand and travel time

Δ : Rate of penalty

A. Decision Variables

To formulate the proposed model, the following binary variables are used: Y_o = 1 if depot o is opened, Z_{io} = 1 if customer i is assigned to depot o, W_{okp}=1 if vehicle k is assigned to depot o in period p, R_{ikp}=1 if vehicle k is assigned to customer i in period p, X_{iknp} = 1 if customer i is visited by vehicle k in sequence n in period p, and β^{+}_{ip}= 1 and β^{-}_{ip} =1 if the vehicle k arrives to customer i before and after time window in period p respectively. Another positive variable are used: ATT_{ip} as reaching time to customer i in period p, AR_{ip} as customer's demand in period p (after deducting penalties), LD_{ip} as lost demand related to customer i in period p.

$$\text{Min} \quad \sum_{o\in O} DFC * Y_o + \sum_{o\in O}\sum_{k\in K}\sum_{p\in P} VFC * W_{okp} + \tag{1}$$

$$\sum_{i\in I}\sum_{p\in P} LRD * LD_{ip} + \sum_{o\in O}\sum_{k\in K}\sum_{p\in P} RR * AT2_{okp}$$

$$+ \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{n\in N}\sum_{p\in P} RR * X_{iknp} * X_{jknp}$$

$$+ \sum_{o\in O}\sum_{k\in K}\sum_{p\in P} RR * AT3_{okp}$$

$$\sum_{i\in I} Z_{io} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{i\in I} Z_{io} \le BM * Y_o \qquad \forall o \in O \tag{3}$$

$$\sum_{i\in I} Z_{io} * D_i \le Qdep \qquad \forall o \in O \tag{4}$$

$$\sum_{o\in O} W_{okp} \le 1 \qquad \forall k \in K , \forall p \in P \tag{5}$$

$$W_{okp} \le Y_o \qquad \forall o \in O, \forall k \in K , \forall p \in P \tag{6}$$

$$\sum_{k\in K} R_{ikp} * W_{okp} = Z_{io} \quad if\ H_{ip} = 1, \forall\, i \in I , o \in O, \forall p \in P \tag{7}$$

$$\sum_{o\in O} W_{okp} \le \sum_{i\in I} R_{ikp} \qquad \forall\, k \in K, \forall p \in P \tag{8}$$

$$\sum_{k\in K} R_{ikp} \le H_{ip} \qquad \forall i \in I , \forall p \in P \tag{9}$$

$$\sum_{i\in I} AR_{ip} * R_{ikp} \le Qveh \qquad \forall k \in K , \forall p \in P \tag{10}$$

$$\sum_{n\in N} X_{iknp} = R_{ikp} * H_{ip} \qquad \forall i \in I, k \in K , \forall p \in P \tag{11}$$

$$\sum_{n\in N} (X_{iknp} - X_{ikn-1p}) \le 0 \quad \forall k \in K, n \in N, n > 1, p \in P \tag{12}$$

$$\sum_{i\in I} X_{iknp} \le 1 \qquad \forall k \in K, n \in N, p \in P \tag{13}$$

$$AT_{iknp} \ge AT_{jkn-1p} + t_{ji} - BM * \left(2 - X_{iknp} - X_{jkn-1p}\right) \quad \forall\, i,j \in I, k \in K, n \in N, n > 1, p \in P \tag{14}$$

$$AT_{iknp} \le AT_{jkn-1p} + t_{ji} + BM * \left(2 - X_{iknp} - X_{jkn-1p}\right) \quad \forall\, i,j \in I, k \in K, n \in N, n > 1, p \in P \tag{15}$$

$$AT_{iknp} \le \sum_{o\in O} tt_{oi} * W_{okp} + BM * \left(1 - X_{iknp}\right) \quad \forall\, i \in I, k \in K, n \in N, n = 1, p \in P \tag{16}$$

$$AT_{iknp} \geq \sum_{o \in O} tt_{oi} * W_{okp} - BM * (1 - X_{iknp}) \quad \forall\, i \in I, k \in K, n \in N, n = 1, p \in P \qquad (17)$$

$$AT_{iknp} \leq BM * X_{iknp} \quad \forall\, i \in I, k \in K, n \in N, p \in P \qquad (18)$$

$$ATT_{ip} = \sum_{k \in K} \sum_{n \in N} AT_{iknp} * H_{ip} \quad \forall\, i \in I, p \in P \qquad (19)$$

$$el_i - ATT_{ip} \leq BM * \beta^{+}_{ip} \quad if\ H_{ip} = 1 \quad \forall i \in I, p \in P \qquad (20)$$

$$ATT_{ip} - ll_i \leq BM * \beta^{-}_{ip} \quad if\ H_{ip} = 1 \quad \forall i \in I, p \in P \qquad (21)$$

$$el_i - ATT_{ip} \geq -BM * (1 - \beta^{+}_{ip}) \quad if\ H_{ip} = 1 \quad \forall i \in I, p \in P \qquad (22)$$

$$ATT_{ip} - ll_i \geq -BM * (1 - \beta^{-}_{ip}) \quad if\ H_{ip} = 1 \quad \forall i \in I, p \in P \qquad (23)$$

$$LD_{ip} = (el_i - ATT_{ip}) * \beta^{+}_{ip} * H_{ip} * \Delta * D_i + (ll_i - ATT_{ip}) * \beta^{-}_{ip} * H_{ip} * \Delta * D_i \quad \forall i \in I, p \in P \qquad (24)$$

$$AR_{ip} = D_i * H_{ip} - \sum_{pp \in P, pp < p} LD_{ipp} * H_{ipp} * H_{ip} \quad if\ H_{ip} = 1,\ \forall i \in I, p \in P \qquad (25)$$

$$AT2_{okp} > tt_{oi} * W_{okp} - BM * (1 - X_{iknp}) + \sum_{j \in I} (X_{jkn-1p} * X_{iknp}) \quad if\ H_{ip} = 1, \forall i \in I, k \in K, n \in N, o \in O, p \in P \qquad (26)$$

$$AT3_{okp} > tt_{oi} * W_{okp} - BM * (1 - X_{iknp}) + \sum_{j \in I} (X_{jkn+1p} * X_{iknp}) \quad if\ H_{ip} = 1, \forall i \in I, k \in K, n \in N, o \in O, p \in P \qquad (27)$$

$$Y_o, Z_{io}, X_{iknp}, W_{okp}, R_{ikp}, \beta^{+}_{ip}, \beta^{-}_{ip} \in \{0,1\} \quad \forall o \in O, \forall i \in I, k \in K, n \in N, p \in P \qquad (28)$$

$$AT_{iknp} > 0, ATT_{ip} > 0, AR_{ip} > 0, LD_{ip} > 0, AT2_{okp} > 0, AT3_{okp} > 0, \quad \forall i \in I, p \in P \qquad (29)$$

In this formulation, the objective function (1) refers to 4 concepts such as minimizing location cost, vehicle cost, lost demand cost and routing cost which have been shown in components 1, 2, 3 and 4 to 6, respectively. Equations (2) and (3) state that each customer must be allocated to only one open depot. Equations (4) and (10) are capacity constraints associated to depot and vehicle respectively. Equations (5) and (6) imply that each vehicle is allocated to maximum one opened depot in each period and it is used once. Equation (7) imposes that each customer's location is visited by only one vehicle in each period. Equations (8) and (9) ensure that vehicle is allocated to depot and customer in each period if it is used for service. Equations (11) - (13) are related to routing and refer to each customer is visited in specific sequence (*n*). Two pair of constraints such as (14) - (15) and (16) - (17) are used to show reaching time to the customer *i* visited in sequence of *n*. The first pair relates to customer *i* expect of the first customer and the second one relates to the first customer. Arriving time to each customer has been shown in (18) and (19). Time window is referred by (20) - (23). Equation (24) shows the penalty for violation of time window and (25) refers to customer's demand after deducting cumulative penalties. Equations (26) and (27) are linked to the routing cost in objective function. The decision variables are defined in (28) – (29).

V. NUMERICAL EXAMPLE

In order to evaluate the efficiency of proposed model, we applied a numerical example introduced in fig. 1 and solved it with Cplex in GAMS 23.6 program. The example is constructed as follows: Coordination of the nodes is according to first instance of cordeou's PLRP data set. Travel times between nodes are defined as coefficient of distance. D_i is drawn from a uniform distribution between 300 and 400. Visit frequency for all customers is 3. Another parameters are defined as follows: el_i=(8,11,10,13,9,8), ll_i=(9,12,11,14,10,9), *Qveh*=1000, *Qdep*=3000, *DFC*=50, *VFC*=50, *LDR*=5, *RR*=30 and Δ=0.08. The scheme of designed routes in each period has been shown in fig. 1 in section I. The main results of solving example are as follows: violation of time window in each period is 6, lost demands in second period is 164 and in third period is 160.

VI. SENSITIVITY ANALYSIS

In order to figure out the sensitivity of the model to some important parameters such as the range of time window, rate of penalty, vehicle cost and depot cost, we considered the example introduced in section I and solved it with different levels of parameters. The proposed model will act closer to the classic PLRP in two scenarios such as extension of the range of time window and decreasing rate of penalty. If the range of time window is extended to el_i=(8,11,9,13,9,8), ll_i=(10,13,12,15,12,12), lost demand is decreased from 324 to 188 according to fig. 2.

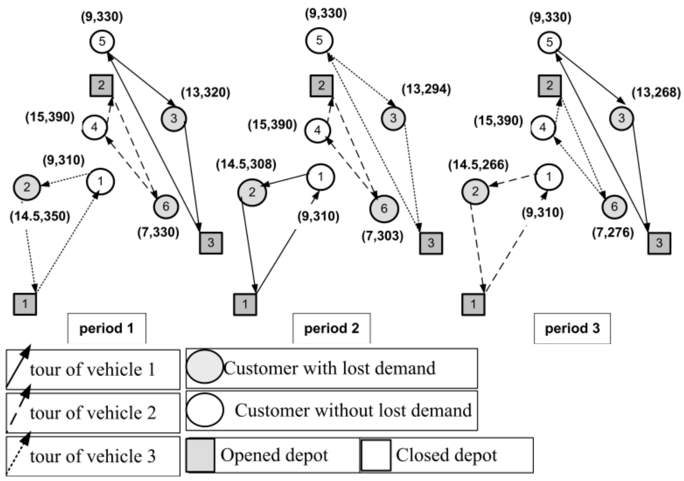

Fig. 2. Effect of extension of time window on problem result

Changing in two other parameters, change the design of routes and affects the total costs. By increasing the *VFC* from initial value 50 to new value 2000, the number of using vehicles is decreased from 9 to 7, so the value of lost demands subsequently increased from 324 to 991. Also increasing of the *DFC* from initial value 5 to new value 500 increases the lost demand 324 to 701. In last case, one of three depots is opened.

VII. COMPARISION of the PLRPTW WITH the CLASSIC PLRP

One of the main aspects of PLRP discussed in proposed model is customer's satisfaction based on delivery time. Table 2 provides a comparison between the proposed model and the classic PLRP [20]. The results show that the proposed model performs better to minimize violation of time windows and lost demands.

TABLE 2
Comparison between the PLRPSTW and the classic PLRP

MODEL	Total time violations	Total lost demands	lost demands cost
Classic PLRP	69	1224	6120
PLRPSTW	18	324	1620

VIII. CONCLUSION

In this paper a new version of PLRP has been introduced such that arriving to customer's place out of time window in each period cause to cumulative decreasing of customer's demand in the next periods. The results of this research can be used in real situations where delivery time is important obligational factor in long term contract between distribution center and customers. Comparison between the proposed model with classic PLRP illustrates that proposed model is more effective to reduce the lost demands. As a future study developing a heuristic algorithm to solve large instances of the problem is suggested.

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