

## Failure mode and effects analysis by a new improved method-based preference solution index under uncertainty

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### Abstract

Competitive nature of global marketing force industries to continually improve their systems' quality and costs to guarantee the customer satisfaction. Failure mode and effects analysis (FMEA) is an analytical approach developed to help organizations reduce their product development time, price and improve the quality, in order to secure their competitive advantages in global marketing. Some important drawbacks in traditional FMEA make it doubtful, such as not to considering the uncertainty of human judgments, considering equal weights for risk factors. To this end, several researchers have proposed new methods and approaches to overcome these drawbacks. This paper presents a new improved method based on preference solution index (PSI) under interval-valued intuitionistic fuzzy (IVIF) environment to conduct FMEA. In this method, there is no need to predetermine the relative weights of risks, and the decision matrix is obtained based on IVIF numbers; and hence, the vagueness handling will be enhanced. A practical example will be applied to show applicability and potential strength of the proposed method.

*Keywords:* Failure mode and effects analysis (FMEA), preference solution index, interval-valued intuitionistic fuzzy sets

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### 1. Introduction

Extensive global competition force industries to reduce their product development time, price and improve the quality, in order to secure the customer satisfaction. Failure mode and effects analysis (FMEA) is a well-known approach, developed to eliminate or avoid potential failure modes before occurring and imposing costs to system. This will be realized by prioritize the failures risks, perform corrective or preventive actions, to reduce the probability of failures and their subsequent drawbacks, i.e., customer dissatisfaction and reduced competitiveness advantages [1-5].

Conventional FMEA is performed through computing and prioritizing the risk priority number (RPN) of failure modes. RPN index is calculated by multiplying three risk factors, i.e. severity of failures (S), likelihood of occurrence (O) and probability of missing the detection of the failures (D). Despite of extensive use of FMEA in different industries, it suffers from some considerable weak points.

These factors are evaluated by a cross-functional FMEA teams including experts from different areas. Considering recent studies in this field reveals that despite extensive use of conventional FMEA in various industries, there are some major debates in RPN computation method, including: 1. it is assumed that O, S and D have an equal importance; 2. despite of different risks hidden in different failures, their combinations of O, S and D may produce the same RPN value, and therefore, it may result in false decisions [6].

In recent years, many researchers have studied the FMEA and have proposed several approaches to overcome traditional FMEA's weak points. There are several studies employing decision making methods in developing new FMEA approaches [7-10]. Recently, Maniya and bhatt [11] have proposed a new method called preference solution index (PSI) to deal with decision making problems. The main strength of this method is that, there is no need to predetermine the risk factors' relative weights. However, it has been not yet applied to FMEA.

One of most important weak points of traditional FMEA is the necessity of precise evaluation of failure modes. FMEA is based on experts' judgments, and hence it is subject to vagueness hidden in human opinions [3, 12,13]. The concept of interval-valued intuitionistic fuzzy sets (IVIFSs) introduced by Atanassov and Gargov [14] is a generalized form of intuitionistic fuzzy sets (IFSs) theory, which can improve decision modelling. It considers membership and non-membership functions as interval values; and hence, it has more potential to handle vague situations rather than IFSs [15].

Based on explanations provided above, a new IVIF-method for FMEA is reported in this article which is based on new decision making method, called PSI method, which has been not yet employed to solve FMEA. In the other hand, the analysis is carried out under an IVIF-environment to manage vagueness and uncertainty hidden in the decisions through considering membership and non-membership functions as interval values. Moreover, since, the risk factors' weights are determined objectively in this method, there is no need to predetermine them by experts' opinions. Finally, an application example is used to illustrate the applicability of proposed method.

**2. Preliminaries**

In this section, the key concepts of IVIFSs theory are explained:

*Definition 1:* Let  $X$  be an ordinary finite, nonempty set. An IVIFS in  $X$  is defined as [16]:

$$\tilde{A} = \{ \langle x, \bar{\mu}_{\tilde{A}}(x), \bar{\nu}_{\tilde{A}}(x) \rangle \}$$

where  $\bar{\mu}_{\tilde{A}}(x) \subset [0,1]$  and  $\bar{\nu}_{\tilde{A}}(x) \subset [0,1]$  are interval-valued membership and non-membership functions, respectively. In fact,  $\bar{\mu}_{\tilde{A}}$  and  $\bar{\nu}_{\tilde{A}}$  are closed intervals instead of real numbers and  $s_1 \bar{\mu}_{\tilde{A}}(x) + s_1 \bar{\nu}_{\tilde{A}}(x) \leq 1, \forall x \in X$ .

In this paper, an IVIF  $\tilde{A}$  is denoted by  $\langle [a_{\tilde{A}}, b_{\tilde{A}}], [c_{\tilde{A}}, d_{\tilde{A}}] \rangle$ , where  $[a_{\tilde{A}}, b_{\tilde{A}}] \subset [0,1], [c_{\tilde{A}}, d_{\tilde{A}}] \subset [0,1]$ . For each element  $x, \tilde{\pi} = [1 - b_{\tilde{A}}(x) - d_{\tilde{A}}(x), 1 - a_{\tilde{A}}(x) - c_{\tilde{A}}(x)]$  is called hesitancy degree of an IVIF of  $x \in X$  in  $\tilde{A}$ . It can be seen that  $\tilde{\pi} \subset [0,1]$ .

Atanassov [14] and Xu [17] defined four operational laws of IVIFNs, each of them is introduced as follows:

$$\tilde{A} \oplus \tilde{B} = \langle [a_{\tilde{A}} + a_{\tilde{B}} - a_{\tilde{A}} \cdot a_{\tilde{B}}, b_{\tilde{A}} + b_{\tilde{B}} - b_{\tilde{A}} \cdot b_{\tilde{B}}], [c_{\tilde{A}} \cdot c_{\tilde{B}}, d_{\tilde{A}} \cdot d_{\tilde{B}}] \rangle \tag{1}$$

$$\tilde{A} \otimes \tilde{B} = \langle [a_{\tilde{A}} \cdot a_{\tilde{B}}, b_{\tilde{A}} \cdot b_{\tilde{B}}], [c_{\tilde{A}} + c_{\tilde{B}} - c_{\tilde{A}} \cdot c_{\tilde{B}}, d_{\tilde{A}} + d_{\tilde{B}} - d_{\tilde{A}} \cdot d_{\tilde{B}}] \rangle \tag{2}$$

$$\lambda \tilde{A} = \langle [1 - (1 - a_{\tilde{A}})^\lambda, 1 - (1 - b_{\tilde{A}})^\lambda], [(c_{\tilde{A}})^\lambda, (d_{\tilde{A}})^\lambda] \rangle, \lambda > 0 \tag{3}$$

$$\tilde{A}^\lambda = \langle [(a_{\tilde{A}})^\lambda, (b_{\tilde{A}})^\lambda], [1 - (1 - c_{\tilde{A}})^\lambda, 1 - (1 - d_{\tilde{A}})^\lambda] \rangle, \lambda > 0 \tag{4}$$

*Definition 2:* Let  $\tilde{A}_1$  and  $\tilde{A}_2$  be two IVIFNs, then score function  $S(\tilde{A})$  and accuracy function  $H(\tilde{A})$  will be obtained by the following form [16]:

$$S(\tilde{A}) = \frac{1}{4} [2 + a_{\tilde{A}} - c_{\tilde{A}} + b_{\tilde{A}} - d_{\tilde{A}}] \tag{5}$$

$$H(\tilde{A}) = a_{\tilde{A}} + b_{\tilde{A}} - 1 + \frac{c_{\tilde{A}} + d_{\tilde{A}}}{2} \tag{6}$$

where  $S(\tilde{A}) \in [0,1]$  and  $H(\tilde{A}) \in [-1,1]$ . The larger value of  $S(\tilde{A})$  the higher the IVIF is. On this basis, the largest and the smallest IVIFN are  $\langle [1,1], [0,0] \rangle$  and  $\langle [0,0], [1,1] \rangle$  respectively. In fact,  $S(\tilde{A})$  and  $H(\tilde{A})$  perform like expectation and variance. First,  $S(\tilde{A})$  is applied to compare to fuzzy number. If expectation values of two numbers are equal, then variances are compared to rank numbers.

*Definition 3:* Let  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_2$  be two IVIFNs, then the Euclidean distance operator can be defined as [18]:

$$d(\tilde{\alpha}_1, \tilde{\alpha}_2) = \sqrt{\frac{1}{4} ((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2)} \tag{7}$$

*Definition 4:* Let  $\tilde{x}_j = \langle [a_j, b_j], [c_j, d_j] \rangle (j \in N)$  be a collection of IVIFNs, and  $\lambda_j = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$  be the weight vector of  $\tilde{x}_j (j \in N)$ , where  $\lambda_j$  indicates the importance degree of  $\tilde{x}_j$ , satisfying  $\lambda_j \geq 0 (j \in N)$  and  $\sum_{j=1}^n \lambda_j = 1$ , and let interval-valued intuitionistic fuzzy weighted averaging (IVIFWA):  $\Phi^n \rightarrow \Phi$  if [18]:

$$\begin{aligned} \tilde{x}_j = IV & (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n) \\ &= \sum_{j=1}^n \lambda_j \cdot \tilde{x}_j \\ &= \left\langle \left[ 1 - \prod_{j=1}^n (1 - a_j)^{\lambda_j}, 1 - \prod_{j=1}^n (1 - b_j)^{\lambda_j} \right], \left[ \prod_{j=1}^n (c_j)^{\lambda_j}, \prod_{j=1}^n (d_j)^{\lambda_j} \right] \right\rangle \end{aligned} \tag{8}$$

**3. Proposed IVIF-PSI method**

Based on IVIF preliminaries, the proposed method can be explained in the following steps:

*Step 1.* Establishment of cross-functional FMEA team

FMEA is a group analysis that requires cross-functional teams from different areas of study. Due to sensitivity of failures detection issue, establishing this team is first and one of the most important steps in FMEA.

*Step 2.* Evaluation of failure modes with respect to risk factors

Supposing there are  $m$  potential failure modes,  $F_i (i = 1, 2, \dots, m)$  assessed with respect to  $n$  risk factors,  $R_j (j = 1, 2, \dots, n)$ , using IVIF numbers,  $\tilde{x}_i = \langle [a_{iA}(x), b_{iA}(x)], [c_{iA}(x), d_{iA}(x)] \rangle$ . Following matrix shows assessment of  $m$  failure modes with respect to  $n$  risk factors:

$$X = (\tilde{x}_i)_{m \times n} = \begin{bmatrix} \langle [a_{11}, b_{11}], [c_{11}, d_{11}] \rangle & \dots & \langle [a_{1n}, b_{1n}], [c_{1n}, d_{1n}] \rangle \\ \vdots & \ddots & \vdots \\ \langle [a_{m1}, b_{m1}], [c_{m1}, d_{m1}] \rangle & \dots & \langle [a_{mn}, b_{mn}], [c_{mn}, d_{mn}] \rangle \end{bmatrix}$$

*Step 3.* Preference variation value ( $PV_j$ ) calculation

$PV_j$  for each risk factor is obtained by Eq. 9:

$$P_j = \sum_{i=1}^m [X_i - IV_j]^2 \tag{9}$$

*Step 4.* Overall preference value calculation

Overall preference value ( $\varphi$ ) is determined using Eq. 10 as follow:

$$\varphi_j = \frac{v_j}{\sum_{j=1}^3 v_j}, \quad \sum_j \varphi_j = 1 \tag{10}$$

where

$$v_j = 1 - P_j \tag{11}$$

*Step 5.* Preference solution index calculation

Finally, the PSI for each alternative is obtained by the following equation:

$$I_i = \sum_{j=1}^3 \theta_i = \theta_{i1} + \theta_{i2} + \theta_{i3} \tag{12}$$

$$\begin{aligned} &= \langle [a_{i1} + a_{i2} - a_{i1} \cdot a_{i2}, b_{i1} + b_{i2} - b_{i1} \cdot b_{i2}], [c_{i1} \cdot c_{i2}, d_{i1} \cdot d_{i2}] \rangle + \theta_{i3} \\ &= \langle [(a_{i1} + a_{i2} - a_{i1} \cdot a_{i2}) + (\theta_{i3}^1) \\ &\quad - (a_{i1} + a_{i2} - a_{i1} \cdot a_{i2})(\theta_{i3}^1), (b_{i1} + b_{i2} - b_{i1} \cdot b_{i2}) + (\theta_{i3}^2) \\ &\quad - (b_{i1} + b_{i2} - b_{i1} \cdot b_{i2})(\theta_{i3}^2)], [(c_{i1} \cdot c_{i2})(\theta_{i3}^3), (d_{i1} \cdot d_{i2})(\theta_{i3}^4)] \rangle \end{aligned}$$

where

$$\theta_i = \varphi_j \tilde{x}_i = \langle [1 - (1 - a_i)^\lambda, 1 - (1 - b_i)^\lambda], [(c_i)^\lambda, (d_i)^\lambda] \rangle \tag{13}$$

*Step 6.* Prioritizing the alternatives.

The more the  $I_i$  index, the more the risk of failure modes and higher priority of the alternatives can be.

**4. Applicable manufacturing example and discussion**

In this section, the applicability of the proposed new IVIF-PSI method in FMEA is shown by a practical example adapted from [19]. In this paper, authors employ interval-valued intuitionistic fuzzy sets theory, and a new method called standard deviation to determine the weights of the risk factors objectively and prioritize the failure modes. The example of this paper is selected to verify the weight determination in two methods.

This example involves evaluation and prioritization of seven failure modes by applying IVIF-PSI method. Table 1 shows the IVIF-decision matrix.

Table 1: interval valued intuitionistic fuzzy decision matrix

Risk Factors		O	S	D
Failure Modes	FM1	$\langle [0.7,0.8], [0.1,0.2] \rangle$	$\langle [0.8,0.9], [0.1,0.1] \rangle$	$\langle [0.7,0.9], [0.1,0.1] \rangle$
	FM2	$\langle [0.5,0.7], [0.1,0.2] \rangle$	$\langle [0.6,0.7], [0.1,0.3] \rangle$	$\langle [0.4,0.5], [0.2,0.4] \rangle$
	FM3	$\langle [0.3,0.5], [0.1,0.3] \rangle$	$\langle [0.4,0.5], [0.1,0.3] \rangle$	$\langle [0.3,0.6], [0.3,0.4] \rangle$
	FM4	$\langle [0.6,0.7], [0.1,0.2] \rangle$	$\langle [0.7,0.8], [0.1,0.2] \rangle$	$\langle [0.5,0.7], [0.1,0.3] \rangle$
	FM5	$\langle [0.5,0.7], [0.2,0.3] \rangle$	$\langle [0.5,0.7], [0.1,0.3] \rangle$	$\langle [0.4,0.6], [0.2,0.3] \rangle$
	FM6	$\langle [0.3,0.4], [0.4,0.6] \rangle$	$\langle [0.2,0.4], [0.5,0.6] \rangle$	$\langle [0.4,0.5], [0.4,0.5] \rangle$
	FM7	$\langle [0.3,0.5], [0.3,0.5] \rangle$	$\langle [0.4,0.6], [0.3,0.4] \rangle$	$\langle [0.4,0.5], [0.2,0.4] \rangle$

Following the procedure described in section 3, the results in Table 2 will be reached.

Table 2: Preference solution index calculations

Failure modes	IVIF-PSI	S	H	IVIF-ranking (Proposed method)	Crisp ranking (Xu, 2011)
1	$\langle [0.74,0.87], [0.00,0.00] \rangle$	0.90	0.61	1	1
2	$\langle [0.50,0.64], [0.00,0.01] \rangle$	0.78	0.15	3	3
3	$\langle [0.33,0.54], [0.00,0.02] \rangle$	0.71	-0.12	5	5
4	$\langle [0.60,0.74], [0.00,0.00] \rangle$	0.83	0.34	2	2
5	$\langle [0.47,0.63], [0.01,0.01] \rangle$	0.77	0.11	4	4
6	$\langle [0.31,0.44], [0.12,0.14] \rangle$	0.62	-0.12	7	7
7	$\langle [0.37,0.53], [0.03,0.06] \rangle$	0.70	-0.05	6	6

As it can be seen in Table 2, the results obtained from the proposed IVIF-PSI method is equal to Xu's ranking. Therefore, the proposed method is applicable and can be employed more serious in FMEA problems. However, this paper considers uncertainty hidden in human judgments by employing interval-valued intuitionistic fuzzy theory. Therefore, it is expected that in more complicated systems and problem, the proposed IVIF-PSI method has a more reliable performance rather than crisp counterpart. Moreover, since the IVIFSs theory considers two dimension of uncertainties, i.e. membership and non-membership functions, in the interval form, the potential of uncertainty handling through the problem solving is more than similar fuzzy theories.

**5. Conclusion**

Improving performance in terms of the quality and costs is essential part of organisations' plans to secure their competitive advantages in global markets. Failure mode and effects analysis (FMEA) is an analytical approach developed to help organizations improve their system reliability and reduce costs due to failure modes occurrence. Due to considerable weak points of traditional FMEA, improving FMEA procedure is main title of several researches in recent years. This paper presents a new method which extends preference solution index (PSI) method under

interval-valued intuitionistic fuzzy environment to conduct FMEA. In this way some weak points of traditional FMEA are removed: first, a new decision making method is employed to analyze and prioritize the failure modes. Second, there is no need to predetermine risks' relative weights; and finally, interval-valued intuitionistic fuzzy environment is used to handle the opinions vagueness. Further research will concentrate on improving this method by considering group decision making concepts and other possibility theories for representing uncertainty and vagueness hidden in human judgments like D-S evidence theory.

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