9th International Iranian Operations Research conference
(IORC 2016)

Shiraz University of technology

Iranian Operations Research Society

IORC, April 27-29, 2016

Multi-objective design of attribute C control chart: an application of differential evolution algorithm

Amirhossein Amiri[a], Samrad Jafarian-Namin[b]*, Mehrdad Mirzabaghi[c]

[a]Associate Professor, Industrial Engineering Department, Shahed University, Tehran, Iran
[b]PhD. Student, Industrial Engineering Department, Faculty of Engineering, Yazd University, Yazd, Iran
[c]Ph.D. Student, School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

Abstract

C control chart is the most well-known control chart for monitoring the number of nonconformities per inspection unit of constant size. Generally, the design of a control chart requires the specification of sample size, sampling interval, and control limits width. Optimally determining these parameters have been considered from statistical and/or economic aspects in the literature. This study proposes Differential Evolution (DE) algorithm in order to solve a multi-objective economic-statistical model of the C control chart. A numerical example is used to illustrate the algorithm procedure.

Keywords: C Control Chart; Multi-objective Economic-statistical Design; Differential Evolution.

1- Introduction

Statistical Process Control (SPC) is an industry-standard methodology for measuring, controlling, and improving the quality and productivity of manufacturing and service enterprises. A Control chart, among seven basic tools of SPC, graphically shows plotted quality data obtained from a process in time order that can be used to maintain the process in the in-control status and improve it through reduction in the variability of the process through analyzing the process changes over time. Generally, there are two classes of control charts: the variables and the attributes. Attribute control charts are used to monitor discrete and/or categorical data. Among them, the classic Shewhart's C control chart is applied to monitor the number of nonconformities per inspection unit, distributed according to a Poisson distribution [8]. Despite the wider application in real practice for the relative simplicity in dealing with attributes than variables, the C control charts have been largely neglected unfortunately. Thus, in this study, the design of attribute C control charts is considered through parameters related to them.

Generally, the design of a control chart requires specifying sample size (n), sampling interval (h), and control limits width (k). For optimally selecting these parameters, Duncan [5] developed the first economic model. Recently, an algorithm using DE was developed for this reason [7]. However, such pure economic models have been criticized due to poor statistical properties of the designed control chart. Thus, Saniga introduced an economic-statistical model, which minimized the Duncan's cost model subject to statistical constraints [9]. Since the importance of statistical properties is of the same as economics, this approach seems ineffective and therefore, simultaneous optimization of both economic and statistical properties has been considered by researchers. In this field, as a multiple criteria decision-making, Chen and Liao [3] applied an approach to design X control chart in which the efficient solutions were selected using Data Envelopment Analysis. Recently, Jafarian-Namin. et al. [6] have implemented this approach with some changes for multi-objective economic-statistical design of C control chart. Moreover in [1], different C control charts named the classic, the adjusted, the regression-based, and the almost exact control limits have been evaluated through the model proposed in [6].

In this study, a solution algorithm using the DE algorithm is proposed to solve the multi-objective economic-statistical model of the C control chart. Then, obtained results of the DE algorithm are compared with those of published in [6].

[b] Corresponding author: *E-mail address*: Samrad.Jafarian@stu.yazd.ac.ir

2- The C control chart

The classic Shewhart's C control chart, as an attribute control chart, is applied to monitor the number of nonconformities per inspection unit, distributed according to a Poisson distribution as follows:

$$(1) \qquad P(x) = \frac{e^{-c}c^{x}}{x!} \qquad x = 0,1,\ldots$$

with parameter c>0 that represents its mean and variance in a preliminary inspection unit of constant size. Commonly, a control chart has a central line (CL) to define the central tendency, and an upper control limit (UCL) and lower control limit (LCL) to exhibit the scope of expected variation of the plotted data. Hence, if the known value of c (obtained using historical information) is considered as the central line, the k-sigma upper control limit and lower control limit are statistically computed for a C control chart as given in the following equations [8]:

$$(2) \qquad \mathrm{UCL} = nc + k\sqrt{nc}$$

$$(3) \qquad \mathrm{LCL} = nc - k\sqrt{nc}$$

where designing of a C control chart based on n inspection unit (not necessarily integer) is of interest. Hence, new inspection unit is obtained with n multiplied by primary inspection unit (notice that the number of items inspected remains the same among all the new subgroups).
In this way, the production process is permanently sampled in specific time intervals, inspected and the results are interpreted to see whether the outcomes are from one consistent and homogeneous process or not. Under the attitude of quality improvement, the assignable cause or causes of variation in the process must be discovered and eliminated so as to reach a stable and predictable process (i.e., 'in-control' state). In this study, a single assignable cause is assumed to occur.
In some cases, when the calculations yield zero or a negative value for LCL, researchers often set it to zero. However, this violates the definition of a lower control limit and Use of run rules. Because of the importance of points plotting below the LCL, indicating a possible reduction in the number of defects, we define a constraint to consider situations in which it is positive.

3- Multi-Objective Design of the C Control Chart

The proposed *C-MOESD* model is based on Duncan's economic model by some adjustments to adapt with the C control chart and consider economic and statistical properties simultaneously. In this section, the main assumptions of the model are introduced. Then, the proposed *np-MOESD* model is presented.

3-1-Assumptions

In order to simplify the mathematical manipulation and analysis, the following assumptions are considered to be hold:

1. The quality characteristic follows a Poisson distribution,
2. The process is either in-control or out-of-control state only and is initially in the 'in-control' state; that is, $c = c_0$,
3. When a random assignable cause of magnitude δ occurs, leads the process mean to shift from c_0 to $c_0 = c_0 + \delta c_0^{0.5}$,
4. The occurrence of an assignable cause possesses an exponential distribution with mean time $1/\lambda$,
5. The process is allowed to continue during the search and repair.

3-2-C-MOESD model

In addition to statistical perspective, designing a control chart has several economic consequences as presented in [6]. So, the C-MOESD model is:

$$
\begin{aligned}
&Min && E_L(S) \\
&Max && ATS_0(S) \\
&Min && ATS_1(S) \\
&s.t. && E_L \le E_L^{\,U} \\
& && ATS_0 \ge ATS_0^{\,L} \\
& && ATS_1 \le ATS_1^{\,U} \\
& && LCL > 0
\end{aligned} \tag{4}
$$

where, E_L is expected hourly cost, $S=(n, h, k)$ is a possible set of design parameters, ATS_0 is the average time to signal when a false alarm occurs, and ATS_1 is the average time to signal when an assignable cause occurs. In addition, E_L^U, ATS_0^L and ATS_1^U are the desired bounds determined by decision maker, and LCL is lower control limit (see [6] for more information about the assumptions in the model).

4- Differential Evolution

We intend to achieve a well-balanced trade-off between the economic and the statistical features. DE is a population-based, direct-search algorithm for globally optimization of the complicated objective functions. In order to change the population and make new generation, mutation and crossover operators are used. The mutation operator related to each parent i is determined using:

$$v(i) = chrom(a) + \beta[chrom(b) - chrom(c)] \tag{5}$$

where a, b and c are chromosomes ($a \neq b \neq c \neq i$), and β is a positive value named constant of differentiation. Then, in crossover, an offspring is made from each parent and its corresponding mutated vector. A random number is taken from (0,1) for each offspring i and gen j. If the number is lower than or equal to the constant of crossover, the gen takes value from the mutated vector. Otherwise, it takes value from the parent [10]. The proposed algorithm is as follows:

Step 0. Set appropriate parameters to implement the algorithm (see Table 1). In this paper, we assume $1 \le n \le 30$, $0.1 \le h \le 4$ and $0.5 \le k \le 4$ as bounds on each design parameter.

Step 1. Generate initial population randomly according to the bounds of design parameters.

Step 2. (Mutation and crossover) Create mutated vector corresponding to each solution (parents) and then combine the matches (mutated vector and its parent) to make a trial vector (offspring).

Step 3. (Evaluation and selection) Evaluate the vectors of parents and offsprings using the cost function, and dedicate penalty for infeasible solutions according to these constraints: $E_L \le 7$, $ATS_0 \ge 100$, $ATS_1 \le 4$, and $LCL > 0$. Afterwards, select a set of the best vectors for the next iteration using NSGA II selection operator and insert the optimal Pareto front of each iteration into a set W [4].

Table1- Parameters used in present DE algorithm

DE Algorithm Parameters	Magnitude/Method
No. of design parameters in a set	3
Population size	50
Search strategy	random
Constant of Differentiation	0.4
Type of Crossover	Binomial
Constant of Crossover	0.5
Selection scheme	NSGA II
Number of generations	1000

Step 4. Repeat steps 2 and 3 for all predetermined number of generations.
Step 5. Eliminate infeasible solutions from *W* and then select efficient solution(s).
As indicated in step 5, it is essential to select the best solution(s). In this paper, we propose the DEA approach for ranking the solutions.

4-1-Data Envelopment Analysis

DEA is a powerful non-parametric approach to evaluate the relative efficiency of a group of decision making units (DMUs) with multiple inputs and outputs. The first DEA approach is known as the CCR model via generalization of the Farrell's single input, single output efficiency measurement [2]. This linear programming formulation can be either input-oriented or output-oriented. Assuming *n DMU*s, each with *m* inputs and *s* outputs, the efficiency of a specific DMU can be obtained by solving the input-oriented *CCR* model:

$$
\begin{aligned}
&Max \quad E_0(D) = \sum_{r=1}^{s} u_r Y_{r0} \\
&s.t. \quad \sum_{i=1}^{m} v_i X_{i0} = 1 \\
&\qquad \sum_{r=1}^{s} u_r Y_{rj}(D) - \sum_{i=1}^{m} v_i X_{ij}(D) \le 0, \quad j = 1,\ldots,n \\
&\qquad u_r \ge 0, \quad r = 1,\ldots,s \\
&\qquad v_i \ge 0, \quad i = 1,\ldots,m
\end{aligned} \tag{6}
$$

where, u_r is the weight of output *r*, v_i is the weight of input *i*, Y_{rj} is the value of output *r* for *j*th DMU, and X_{ij} is the value of input *i* for *j*th DMU. The performance of each DMU measured, is relative to the remaining DMUs. A DMU is relatively inefficient if $E_0^*<1$ and relatively efficient, strictly or weakly, if $E_0^*=1$. In designing control charts, DMUs refer to feasible combinations of design parameters.
In the *C-MOESD* model, the objectives including E_L and ATS_1 are considered as inputs because of their minimizing nature, and ATS_0 is probed as output. The model should be formulated for each DMU in step 5 of the proposed algorithm to find the set of weights, as decision variables, that maximize the relative efficiency of considered DMU. As a result, at least one of the DMUs will be efficient.

5- A Numerical Example

In order to illustrate the results of the model, we consider the example presented in [6]. Accordingly, the number of nonconformities in a unit of size *n* is supposed to follow a Poisson distribution with mean c_0=4. Moreover, when an assignable cause with the rate of λ=0.01 occurs, it provides a shift of size δ=2 in the process mean (and so c_1=8). The values of other parameters are listed in Table 2.
Table 3 shows the determined efficient units in addition to comparison with pure economic design in [5]. In spite of increasing in cost, statistical performance is improved substantially using C-MOESD model. Moreover, all the objectives of our designs are in the desired limits, while those of pure economic design are not satisfactory at all.
Finally, in Table 4 the efficiency scores of optimal designs using DE are compared with those of published in [6] using Jafarian-Namin. et al.'s algorithm. These results reveal the insufficiency of the pure economic design in the multi-objective space. Moreover, the efficiency of DE algorithm is entirely explicit.

Table2- Input values of parameters

Cost factors	Time factors

M	T	W	f	v	g	D
20	25	12.5	1	0.1	0.05	2

Table3- Efficient design parameters for Multi-Objective model in comparison with Pure-Economic model Using DE

Design	n	h	k	ATS_0	ATS_1	E_L
P-Economic	1.79	1.74	2.54	81.02	4.68	2.20
M-Objective	3.62	0.25	3.90	1039.06	3.98	6.70
		Improvement Rate %		92.20	-17.59	67.16

Table4- Comparison of efficiency values using DE and Jafarian's algorithm

	DE		Jafarian-Namin. et al.'s algorithm		
Design	M-Objective	P-Economic	M-Objective	M-Objective	P-Economic
Efficiency	1.00	0.24	0.86	0.63	0.18

6- Conclusion

In this study, we applied the DE approach to specify efficient design parameters in the proposed C-MOESD model. To do so, all calculations have been facilitated under a program coded in the MATLAB (version R2013b) environment. The algorithm procedure was investigated in addition to comparison with pure economic design. The results confirmed the better statistical properties of the proposed model. In addition, the results obtained by DE were more efficient than the results obtained in [6].

References

[1] A. Amiri, and S. Jafarian-Namin, "Evaluating multi-objective economic-statistical design of attribute C control charts for monitoring the number of non-conformities," International Journal of Quality Engineering and Technology, vol. 5, pp. 145-161, Appril 2015.

[2] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," European Journal of Operational Research, vol. 2, pp. 429-444, November 1978.

[3] Y. K. Chen, and H. C. Liao, "Multi-criteria design of an x̄ control chart," Computers & Industrial Engineering, vol. 46, pp. 877-891, July 2004.

[4] K. Deb, A. Pratap, S. Agarwal, and T. A. M. T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," IEEE Transactions. Evolutionary Computation, vol. 6, pp. 182-197, April 2002.

[5] A. J. Duncan, "The economic design of x charts used to maintain current control of a process," Journal of the American Statistical Association, vol. 51, pp. 228-242, Jun 1956.

[6] S. Jafarian-Namin, A. Amiri, and E. Najafi, "Efficient selection of design parameters in multi-objective economic-statistical model of attribute c control chart," International Journal of Data Envelopment Analysis, vol. 2, pp. 357-367, February 2014.

[7] R. V. Kasarapu, and V. B. Vommi, "Economic design of x̄ control chart using differential evolution," International Journal of Emerging Technology and Advanced Engineering, vol. 3, pp. 541-548, April 2013.

[8] D. C. Montgomery, Introduction to statistical quality control, New York: Wiley, 2008.

[9] E. M. Saniga, "Economic statistical control chart designs with an application to x̄ and R charts," Technometrics, vol. 31, pp. 313-320, August 1989.

[10] R. Tavakkoli Moqaddam, N. Norouzi, S-M. kalami, and A. Salamat-bakhsh, Meta-heuristic algorithms theory and implementation in MATLAB (In Persian), Tehran: Islamic Azad University-South Tehran Branch, 2013.