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Forward Interference Cancellation in Spectrally Efficient Frequency Division Multiplexing Systems

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Abstract—**OFDM is widely known as an effective modulation in communication systems. Recent studies have shown that the bandwidth can be reduced further by violating the orthogonality criteria- Spectrally Efficient FDM systems. Various studies are based on extracting the transmitted data in the presence of inter-channel-interference at the receiver side. The aim of this paper is to propose a new method for interference cancellation at the transmitter side through active constellation extension. Experimental results regarding the proposed method has shown that interference-free communication is possible in high SNRs. Also the performance of the algorithm is acceptable as compared to receiver-based methods at low-SNR AWGN channels.**

INTRODUCTION

Orthogonal Frequency Domain Multiplexing (OFDM) as a robust multi-carrier modulation in multipath fading channels has attracted many research interests [1]. Also development of media and communication devices in addition to the growth of the number of users magnify the necessity of efficient bandwidth usage. Therefore it would be of paramount importance to find modulations with as many users as possible.

The main structure of an OFDM system is based on modulating the narrowband signals on a set of orthogonal carriers in order to obtain zero Inter Channel Interference (ICI) [2]. In a recent method introduced as Spectrally Efficient Frequency Division Multiplexing (SE-FDM), the frequency differences among adjacent carriers are being reduced in order to increase the number of users being supported with a fixed total bandwidth [1], [3], [4]. Clearly, such systems suffer from the interference occurred in each sub-channel due to other user data since the orthogonality of their carriers has been deliberately violated [5]. The input data in this case may not be extracted via a simple correlative detection. More complex decoding algorithms such as Maximum-Likelihood (ML) leads to an acceptable performance at the expense of highly complex and impractical implementations [6], [7], [8]. It has been experimentally observed that for both QAM and PAM data mapping, the bandwidth reduction lower than a specific amount violates the one-to-one condition; in this case the original data even in the noiseless case cannot be obtained without ambiguity [1].

The aim of this paper is to propose a new method which transfers the implementational complexity from the receiver to the transmitter in an SE-FDM system. Our proposed method is based on Active Constellation Extension (ACE) that has been used for Peak-to-Average-Power-Ratio (PAPR) reduction [9]. Simulation results have shown that by using conventional OFDM receivers and at high Signal-to-Noise Ratios, the proposed method achieves similar performance to that of an OFDM system up to 33% spectral overloading.

The paper is organized as follows: In section II the structure of an SE-FDM system is discussed. Section III is devoted to introduce and analyze the proposed method while section IV demonstrates the simulation results, and finally the conclusions are made in section V.

SPECTRALLY EFFICIENT FREQUENCY DIVISION MULTIPLEXING (SE-FDM)

In an OFDM communication system, N channels with the bandwidth of $\frac{1}{NT}$ are supported by N respective sinusoidal carriers through the following procedure:

$$s(t) = \sum_{n=0}^{N-1} x_n(t) e^{j\frac{2\pi nt}{NT}}, \quad 0 \le t < T \qquad (1)$$

where $x_n(t)$ and $e^{j\frac{2\pi nt}{NT}}$ for $n \in 0, 1, ..., N-1$ are respectively the data and carrier associated to the nth channel [1]. The OFDM signal $s(t)$ occupies a bandwidth of $\frac{1}{T}$. We can sample the OFDM symbol defined in (1) at the rate of $\frac{1}{T}$ to obtain $\tilde{s}_m$:

$$\tilde{s}_m = s(mT) = \sum_{n=0}^{N-1} x_n e^{j\frac{2\pi nm}{N}}, \quad m = 0, 1, ..., N-1 \qquad (2)$$

In other words (2) can be re-written as $\tilde{s}_m = DFT^{-1}\{x_n\}$.

In SE-FDM system, unlike an OFDM system, the sinusoidal carriers are chosen as $e^{j\frac{2\pi nt}{NT}\alpha}$ for $n \in 0, 1, ..., N-1$ where α is a real number called the bandwidth compactness $(0 < \alpha \le 1)$. This definition results in frequency distances among adjacent carriers $(\frac{\alpha}{NT})$ and the overall bandwidth reduces to $\frac{\alpha}{T}$. As discussed before, the achieved efficiency in

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bandwidth occupation is at the expense of ICI at the receiver end [2]. Assuming that a simple correlative detector (similar to an OFDM receiver) is used during the demodulation, we can write:

$$\tilde{x}_{n_0} = \frac{1}{N}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} x_n e^{j\frac{2\pi m(n-n_0)}{N}\alpha} = x_{n_0} + \sum_{\substack{n=0\\ n\neq n_0}}^{N-1} x_n G_{n,n_0} \tag{3}$$

The second term at the right side of (3) is the interference caused by other users; we thus have:

$$G_{m,n} = \frac{sin(2\pi N\alpha(m-n))}{Nsin(2\pi\alpha(m-n))} e^{j\pi(m-n)\frac{N-1}{N}\alpha} \tag{4}$$

$$m, n \in \{0, 1, ..., N-1\}$$

$G_{m,n}$ is the inner product of the mth and the nth carriers. The matrix form of the modulation-demodulation procedure in an SE-FDM system can be shown as:

$$\tilde{x} = Gx \tag{5}$$

with $G_{n,n} = 1$ for $n \in \{0, 1, ..., N-1\}$.

For the sake of simplicity, we may assume that $\alpha = \frac{M}{N}$ with $M \in \mathcal{N}$ and $M \leq N$. In this case it can be easily shown that:

$$G = \frac{1}{N}\psi^H\psi \tag{6}$$

where ψ is an $M \times N$ matrix whose rows are the first M rows of an N-point IDFT matrix and H is the Hermitian operator. In other words, the recovering of the initial data in a reduced bandwidth is equivalent to the reconstruction of an N-point complex signal using only its first M IDFT samples ($M \leq N$).

Obviously for an arbitrary N-point complex signal, the above recovery is impossible since $rank(G) = M$ will be chosen less than N in the overloaded scenarios. That is the matrix G is a non-invertible matrix. However, the input vector x is assumed to have all its components in a discrete form, e.g., $x_n = \pm 1 \pm j$ for $\forall n \in \{0, 1, ..., N-1\}$ in a QPSK mapping. Thus the set of all possible vectors for the input signal has a finite number of elements and the input vector may be extracted using a variety of decoding schemes such as the ML method:

$$x^*_{ML} = \underset{x\in\{\pm 1\pm j\}^N}{\text{argmin}} ||y - \psi x||_2^2 \tag{7}$$

where y is the SE-FDM received symbol at the receiver side. In a more realistic scenario it can be assumed that the SE-FDM symbols are being transmitted through an AWGN channel, resulting in a general model for the received symbols to be in the form of:

$$y = \psi x + n \tag{8}$$

In (8), n is an $M \times 1$ white Gaussian noise vector with zero mean and covariance matrix of $\sigma^2 I$.

Decoding algorithms like (7) are computationally expensive and thus impractical in real world applications. The proceeding sections will introduce a methodology that achieves interference free communication in SE-FDM systems even by using simple correlative decoders (conventional OFDM receivers). The core idea is to deliberately manipulate the input constellation in order to achieve a new input vector, namely $\hat{x}$, which should be modulated and transmitted instead of x. This manipulation should occur in a way that $G\hat{x} + \psi^H n$ has the same symbols with those of x when $\sigma^2 \to 0$ (high Signal-to-Noise Ratios).

Interference Cancellation via Active Constellation Extension (ACE)

In this section we show that by using proper pre-processing algorithms in the transmitter of an SE-FDM system, one can alleviate the ICI at the expense of extra computational load and a relative increase in power [9].

Assume an SE-FDM transmitter-receiver with N users and bandwidth compactness of $\frac{M}{N}, M < N$. According to the previous section, such a system can be modeled by a MIMO interference channel with the channel matrix G. The Non-diagonal entries of G represent the interfering coefficients of other sub-carriers in a particular sub-channel. Also assume that a QPSK modulation has been used for mapping the input binary data into the input complex constellation. Therefore given an input data x_0, we have:

$$x_0 \in \{\pm 1 \pm j\}^N, \quad \hat{x}_0 = Gx_0 \tag{9}$$

where $\hat{x}_0$ represents the received input data at the receiver in the absence of noise and by using an OFDM receiver. According to the linear nature of interference, we can replace an arbitrary component in x_0, e.g., $(x_0)_n$ with a value $(x'_0)_n$ defined as:

$$(x'_0)_n = (x_0)_n - \sum_{\substack{m=0\\ m\neq n}}^{N-1} (x_0)_m G_{m,n} \tag{10}$$

All other components in x'_0 are left unchanged and are equal to the corresponding components in x_0. Therefore x'_0 is a manipulated version of the input vector. It can be easily shown that through the above replacement, the nth component of Gx'_0 contains zero interference, which means:

$$(\hat{x'_0})_n = (Gx'_0)_n = (x_0)_n \tag{11}$$

Where $\hat{x'_0}$ is the estimate of x'_0 at the receiver. It should be noted that the other components in $\hat{x'_0} = Gx'_0$ still suffer from interference. The repetition of this algorithm for the remaining components does not obviate the problem since each time an entry is altered to cancel its interference, the interferences in other entries would regrow.

The proposed strategy to deal with the above problem is to make conditional replacements considering the following facts:

- Not all the interferences are causing errors during the symbol decoding.
- The interferences are known at the transmitter as long as both x_0 and G are previously determined.

Based on all the previous discussions and under the assumption of QPSK modulation, the proposed algorithm could be summarized as the iterative ACE pseudo-code.

Algorithm: Iterative ACE Method

$k \leftarrow 0$
while $x_k \neq x_{k-1}$ **do**
  $v_k \leftarrow x_0 - Gx_k$.
  for $n = 1, 2, ..., N$ **do**
    if $sign\left(\Re\left\{(x_0)_n\right\}\Re\left\{(Gx_k)_n\right\}\right) = 1$ then $\Re\left\{(v_k)_n\right\} \leftarrow 0$.
    if $sign\left(\Im\left\{(x_0)_n\right\}\Im\left\{(Gx_k)_n\right\}\right) = 1$ then $\Im\left\{(v_k)_n\right\} \leftarrow 0$.
  end
  $x_{k+1} \leftarrow x_k + \lambda v_k$
  $k \leftarrow k + 1$
end

The $\Re$ and $\Im$ operators serve as real and imaginary parts of a complex number respectively. Parameter λ is called the relaxation parameter and must be tuned to avoid instability. The above iterations should continue until each component in the manipulated input vector x_k and its corresponding component in the received vector Gx_k are located in the same quadrants of the constellation. So the manipulated input vector x' then could be found as:

$$x' = \lim_{k\to\infty} x_k \tag{12}$$

The achieved input vector results in an increase in the transmitted power, and thus makes the whole system sensitive to noise in a realistic AWGN channel. Besides, the interferences occurring in various sub-channels would spread the constellation points resulting into more noise sensitivity. Further modifications can be made in order to improve the above problem. In this work the criteria to move any point in the constellation are extended to modified conditions as follows:

Omit the real part in $(v_k)_n$ if:

$$sign(\Re\left\{(x_0)_n\right\})\Re\left\{(Gx_k)_n\right\} \geq \eta$$

Omit the imaginary part in $(v_k)_n$ if:

$$sign(\Im\left\{(x_0)_n\right\})\Im\left\{(Gx_k)_n\right\} \geq \eta$$

η is a non-negative parameter called the guard distance. The above modification dictates each component in the new input vector to be mapped beyond a guard distance of η from both real and imaginary axes while it is placed in the same quadrant with its corresponding component in x_0. The numerical results of using this method are shown and analysed in the next section.

The whole idea of ACE in SE-FDM systems can be investigated from a different point of view. Assume x to be the manipulated input vector transmitted instead of the original input vector x_0. Let us define:

$$\mathcal{G} = \begin{pmatrix} G_R & -G_I \\ G_I & G_R \end{pmatrix}, X_0 = \begin{pmatrix} x_{0_R} \\ x_{0_I} \end{pmatrix} \quad and \quad X = \begin{pmatrix} x_R \\ x_I \end{pmatrix} \tag{13}$$

where the R and I indices indicate the real and imaginary components of a complex number, respectively. Also let us define $\mathcal{D}_{X_0}$ as a $2N \times 2N$ diagonal matrix whose diagonal entries consist of X_0 components. It can be easily shown that the necessary constraints on x which have been mentioned in the proposed algorithm can be written in the following form:

$$(\mathcal{G}\mathcal{D}_{X_0})X \geq \eta(\mathbf{1}_{2N\times 1}) \tag{14}$$

where the inequality is element by element. Any X satisfying (14) guarantees error free communication in the absence of noise with the consideration of the guard distances to reduce the noise sensitivity. In order to address the power increase problem, we may look for the minimum norm solution of the above inequalities. In other words:

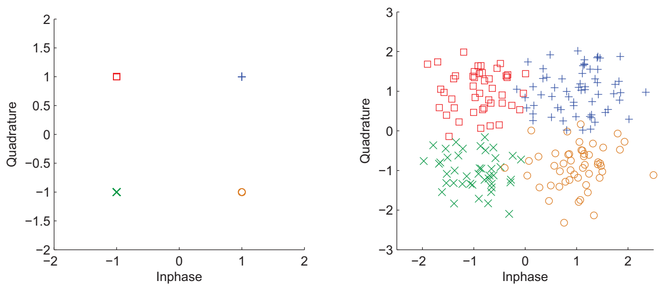
Quadratic Programming Method

$$X^* = \underset{X\in\mathcal{R}^{2N}}{\text{argmin}} \ \frac{1}{2}||X||_2^2 \quad \text{subject to} \quad (\mathcal{G}\mathcal{D}_{X_0})X \geq \eta(\mathbf{1}_{2N\times 1}) \tag{15}$$

The optimization problem stated in (15) could be solved using Quadratic Programming (QP). The performance of the introduced method will be shown in the next section.

Experimental Results

Fig. 1 demonstrates what happens to the received constellation in an SE-FDM communication system with $\alpha = 0.8$ when a simple OFDM receiver is employed. In Fig. 1(b) it can be clearly observed that not only some symbols have been mapped into the erroneous quadrants, but also the points are heavily scattered and therefore are sensitive to noise. Fig. 2(a) shows the new input constellation achieved using the proposed iterative ACE algorithm in section III for the original input vector shown in Fig. 1a. The guard distance η and the relaxation parameter λ are chosen as 0.7 and 1 respectively. As it is shown on Fig. 2(b), the received constellation shows perfect accuracy and more robustness to noise. Fig. 3 represents the same procedure using the proposed QP method. From Fig. 2



(a) Input Constellation (b) Received Constellation

Fig. 1: SE-FDM system performance via simple receiver

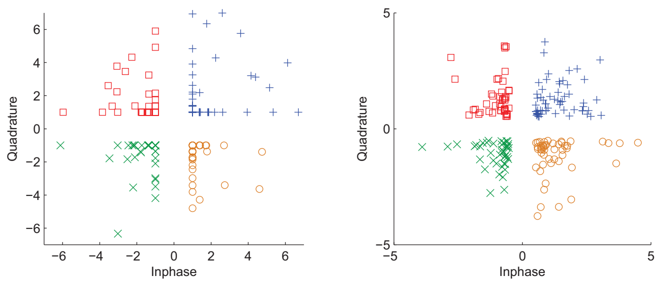
and 3 we conclude that the QP method has better performance comparing to the iterative ACE algorithm. However, the QP method leads to more computational complexity.

Fig. 4 demonstrates the performance of the proposed algorithms in an AWGN channel, comparing with two receiver-based decoding schemes (the curves with the square markers). The methods that are employed for comparison are simple correlative decoder and a Parallel Interference Cancellation (PIC) receiver with 30 consecutive cancellations. It is shown that the iterative PICs will not reduce the interference to lower than a specified value while the proposed transmitter-based methods (the curves with circle markers) reduce the error by a much less complex receiver.

Conclusion

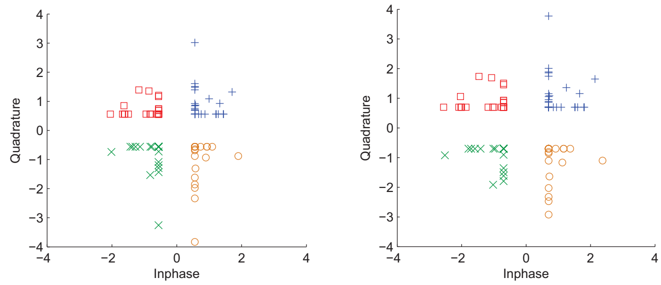
In this paper we have proposed a new method to alleviate the interference occurred in SE-FDM systems at the transmitter side. This scheme can significantly reduce the implementational costs in many communication systems such as broadcast links since the interference cancellation is performed at the transmitter and the infrastructures at the receivers are remained unchanged. Experimental results reveal that without using any interference cancellation methods at the receiver, we can achieve error free communication for high Signal-to-Noise Ratios. Also the performance of the proposed methods are acceptable for low SNRs.

The mapping type considered in this paper has been chosen as QPSK for simplicity, which can be extended to more complicated forms. Also the mathematical proofs for the heuristic achievements in addition to further analytic optimization should be investigated in future works.



(a) Manipulated Input Constellation (b) Received Constellation

Fig. 2: SE-FDM system performance via Interference Cancellation in the transmitter side (Iterative ACE).



(a) Manipulated Input Constellation (b) Received Constellation

Fig. 3: SE-FDM system performance via Interference Cancellation in the transmitter side (QP method).

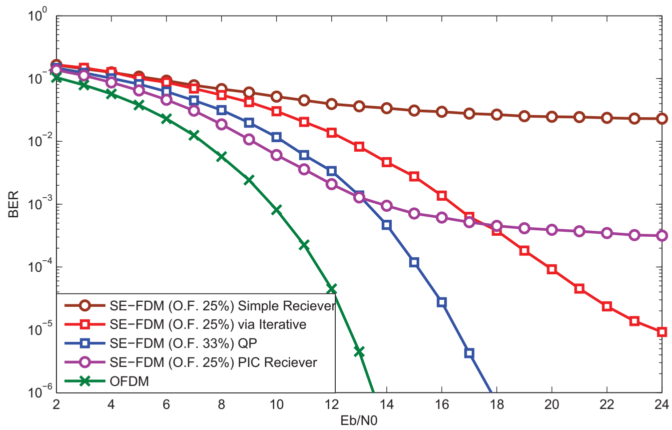


Fig. 4: BER vs. $\frac{Eb}{N_0}$ for the proposed iterative and QP methods with respective 25% and 33% overloading factors (square marked curves) as compared to the two receiver-based methods (circle marked curves).

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