Selected Mapping Algorithm for PAPR Reduction of Space-Frequency Coded OFDM Systems without Side Information

Mahmoud Ferdosizadeh Naeiny, Farokh Marvasti

***Abstract*—Selected Mapping (SLM) is a well-known technique for Peak to Average Power Ratio (PAPR) reduction of Orthogonal Frequency Division Multiplexing (OFDM) systems. In this technique, different representations of OFDM symbols are generated by rotation of the original OFDM frame by different phase sequences and the signal with minimum PAPR is selected and transmitted. To compensate for the effect of the phase rotation at the receiver, it is necessary to transmit the index of the selected phase sequence as Side Information (SI). In this paper, an SLM technique is introduced for the PAPR reduction of Space Frequency Block Coded (SFBC) OFDM systems with Alamouti coding scheme. Additionally, a suboptimum detection method that does not need SI is introduced at the receiver side. Simulation results show that the proposed SLM method reduces the PAPR effectively, and the detection method has performance very close to the case that the correct index of the phase sequence is available at the receiver side.**

***Index Terms*—SLM, OFDM, PAPR, Spatial Diversity, SFBC**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a well-known technique for transmission of high rate data over broadband frequency selective channels [1]. One of the drawbacks of OFDM systems is high PAPR, which leads to the saturation of the high power amplifier. Thus, a high dynamic range amplifier is needed, which increases the cost of the system. The frequency domain symbols of an OFDM frame is denoted by $\mathbf{X} = [X(0), X(1), ..., X(N_c-1)]^T$, where N_c is the number of subcarriers. It is assumed that $X(k) \in \mathcal{C}$, where $\mathcal{C}$ is the set of the constellation points. The vector $\mathbf{x} = [x(0), x(1), ..., x(N-1)]^T$ contains the time domain samples of the complex baseband OFDM signal as given below:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N_c-1} X(k) e^{j\frac{2\pi nk}{N}} \quad (1)$$

where $j = \sqrt{-1}$ and N/N_c is the oversampling ratio. It is clear that $\mathbf{x} = IFFT_N\{\mathbf{X}\}$, where $IFFT_N\{\}$ is the N points IFFT operation. The PAPR of the OFDM frame is defined by

$$PAPR(\mathbf{x}) = \frac{\max_n\{|x(n)|^2\}}{E\{|x(n)|^2\}} \quad (2)$$

where $E\{.\}$ is the mathematical expectation. According to (1), the time domain samples are the sum of N_c independent terms. When N_c is large, based on the central limit theorem, the time domain samples have a Gaussian distribution and, thus, they may have large amplitudes [2]. To overcome this problem, some algorithms have been proposed which reduce the PAPR of the baseband OFDM signal [3]-[16]. Some of these methods need Side Information (SI) to be transmitted to the receiver, such as Partial Transmit Sequence (PTS) [3], [4] and Selected Mapping (SLM) [5], [6], [7]. Some other methods do not need SI, such as clipping and filtering [8], [9], tone reservation [10], [11], block coding [12], [13], and Active Constellation Extension (ACE)[14].

In the SLM method, D different representations of the OFDM frame are generated and the one with minimum PAPR is transmitted. If the vectors $[\phi^d(0), \phi^d(1), ..., \phi^d(N_c-1)]^T, d = 0, 1, ..., D-1$ are D random phase sequences with the length of N_c, and $\mathbf{b}^d = [e^{j\phi^d(0)}, e^{j\phi^d(1)}, ..., e^{j\phi^d(N_c-1)}]^T$, then D representations of the signal $\mathbf{x}$ are

$$\mathbf{x}^d = IFFT_N\{\mathbf{X} \otimes \mathbf{b}^d\}, \quad 0 \le d \le D-1 \quad (3)$$

where $\otimes$ is element by element production. The index of the optimum phase sequence is

$$\bar{d} = \underset{d \in \{0,1,...,D-1\}}{argmin} \left\{ PAPR\{\mathbf{x}^d\} \right\} \quad (4)$$

To reduce the complexity of the application of different phase sequences often the phases $\phi^d(k)$ is chosen randomly from $\{0, \pi\}$. This means that $b^d(k) \in \{\pm 1\}$ and it is enough to change the sign of the symbols $X(k)$ before IFFT operation. The signal $\mathbf{x}^{\bar{d}}$ is transmitted and the index of selected phase sequence ($\bar{d}$) is sent to the receiver as SI. If the SI is received with an error, the OFDM frame will be lost; thus, this information must be protected by coding. Several SLM methods have been proposed for single antenna OFDM systems which do not need explicit SI [15], [16], [17]. Some of these algorithms pay a penalty for the transmission power [15], [16]. The drawback of the method introduced in [17] is that the phase sequences must be chosen such that $b^d(k)X(k) \notin \mathcal{C}$. This leads to the increase of the complexity of transmitter because the coefficients $b^d(k)$ can no longer be chosen from the set $\{\pm 1\}$.

Using several transmitter antennas, one can improve the data rate or Bit Error Rate (BER) of wireless systems. In spatial multiplexing systems, independent symbols are transmitted from several antennas and this leads to the increase of data rate. In [18], simplified SLM method has been introduced for PAPR reduction of spatially multiplexed OFDM systems. If spatial diversity techniques are used in wireless systems with several transmitter antennas, the BER can be reduced. The space-time codes to achieve the full transmission diversity have been introduced in [19], [20]. Through a combination of spatial diversity and OFDM techniques, a higher capacity can be achieved over broadband multipath fading wireless channels [21], [22]. Two possible combinations of spatial diversity and OFDM techniques are Space Time Block Coded (STBC) OFDM and Space Frequency Block Coded (SFBC) OFDM systems. Both combinations suffer from high PAPR problem. In [23], clipping and filtering method has been used for PAPR reduction in SFBC-OFDM systems with two transmitter antennas and an iterative method has been proposed to compensate for the effect of clipping noise at the receiver. In [24] Polyphase Interleaving and Inversion (PII) method has been proposed for SFBC-OFDM systems with two transmitter antennas and Alamouti space frequency coding scheme. Also in [25] a low complexity version of SLM method has been proposed for PAPR reduction of SFBC-OFDM systems.

In this paper, it is shown that the simplified SLM technique can be applied to SFBC-OFDM systems with two transmitter antennas and Almouti coding scheme without changing the orthogonality of space frequency coding. In this method, the optimum phase sequence is applied to the OFDM frames of two antennas such that the SFBC structure remains constant. Then, it will be shown that at the receiver side, the optimum phase sequence can be detected without SI. Detection of phase sequence index does not need any extra transmission power or extra computation complexity of transmitter and only uses the intrinsic redundancy of SFBC code. In this paper, the optimum receiver for blind detection of the phase sequence index ($\bar{d}$) is introduced and a low-complexity suboptimum receiver is proposed. Simulation results show that the performance of the proposed blind detection method depends on the OFDM frame length and its Symbol Error Rate (SER) is very close to the case where the

M. Ferdosizadeh Naeiny and F. Marvasti are with Advanced Communication research Institute (ACRI) and the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran, e-mails: mferdosi@ee.sharif.edu, marvasti@sharif.edu.

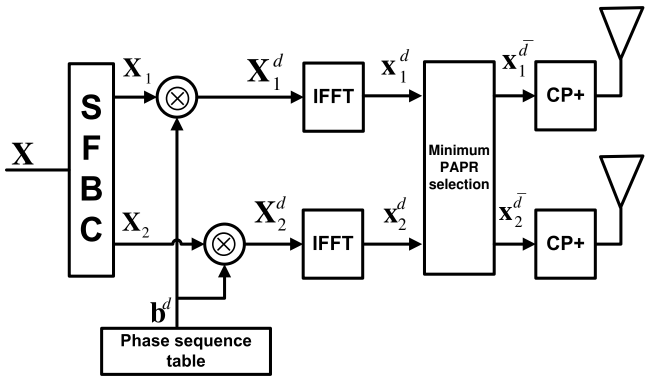

Fig. 1. Block diagram of SFBC-OFDM transmitter with two transmitter antennas and SLM method for PAPR reduction.

phase sequence index is available at the receiver side.

The remainder of this paper is organized as follows: In section 2, the system model of the SFBC-OFDM system with two transmitter antennas is introduced. In section 3, an SLM technique for PAPR reduction of this system is proposed. In section 4, the optimum method for detection of the phase sequence index is introduced and a suboptimum algorithm with lower complexity is proposed. Section 5 includes the simulation results that show the effect of the proposed method in PAPR reduction and SER.

II. SFBC-OFDM SYSTEM MODEL

In SFBC-OFDM systems with two transmitter antennas, the frequency domain vector transmitted from the pth antenna is denoted by $\mathbf{X}_p = [X_p(0), X_p(1), ..., X_p(N_c-1)]$. The vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ can be generated from the original OFDM frame, $\mathbf{X}$, as follows [21]:

$$\begin{aligned} \begin{pmatrix} X_1(2\nu) & X_1(2\nu+1) \\ X_2(2\nu) & X_2(2\nu+1) \end{pmatrix} &= \underbrace{\begin{pmatrix} X(2\nu) & X(2\nu+1) \\ X^*(2\nu+1) & -X^*(2\nu). \end{pmatrix}}_{\mathbf{C}} \\ \nu &= 0,1,...,N_c/2-1 \end{aligned} \quad (5)$$

As shown in Fig.1, the vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ are passed through the IFFT operation to yield the time domain samples $x_1(n)$ and $x_2(n), 0 \le n \le N-1$. It is noteworthy that the orthogonality of the space frequency matrix $\mathbf{C}$ in (5) leads to full diversity at the receiver side [20], i.e.,

$$\mathbf{C}\mathbf{C}^H = \left(|X(2\nu)|^2 + |X(2\nu+1)|^2\right)\mathbf{I}_2 \quad (6)$$

where $\mathbf{I}_n$ is the $n \times n$ identity matrix. The PAPR of the pth antenna is defined by

$$PAPR\{\mathbf{x}_p\} = \frac{\max_n\{|x_p(n)|^2\}}{E\{|x_p(n)|^2\}}, \quad p = 1,2 \quad (7)$$

where $E\{.\}$ is the mathematical expectation. The overall PAPR of the SFBC-OFDM system is defined by

$$PAPR = \max_{p\in\{1,2\}} PAPR\{\mathbf{x}_p\} \quad (8)$$

III. SELECTED MAPPING FOR PAPR REDUCTION OF SFBC-OFDM SYSTEMS

The vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ can be multiplied by D different phase sequences to yield the minimum PAPR representation, but the SFBC structure shown in (5) must remain constant so that full diversity can be achieved. In [18], simplified SLM for PAPR reduction of spatial multiplexed OFDM has been proposed. In this scheme, the OFDM frames of the antennas are modified simultaneously with the same single phase sequence. This leads to reduction of the number of bits that must be transmitted as SI. In this paper, this approach is used for SFBC-OFDM systems. It is shown that if this method is used for SFBC-OFDM system, then the intrinsic redundancy of space frequency coding can be used to detect the index of the phase sequence at the receiver side without SI. Based on this approach, D different representations of the signals $\mathbf{x}_1$ and $\mathbf{x}_2$ are generated as follows:

$$\begin{aligned} \mathbf{x}_1^d &= IFFT_N\{\mathbf{X}_1 \otimes \mathbf{b}^d\} \\ \mathbf{x}_2^d &= IFFT_N\{\mathbf{X}_2 \otimes \mathbf{b}^d\}, \quad 0 \le d \le D-1. \end{aligned} \quad (9)$$

In [26], it has been shown that a simple and optimal choice for the phase sequences is a random selection of 0 and π with equal probabilities. In this case, the complexity of the phase rotation is very low because $b^d(k) = e^{j0} = +1$ or $b^d(k) = e^{j\pi} = -1$ and, thus, the multiplication of the frequency domain vectors by the vector $\mathbf{b}^d$ is via the sign change of some of the symbols. The pairs $[b^d(2\nu), b^d(2\nu+1)], 0 \le \nu < N_c/2, 0 \le d < D-1$ can take the values $[+1,-1]$, $[-1,+1]$, $[-1,-1]$ and $[+1,+1]$ with equal probabilities. After the multiplication of $\mathbf{X}_1$ and $\mathbf{X}_2$ by $\mathbf{b}^d$, the matrix $\mathbf{C}$ in (5) is changed to

$$\mathbf{C}^d = \begin{pmatrix} b^d(2\nu)X(2\nu) & b^d(2\nu+1)X(2\nu+1) \\ b^d(2\nu)X^*(2\nu+1) & -b^d(2\nu+1)X^*(2\nu). \end{pmatrix} \quad (10)$$

It is noteworthy that the matrix $\mathbf{C}^d$ is also orthogonal, i.e.,

$$\mathbf{C}^d(\mathbf{C}^d)^H = \left(|X(2\nu)|^2 + |X(2\nu+1)|^2\right)\mathbf{I}_2 \quad (11)$$

By keeping the space frequency code orthogonal, full diversity can be achieved at the receiver side. It is shown in the next section that it is possible to detect the index of the optimum phase sequence ($\bar{d}$) at the receiver without SI.

IV. BLIND DETECTION OF $\bar{d}$

At the receiver side, it is necessary to determine the index of the optimum phase sequence ($\bar{d}$) to detect the transmitted symbols $X(k), 0 \le k \le N_c-1$ correctly. After removing the cyclic prefix and applying FFT, the received vector $\mathbf{Y} = [Y(0), Y(1), ..., Y(N_c-1)]^T$ can be described as follows:

$$Y(k) = H_1(k)X_1^{\bar{d}}(k) + H_2(k)X_2^{\bar{d}}(k) + V(k), \quad 0 \le k \le N_c-1 \quad (12)$$

where $H_p(k)$ and $V(k)$ are the baseband equivalent coefficient of the channel between the pth transmitter antenna and the receiver antenna and the noise component at the kth subcarrier, respectively. The complex baseband noise is assumed to be white zero-mean Gaussian. It is assumed that the channel coefficients are known at the receiver side. Our goal is to determine $\bar{d}$ from $\{Y(k)\}_{k=0}^{N_c-1}$. SFBC-OFDM systems are often used in the channels which are flat over several subcarriers. If the channel coefficients are assumed to be the same for two adjacent subcarriers $(H_p(2\nu) = H_p(2\nu+1), p = 1,2, 0 \le \nu \le N_c/2-1)$, then from (5), (9) and (12) it can be seen that

$$\begin{aligned} Y(2\nu) &= H_1(2\nu)b^{\bar{d}}(2\nu)X(2\nu) \\ &+ H_2(2\nu)b^{\bar{d}}(2\nu)X^*(2\nu+1)+V(2\nu), \\ Y(2\nu+1) &= H_1(2\nu)b^{\bar{d}}(2\nu+1)X(2\nu+1) \\ &- H_2(2\nu)b^{\bar{d}}(2\nu+1)X^*(2\nu)+V(2\nu+1) \end{aligned} \tag{13}$$

The Maximum Likelihood (ML) detection for $\bar{d}$ and $\mathbf{X}$ is

$$\begin{aligned} \left[d^{(ML)}, \mathbf{X}^{(ML)}\right] = \underset{\substack{0 \le \hat{d} < D \\ \hat{\mathbf{X}} \in \mathcal{C}^{N_c}}}{argmin} \sum_{\nu=0}^{\nu=N_c/2-1} \Bigg\{ \Big| Y(2\nu) \\ - H_1(2\nu)b^{\hat{d}}(2\nu)\hat{X}(2\nu) - H_2(2\nu)b^{\hat{d}}(2\nu)\hat{X}^*(2\nu+1)\Big|^2 \\ + \Big| Y(2\nu+1) - H_1(2\nu)b^{\hat{d}}(2\nu+1)\hat{X}(2\nu+1) \\ + H_2(2\nu)b^{\hat{d}}(2\nu+1)\hat{X}^*(2\nu)\Big|^2 \Bigg\} \end{aligned} \tag{14}$$

The new variables $Z^d(2\nu), Z^d(2\nu+1)$ and $f^d(\nu)$ are defined by

$$\begin{aligned} Z^d(2\nu) &= b^d(2\nu)X(2\nu) \\ Z^d(2\nu+1) &= b^d(2\nu)X(2\nu+1) \\ f^d(\nu) &= \frac{b^d(2\nu+1)}{b^d(2\nu)} = b^d(2\nu)b^d(2\nu+1) \end{aligned} \tag{15}$$

Based on these definitions, (13) can be rewritten as

$$\begin{aligned} Y(2\nu) &= H_1(2\nu)Z(2\nu) + H_2(2\nu)Z^*(2\nu+1) + V(2\nu), \\ Y(2\nu+1) &= H_1(2\nu)f^{\bar{d}}(\nu)Z(2\nu+1) - H_2(2\nu)f^{\bar{d}}(\nu)Z^*(2\nu) \\ &+ V(2\nu+1) \end{aligned} \tag{16}$$

and the ML detection can be modified as

$$\begin{aligned} \left[d^{(ML)}, \mathbf{X}^{(ML)}\right] = \underset{\substack{0 \le \hat{d} < D, \hat{\mathbf{Z}} \in \mathcal{C}^{N_c} \\ f^{\hat{d}}(\nu) = b^{\hat{d}}(2\nu)b^{\hat{d}}(2\nu+1)}}{argmin} \sum_{\nu=0}^{\nu=N_c/2-1} \Bigg\{ \\ \Big| Y(2\nu) - H_1(2\nu)\hat{Z}(2\nu) - H_2(2\nu)\hat{Z}^*(2\nu+1)\Big|^2 + \Big| Y(2\nu+1) \\ - H_1(2\nu)f^{\hat{d}}(\nu)\hat{Z}(2\nu+1) + H_2(2\nu)f^{\hat{d}}(\nu)\hat{Z}^*(2\nu)\Big|^2 \Bigg\} \end{aligned} \tag{17}$$

The complexity of ML detection is in the order of DA^{N_c}, where A is the number of the constellation points ($A = 16$ for 16-QAM modulation). To reduce the complexity of this detection method, a suboptimum algorithm is proposed in this paper. In the first step of the proposed method, the vectors $[Z(2\nu), Z(2\nu+1), f(2\nu)]$ are detected independently for different ν's. The detected vector,$[\dot{Z}(2\nu), \dot{Z}(2\nu+1), \dot{f}(2\nu)]$, is given below:

$$\begin{aligned} \left[\dot{Z}(2\nu), \dot{Z}(2\nu+1), \dot{f}(nu)\right] = \underset{\substack{\hat{f}(\nu) \in \{\pm 1\} \\ \hat{Z}(2\nu) \in \mathcal{C} \\ \hat{Z}(2\nu+1) \in \mathcal{C}}}{argmin} M_\nu \Big(Y(2\nu), \\ Y(2\nu+1), \hat{Z}(2\nu), \hat{Z}(2\nu+1), \hat{f}(\nu) \Big) \end{aligned} \tag{18}$$

where

$$\begin{aligned} M_\nu \Big(Y(2\nu), Y(2\nu+1), \hat{Z}(2\nu), \hat{Z}(2\nu+1), \hat{f}(\nu) \Big) = \\ \Big| Y(2\nu) - H_1(2\nu)\hat{Z}(2\nu) - H_2(2\nu)\hat{Z}^*(2\nu+1)\Big|^2 + \\ \Big| Y(2\nu+1) - H_1(2\nu)\hat{f}(\nu)\hat{Z}(2\nu+1) + H_2(2\nu)\hat{f}(\nu)\hat{Z}^*(2\nu)\Big|^2. \end{aligned} \tag{19}$$

It can easily be seen that:

$$\begin{aligned} M_\nu \Big(Y(2\nu), Y(2\nu+1), \hat{Z}(2\nu), \hat{Z}(2\nu+1), \hat{f}(\nu) \Big) = \\ \Big(|H_1(2\nu)|^2 + |H_2(2\nu)|^2 - 1 \Big) |\hat{Z}(2\nu)|^2 + \\ \Big| \hat{Z}(2\nu) - H_1(2\nu)Y^*(2\nu) + H_2^*(2\nu)\hat{f}(\nu)Y(2\nu+1)\Big|^2 + \\ \Big(|H_1(2\nu)|^2 + |H_2(2\nu)|^2 - 1 \Big) |\hat{Z}(2\nu+1)|^2 + \\ \Big| \hat{Z}(2\nu+1) - H_2^*(2\nu)Y(2\nu) - H_1^*(2\nu)\hat{f}(\nu)Y(2\nu+1)\Big|^2 \end{aligned} \tag{20}$$

The first two terms of the above equation depend on $\hat{Z}(2\nu)$ and $\hat{f}(\nu)$ and the last two terms depend on $\hat{Z}(2\nu+1)$ and $\hat{f}(\nu)$. To minimize M_ν in (20), $\dot{Z}(2\nu)$ and $\dot{Z}(2\nu+1)$ must be chosen as:

$$\begin{aligned} \dot{Z}(2\nu) &= Q\Bigg(\underbrace{\frac{H_1(2\nu)Y^*(2\nu) - H_2^*(2\nu)\hat{f}(\nu)Y(2\nu+1)}{|H_1(2\nu)|^2 + |H_2(2\nu)|^2}}_{\zeta(2\nu)} \Bigg) \\ \dot{Z}(2\nu+1) &= Q\Bigg(\underbrace{\frac{H_2^*(2\nu)Y(2\nu) + H_1^*(2\nu)\hat{f}(\nu)Y(2\nu+1)}{|H_1(2\nu)|^2 + |H_2(2\nu)|^2}}_{\zeta(2\nu+1)} \Bigg) \end{aligned} \tag{21}$$

where $Q(.)$ is the mapping to the nearest constellation point. The two equations in (21) depend on $\hat{f}(\nu)$; thus, $\dot{Z}^+(2\nu)$ and $\dot{Z}^+(2\nu+1)$ are defined by inserting $\hat{f}(\nu) = +1$, and $\dot{Z}^-(2\nu)$ and $\dot{Z}^-(2\nu+1)$ are defined by inserting $\hat{f}(\nu) = -1$ in (21). It can easily be seen that for the minimization of M_ν, the parameter $\dot{f}(\nu)$ must be chosen as

$$\begin{aligned} \dot{f}(\nu) = sign \Bigg(M_\nu \Big(Y(2\nu), Y(2\nu+1), \dot{Z}^-(2\nu), \dot{Z}^-(2\nu+1), -1 \Big) \\ - M_\nu \Big(Y(2\nu), Y(2\nu+1), \dot{Z}^+(2\nu), \dot{Z}^+(2\nu+1), +1 \Big) \Bigg) \end{aligned} \tag{22}$$

where $sign(.)$ is the sign function. This is equivalent to

$$\dot{f}(\nu) = sign\left(\left|\dot{Z}^{-}(2\nu)-\zeta^{-}(2\nu)\right|^2+\left|\dot{Z}^{-}(2\nu+1)-\zeta^{-}(2\nu+1)\right|^2 - \left|\dot{Z}^{+}(2\nu)-\zeta^{+}(2\nu)\right|^2-\left|\dot{Z}^{+}(2\nu+1)-\zeta^{+}(2\nu+1)\right|^2\right) \quad (22)$$

where $\zeta(2\nu)$ and $\zeta(2\nu+1)$ are defined in (21). After the detection of $\dot{f}(\nu)$ for $\nu = 0,1,...,N_c/2-1$, the vector $\dot{\mathbf{f}} = [\dot{f}(0),\dot{f}(1),...,\dot{f}(N_c/2-1)]^T$ is constructed. In the second step, this vector must be mapped to the closest sequence among $\mathbf{f}^d, 0 \le d \le D-1$, i.e.,

$$\widetilde{d} = \underset{0\le d\le D-1}{argmin} \; dist\left(\dot{\mathbf{f}},\mathbf{f}^d\right) \quad (24)$$

where $dist(\mathbf{A},\mathbf{B})$ is the Hamming distance between the vectors $\mathbf{A}$ and $\mathbf{B}$ and the vectors $\mathbf{f}^d, d = 0,1,...,D-1$ have been calculated and stored at the receiver as

$$\mathbf{f}^d = [f^d(0), f^d(1),...,f^d(\frac{N_c}{2}-1)]^T,$$
$$f^d(\nu) = b^d(2\nu)b^d(2\nu+1) \quad (25)$$

Now, based on the definitions of (15), the detected symbols $\widetilde{X}(k)$ can be derived as

The steps of the proposed algorithm for the blind detection of $\bar{d}$ and the OFDM frame $\mathbf{X}$ can be summarized as

- Calculate $\dot{Z}^+(2\nu)$, $\dot{Z}^-(2\nu)$, $\dot{Z}^+(2\nu+1)$ and $\dot{Z}^-(2\nu+1)$ for $\nu = 0,1,...,N_c/2-1$ using (21).
- Evaluate $\dot{f}(\nu), \nu = 0,1,...,N_c/2-1$ using (22) and construct the vector $\dot{\mathbf{f}}$.
- Determine $\widetilde{d}$ using (24).
- Determine the transmitted symbols $\{\widetilde{X}(k)\}_{k=0}^{k=N_c-1}$ using (26).

It is noteworthy that in the proposed method, the complexities of FFT operation, channel estimation and synchronization do not change and only the SFBC decoding is done twice; once for the calculation of the $\dot{Z}^+(k)$ and the other one for that of $\dot{Z}^-(k)$. To calculate FFT with length Nc, $3N_c\log_2(N_c)$ Real Additions (RA) and $2N_c\log_2(N_c)$ Real Multiplications (RM) are required [27]. Based on (21), the SFBC decoder for every ν requires 26 RAs and 32 RMs. Thus the SFBC decoding for all the subcarriers needs $13N_c$ RAs and $16N_c$ RMs. Therefore, the additional complexity at the receiver can be calculated as:

$$\begin{aligned}
\textit{Percentage of additional RAs}(\%) &= \frac{\textit{SFBC RAs}}{\textit{SFBC RAs}+\textit{FFT RAs}} = \frac{13}{13+3\log_2(N_c)}\times 100\% \\
\textit{Percentage of additional RMs}(\%) &= \frac{\textit{SFBC RMs}}{\textit{SFBC RMs}+\textit{FFT RMs}} = \frac{16}{16+2\log_2(N_c)}\times 100\%
\end{aligned} \quad (27)$$

For example for $N_c = 512$, 35% RAs and 50% RMs are added to the receiver. If the complexity of channel estimation and synchronization algorithms are taken into account, the denominators in (27) are increased and the additional complexity percentage of the receiver is reduced.

V. Simulation Results

The performance of the proposed method has been evaluated for two different OFDM frame lengths; $N_c = 128$ and $N_c = 512$. The symbols $X(k)$ are chosen from the 16-QAM constellations.

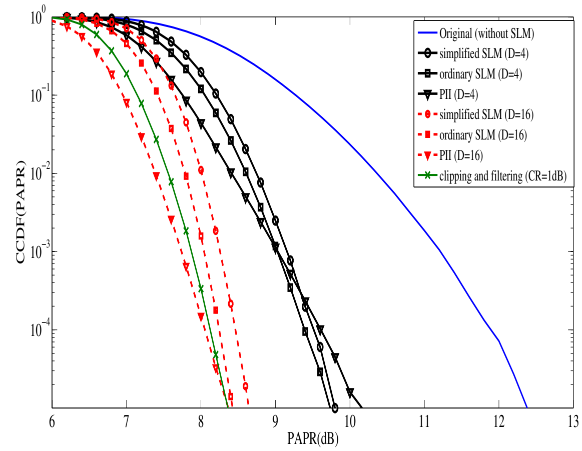

Fig. 2. PAPR reduction performance of the ordinary SLM, simplified SLM, PII and clipping and filtering methods for SFBC-OFDM system with two transmitter antennas and $N_c = 128$ for different values of D.

A. Performance in PAPR reduction

The performance of the proposed method in PAPR reduction is evaluated by the Complementary Cumulative Density Function (CCDF) of the PAPR which is defined as

$$CCDF(PAPR_0) = Pr\{PAPR \ge PAPR_0\} \quad (28)$$

where $Pr\{A\}$ is the probability of the event A. To find the peak values and to estimate the PAPR of the analogue signal, the over-sampling ratio of 4 has been used. To estimate the CCDF of the PAPR, 10^6 OFDM frames have been generated in a Monte-Carlo simulation. In this section, non-blind ordinary SLM and PII methods [24] have also been simulated. In the ordinary SLM method, the frame $\mathbf{X}(k)$ is multiplied by the sequences $\mathbf{b}^d$ before the SFBC encoder. In the PII method, the frame $\mathbf{X}(k)$ is partitioned into D subblocks called polyphases. Then, the even and odd elements of each subblock are interleaved and their signs are inverted such that the PAPR is minimized. Figures 2 and 3 show the performances of the proposed method for $N_c = 128$ and $N_c = 512$, respectively. As can be seen from Figure 2, at the probability of 10^{-5} the simplified SLM method reduces the PAPR by 2.6 and $3.7dB$'s for $D = 4$ and 16, respectively. As can be seen, the performance of the simplified SLM is very close to that of PII and ordinary SLM methods (for $D = 4$ even its performance is better than that of PII method). For $N_c = 512$, the application of the simplified SLM method leads to PAPR reductions of 2.5 and $3.5dB$'s for $D = 4$ and 16, respectively. In this case also the performance degradation in comparison to PII and ordinary SLM methods is less than $0.3dB$ (for $D = 4$, the simplified SLM method outperforms PII method). In Figures 2 and 3, the performance of the clipping and filtering introduced in [23] is also plotted. As shown in the figures, the clipping and filtering method is 0.3 and $0.9dB$'s better than the simplified SLM method in PAPR reduction for $N_c = 128$ and $N_c = 512$, respectively. In the next subsection. it is seen that the clipping and filtering method leads to a considerable increase in SER at the receiver side.

B. SER Performance

To evaluate the SER performance, five tap frequency selective channel with impulse response of $h_p(t) = \sum_{l=1}^{L=5} h_p(l)\delta(t-\Delta(l))$ is assumed between the pth transmitter antenna and

$$\left(\widetilde{X}(2\nu), \widetilde{X}(2\nu+1)\right) = \begin{cases} \left(\dot{Z}^{+}(2\nu), \dot{Z}^{+}(2\nu+1)\right) & \text{if } b^{\tilde{d}}(2\nu) = +1, f^{\tilde{d}}(\nu) = +1 \\ \left(-\dot{Z}^{+}(2\nu), -\dot{Z}^{+}(2\nu+1)\right) & \text{if } b^{\tilde{d}}(2\nu) = -1, f^{\tilde{d}}(\nu) = +1 \\ \left(\dot{Z}^{-}(2\nu), \dot{Z}^{-}(2\nu+1)\right) & \text{if } b^{\tilde{d}}(2\nu) = +1, f^{\tilde{d}}(\nu) = -1 \\ \left(-\dot{Z}^{-}(2\nu), -\dot{Z}^{-}(2\nu+1)\right) & \text{if } b^{\tilde{d}}(2\nu) = -1, f^{\tilde{d}}(\nu) = -1 \end{cases} \quad (26)$$

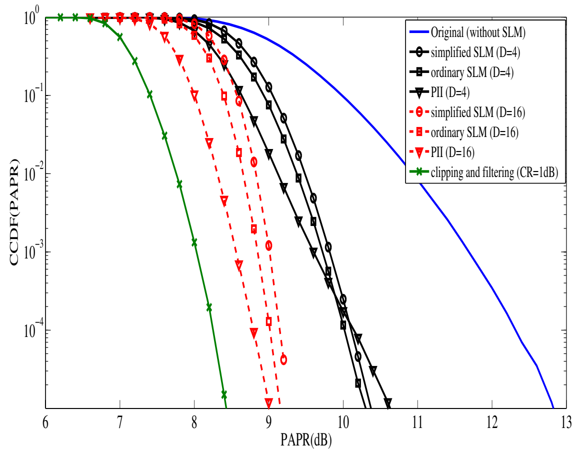

Fig. 3. PAPR reduction performance of the ordinary SLM, simplified SLM, PII and clipping and filtering methods for SFBC-OFDM system with two transmitter antennas and $N_c = 512$ for different values of D.

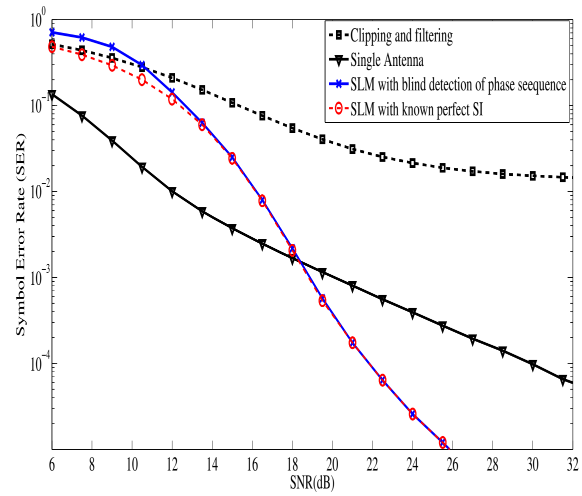

Fig. 4. The SER performance of SFBC-OFDM system with two transmitter antennas and $Nc = 128$ with three different PAPR reduction methods: 1) the simplified SLM with perfect SI ($D = 16$) , 2) the simplified SLM ($D = 16$) with blind detection of SI, 3) the clipping and filtering, and SER performance of a single antenna OFDM system.

the receiver antenna [17]. The tap delays, $\Delta(l)$, are $[0, 0.0025, 0.005, 0.01, 0.01, 0.015, 0.025] \times T$ and the average path gains, $10 \log E\{|h_p(l)|^2\}$, are $[0, -4, -8, -16, -24, -39]dB$, where T is the OFDM symbol duration. The error probabilities $\dot{P}$, $\widetilde{P}\left(d|\bar{d}\right)$ and $\widetilde{P}$ are defined by

$$\begin{aligned} \dot{P} &= Pr\left\{\dot{f}(\nu) \neq f^{\bar{d}}(\nu)\right\} \\ \widetilde{P}\left(d|\bar{d}\right) &= Pr\left\{\tilde{d} = d\middle|\bar{d}\right\} \\ \widetilde{P} &= Pr\left\{\tilde{d} \neq \bar{d}\right\} \end{aligned} \quad (29)$$

It is clear that

$$\begin{aligned} \widetilde{P} &= \sum_{\bar{d}=0}^{\bar{d}=D-1} Pr\left\{\tilde{d} \neq \bar{d}\middle|\bar{d}\right\} Pr\left\{\bar{d}\right\} = \frac{1}{D} \sum_{\bar{d}=0}^{\bar{d}=D-1} \sum_{d=0, d\neq\bar{d}}^{D-1} \widetilde{P}\left(d|\bar{d}\right) \\ &= \frac{1}{D} \sum_{\bar{d}=0}^{\bar{d}=D-1} \sum_{d=0, d\neq\bar{d}}^{D-1} \left(\dot{P}\right)^{dist(\mathbf{f}^{d}, \mathbf{f}^{\bar{d}})} \end{aligned} \quad (30)$$

Thus, the probability of error detection of $\bar{d}$ depends on the Hamming distance of the sequences $\mathbf{f}^{d}, 0 \leq d \leq D-1$. If these vectors are generated randomly as described in section 3, then the Hamming distance of the sequences increases when the OFDM frame length is increased, hence the detection error is reduced.

Figure 4 shows the SER versus Signal to Noise Ratio (SNR) for $N_c = 128$ and $D = 16$ for the case when we have perfect SI at the receiver and when the sequence index is detected based on the proposed method. As can be seen from this figure, for SNRs higher than $14dB$ there is no loss due to error detection of $\bar{d}$. In this figure also the SER of a single antenna OFDM system has been plotted. The slope of two antenna system at high SNRs is twice that of single antenna system. This is due to full diversity of space frequency coding. Also the SER of the clipped and filters space frequency coded system proposed in [23] has been plotted. As it is apparent, while the clipping and filtering method reduces the PAPR effectively however SER performance is degraded at the receiver. In [23], a complex iterative algorithm has been proposed to compensate for clipping effect at the receiver side. Figure 5 shows the same plots for $N_c = 512$. As can be seen from these figures, in the SNR values of higher than $10dB$ there is no loss due to error detection of $\bar{d}$.

VI. Conclusion

In this paper, it is shown that the simplified method that has been previously proposed for spatially multiplexed OFDM systems is suitable for PAPR reduction of SFBC-OFDM systems. In fact, the simplified SLM does not change the orthogonality of space frequency codes. In this method, the same phase sequence is concurrently applied to the frequency domain signals for both antennas, and the signal with minimum PAPR is found and transmitted. The optimum ML detection for the transmitted symbols and the phase sequence index was introduced and then a low-complexity suboptimum detector was proposed to detect the phase sequence index without side information. Simulation results show that the simplified SLM method effectively reduces the PAPR of SFBC-OFDM system and the error rate of blind detection of the phase sequence decreases when the number of subcarriers is increased. The detection errors of the proposed method for SNR values of higher than 10 and $14dB$'s are negligible for the OFDM frame lengths of 512 and 128, respectively.

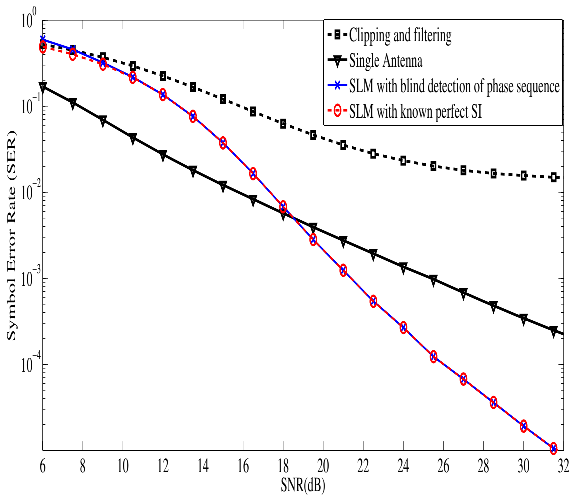

Fig. 5. The SER performance of SFBC-OFDM system with two transmitter antennas and $Nc = 512$ with three different PAPR reduction methods: 1) the simplified SLM with perfect SI ($D = 16$) , 2) the simplified SLM ($D = 16$) with blind detection of SI, 3) the clipping and filtering, and SER performance of a single antenna OFDM system.

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Mahmoud Ferdosizadeh Naeiny was born in Iran in 1979. He received B.Sc. (with highest honors) form Amirkabir university of technology (Tehran Polytechnic) in 2002 and Ms.C from Sharif university of technology in 2004. He is now the Ph.D student in electrical engineering department of Sharif university of technology under the supervision of professor Farrokh Marvasti. He is also the member of Advance Communication Research Institute (ACRI) in Sharif university from 2006. His research interests are design and implementation of OFDM systems including the synchronization issues, software defined radio, MIMO systems and the application of signal processing techniques in communications.

Farokh Marvasti received his BS, Ms and PhD degrees all from Renesselaer Polytechnic Institute in 1970, 1971 and 1973, respectively. He has worked, consulted and taught in various industries and academic institutions since 1972. Among which are Bell Labs, University of California Davis, Illinois Institute of Technology, University of London, King's College. He was one of the editors and associate editors of IEEE Trans on Communications and Signal Processing from 1990-1997. He has about 60 Journal publications and has written several reference books. His last book is on Nonuniform Sampling: Theory and Practice by Kluwer in 2001. He is also a guest editor on Special Issue on Nonuniform Sampling for the Sampling Theory and Signal and Image Processing journal. Dr. Marvasti is currently a professor at Sharif University of Technology and the director of Advanced Communications Research Institute (ACRI).