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Study of Mathematical Models of Particles Transmission Through Artery Wall

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Abstract

Atherosclerosis is one of the most important cardiovascular diseases. It originates from accumulation of particles containing fat on the arterial wall. Atherosclerosis is a form of sudden obstruction in the physical form of a arterial and leads to changes in the entering forces onto arterial wall. In this study, we investigate different models of mass transmission through the walls of vessels, and finally we will introduce the complete model.

Keywords: Atherosclerosis disease, Kedem-katchalsky equations, Navier-stokes equations.

1 Introduction

Every year America Heart Association in collaboration with the Center for Disease Control and Prevention, National Institutes of Health and other government agencies offer the most updated statistics in relation to cardiovascular disease, stroke and $\cdots$. The updated data is a valuable resource for researchers, doctors, general public and many others who are looking for the best data on heart disease, stroke and other complications of the this disease. In 2007 cardiovascular disease is the leading cause of death in the United States America. America Heart Association says that nearly 80 million American adults to one or more than one type of cardiovascular disease are affected. Only 47 percent of the population aged 65 or above have been estimated. Also according to the statistics in the new year and on 25 October from the Iranian Ministry of Health, cardiovascular diseases caused approximately 50 percent of the total death in Iran.
For the first time Keller [1] proceeded to investigate the phenomenon of mass transport of particles in the macromolecular level. Beck [2] theoretically studied mass transfer to the arterial walls in different flow regimes.

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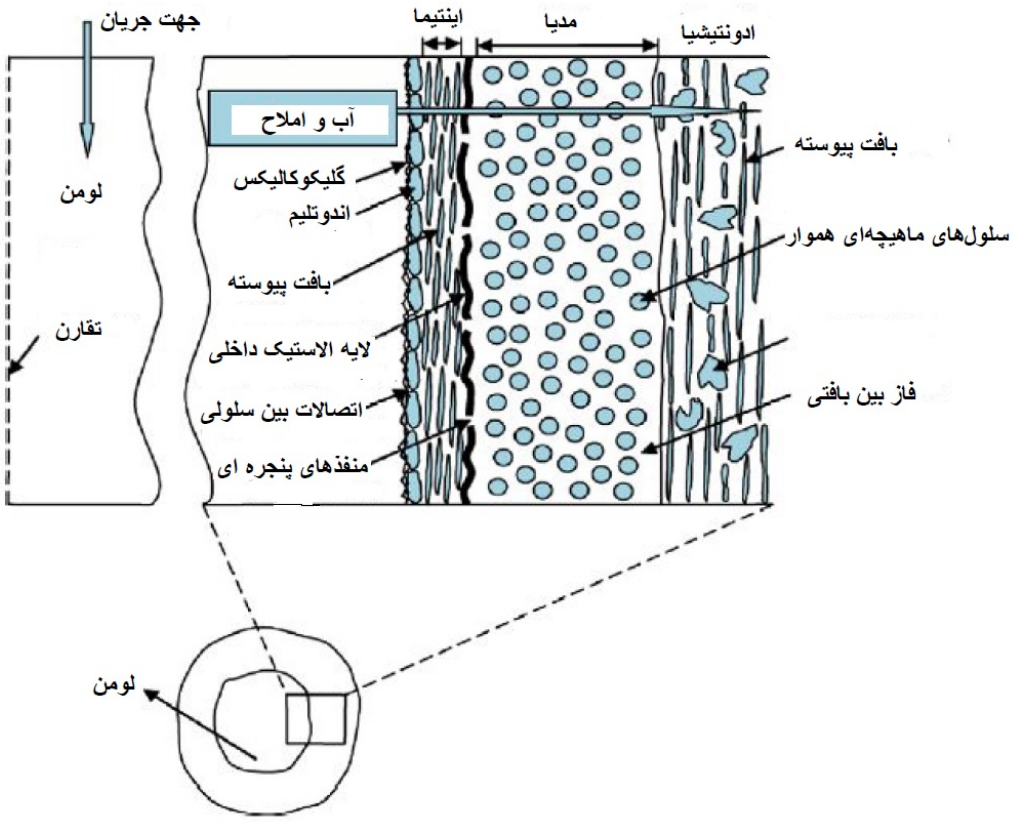
In the following for a better understanding of cardiovascular diseases we proceeding to explain and discuss the structure of the arterial.
The blood vesseles that the part of the circulatory system to transport blood throughout the body in succession to smaller vessels branching until they reach a capillary bed. Capillaries are the smallest part of the circulatory system. Blood and cells with specific and selective permeability are exchanged materials through two different membranes. One cell membranes and other is capillary walls. Vessels usually have been cleaving into two categories arterial (artery) and venous (vein) which the arteries abduct blood from the heart and veins return blood to the heart.
Now, in order to check the structure of the arterial wall we discussing a kind of anatomy of this structure that provided by Yang and Vafaei [3].
In a large artery with move from lumen side to the outermost layer, six made layers exists that include: glycocalyx, Endothelial, Intima, Internal elastic layer(IEL), media, adventitia. The following figure shows this anatomy:



Mass transfer in the arterial wall is performed according to two methods: convection associated with the differential pressure of transition flow and the other mass diffusion created by the con-

centration gradient. Molecular diffusion caused by changes in the concentration of solute in the arterial walls. This changes resulting from the process of absorption and production of protein in tissue cells. Proteins in the blood while being transferred through layers of endothelial and intima usually encounter with some of the resistance to a mass transfer that these resistance greatly depend to the size and weight of protein. Proteins can be impressed by reactions and penetrate to media.

In order to examine how the accumulation of fat particles on the arterial wall several mathematical model to study the movement of macromolecules such as low-density lipoproteins in arterial has been developed that in the following we describes the models based on describing the arterial wall.

In this course recently several mathematical models to study the movement of macromolecules such as low-density lipoproteins in arterial by Prosi and him colleagues [4] developed that these models based on the description of the arterial walls are divided into three categories:

For the simplest model, model without porosity wall, the arterial wall using appropriate boundary condition at the inner surface of the wall (lumens-Endothelial boundary) is described. This model is used to evaluate hemodynamic blood in the arterial lumen area that in this field Fazli and colleagues [5] and Nematollahi and colleagues [6] deals to examine the distribution of low density lipoprotein particles in the arterial wall.

Model of homogeneous walls verify the walls of arterial that describes the mass transfer in the blood and in the wall using the appropriate laws of physics for the modeling of the interactions between blood flow and movement of chemicals. It should be noted that the heterogeneous structure of the wall estimated by a homogeneous layer of porous. This model is used to describe the dynamics of solutes in healthy arterial. Ethier and Moore [7] were examined distribution oxygen in the arterial wall with the use of this model.

The most complex arterial transport models that proposed so far, is multi layers model that used to several of homogeneous layers, endothelium, intima, internal elastic layer and the media and acceptably indicated distribution concentration of fat particles in the arterial wall. In this regard, Yang and vafai [8] have done investigation numerical model of multilayer wall and the distribution of concentration in the different layers. Ai and Vafai [9] the effect of congestion on distribution lipoproteins have examined with multilayer model. Yang and Vafai [10] and Khakpoor and Vafai [11] analytically have solved the governing equations on the multilayer model. Chang and Vafai [12] have examined particle distribution in a multi-layer model.

Multi-layer wall model expresses the actual data on the chemical dynamics (macromolecules) in the wall.

In the following, proceeded to studies these models with review the brief of blood rheology and Hemodynamic role in formation of Atheroscleros.

From the perspective of rheology, blood can be considered as a suspension Solid-liquid. Also the blood includes the cell elements that there are in the solid state. In a slow blood flow, cellular elements contribute in disturbed the flow lines and thus disturbed the blood flow. With increasing amounts of cells, flow lines gradually impaired and compared to plasma viscosity blood viscosity increases. So degree disruption of the flow lines and thus blood viscosity depends to concentration red blood cells that shown by hematocrit. Blood viscosity almost four times the

viscosity of water that this viscosity is not constant at all flow rates. Also the blood is in the circulatory system is a non-Newtonian fluid. non-Newtonian behavior of blood at shear rates is obviously very low [13]. Several models for the study of non-Newtonian behavior of blood exist as follows:

1. power-law model
2. model Casson
3. model Quemada
4. Viscoelastic model

One of relationships that are often expressed to express blood viscosity is a power- law model which as follows:

$$\mu = K_p|\dot{\gamma}|^{n_p-1} \tag{1.1}$$

that K_p n_p and $\dot{\gamma}$ are the consistency coefficient, the power law index, and the shear rate, respectively. Casson model explains a nonlinear relationship between shear stress and shear strain as follows:

$$\sqrt{\bar{\tau}} = \sqrt{\dot{\gamma}\mu_\infty} + \sqrt{\tau_y} \tag{1.2}$$

that $\bar{\tau}$ τ_y and μ_∞ are the fluid shear stress, fluid yield stress and viscosity at high shear rate (Casson viscosity or asymptotic viscosity), respectively. Quemada model useful for study of viscosity concentrated diffuse systems that based on shear rate and Homatocriet that this model is defined as follows:

$$\mu = \mu_0\left(1 - \frac{\varphi}{2}\left(\frac{k_0 + k_\infty\sqrt{\frac{\dot{\gamma}}{\dot{\gamma}_c}}}{1+\sqrt{\frac{\dot{\gamma}}{\dot{\gamma}_c}}}\right)\right)^{-2} \tag{1.3}$$

that μ_0 and φ are the plasma viscosity and the hematocrit, respectively. In the case of viscoelastic model must consider commonly the viscosity determined by blood plasma if the blood is too viscous and elastic and elasticity of blood because elasticity of red blood cells that almost occupy 40 to 50 percent of the blood, is considerable.
In the case of medium and large arteries, Newtonian model for rheology Blood is taken. So in most arteries, blood with a fix viscosity is considered for a normal Homatocriet of Newtonian fluid.

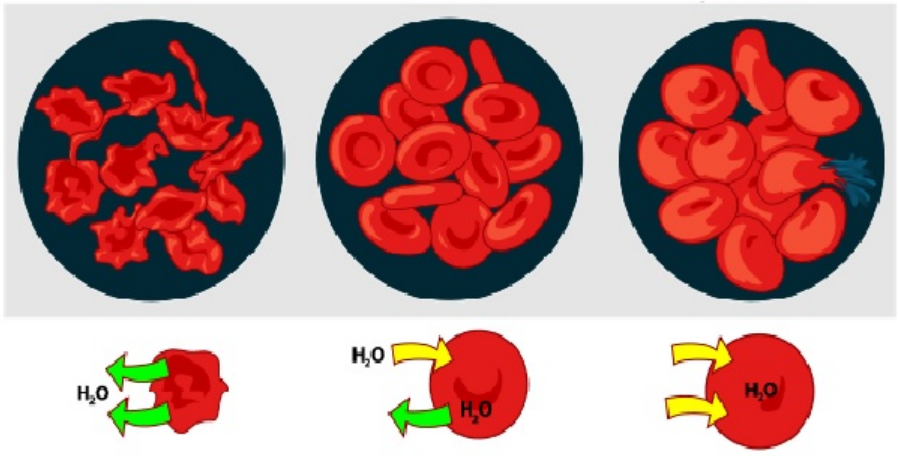
Application of porous medium in the mass transport modeling from path of biological tissue

Study of Atheroscleros includes the hemodynamic in arterial and mass transfer is across of arterial walls. As mentioned, hemodynamic conditions, plays a very important role in the

development and the formation of Atherosclerosis. Study of mass transfer in the across of arterial wall to the information needs in the field of vascular anatomy. arterial walls like other human tissues can be considered as a porous medium. In this way, the porous medium in describe the biological phenomena is very important. A porous medium determined by porosity which indicates the ratio the empty space to total volume environment. Most muman tissue can be considered as a porous medium that this tissue has been established from scattered cells by holes interconnected in which blood is flows.
However, in order to examine how transfer of particles from sporadic arterial layers, focus to describe the processes of displacement.

Processes of transport

Consider two solutions that separates by a porous thin membrane [14], this membrane is a selective permeable membranes to molecules with certain size allowed passing that this phenomenon performed among the lumen and wall and the various layers wall. With a semi-permeable membrane apart from two solutions concentraton c in a suitable solvent, we define speed of solvent filtration across the membrane (J_v) and chemical mass flux per unit $(J)_s$. Factor that causes fluid moving from one side to the other side is hydrostatic pressure force and another osmotic force pressure. In the case of osmotic pressure force, first proceeds to general definition osmosis that said to the process that duration of it the solvent through a semi-permeable membrane from where it is to where the thinner solution to the higher of concentration solution influence. Increasing concentration with increasing pressure osmotic directly related together. This phenomenon exists in the red blood cell of body. This means that if the red blood cells in put pure water, water molecules traverse from the membrane of the semi-permeable red blood cells and inter into the red blood cells. Water solvent concentration in the blood cells had more and to have found their way into the blood cells, so the amount of water in the blood cell gradually increased and will leading to a rupture of the wall of blood cells. But if this cell used in water and salt, because it has higher solvent water from cell to influence the environment and it causes the cells to shrink. Thus, intravenous injection should use the solvable (isotonic) that have the same osmotic pressure with osmotic pressure of blood. The following figure shows the phenomenon of red blood cells:



If L_p is hydraulic conductivity , ρ membrane permeability , s sieving coefficient and σ reflection coefficient that is the its complementary of s and according to that exist two types of ieving coefficient, one asthmatic s_d and another frictional

s_f and as a result two types of reflection coefficient, one asmatic σ_d and one frictional σ_f , an accepted mathematical model expressed for solvent and solute fluxes by equations Kedem-Katchalsky [15, 16]:

$$J_v = L_p(\delta_p - \sigma_d \delta_\pi) \tag{1.4}$$

$$J_s = \rho \delta_c + J_v(1 - \sigma_f)\bar{c} \tag{1.5}$$

that $\bar{c}$ mean concentration

$$\delta_p = RT\delta_c \tag{1.6}$$

that R and T are the gas constant and the absolute temperature.
Equation

$$J_v = L_p(\delta_p - \sigma_d \delta_\pi) \tag{1.7}$$

Sterling law of filtration called which states that the solvent flux in across the membrane with pressure drop is proportional between the two chambers that the pressure drop divided to the two osmotic pressure drop (σ_π) and pressure drop astatic σ_p . Osmotic pressure drops in the two chambers according to the Vant hoff law ($\delta_\pi = RT\delta_c$) , due to the concentration difference between the two chambers. Thus, the Kdm-Katchalsky equations can be stated as described the effect of the driving forces through the membrane (σ_π) and (σ_p) on the physical quantities J_v and J_s . A critical parameter that appears in the Kdm-Katchalsky equations is the mean concentration in membrane $\bar{c}$. In fact, several models can be assumed to estimate this quantity.

If we assume that the dynamics of solute in membrane under diffusion and transfer equations, the concentration in the membrane obtained under the following equation:

$$\begin{aligned} -a\ddot{c}(x) + b\dot{c}(x) = 0 \quad x \in (0,l) \\ c(0) = c_1 \ , \ c(l) = c_2 \end{aligned} \tag{1.8}$$

that $(0,l)$ cross section of the membrane shown by that $a := p$ and $b := L_p(1-\sigma_f(p_1-p_2))$ that the answer of this problem as follows:

$$c(x) = \frac{1}{1-exp(Pe)}\left[c_2 - exp(Pe)c_1 + (c_1-c_2)exp\left(\frac{bx}{a}\right)\right] \tag{1.9}$$

That P is the global Peclet number that equal to $Pe := \dfrac{bl}{a}$ Then the average concentration in the membrane is defined by the following expression:

$$\bar{c} := (\frac{1}{l})\int_a^b c(x)d(x) \tag{1.10}$$

This equation obtained from the following equation:

$$\begin{aligned} \bar{c} = f_w(c_1,c_2) = w_1c_1 + w_2c_2 \\ w_1 = \frac{exp(Pe)}{exp(Pe)-1} - \frac{1}{Pe}, \quad w_2 = \frac{1}{Pe} - \frac{1}{exp(Pe)-1} \end{aligned} \tag{1.11}$$

That $f_w(c_1,c_2)$ was called the average weight of math. This selection of mean concentration $\bar{c}$ is suitable for membranes that their thickness presumed according to characteristic of molecules size that filtered through them. In this framework, the average of concentration in the membrane can be defined by using the Nerst-Planck equation for the equilibrium chemical potential that:

$$\bar{c} = f_l(c_1,c_2) = \frac{(c_1-c_2)}{ln(\frac{c_1}{c_2})} \tag{1.12}$$

Physical experiments show that this model appropriate for very thin and selective permeable membranes. These models lead to different values of mean concentration. Finally, in the total state view that the concentration in the membrane defined by $\bar{c} = f(c_1,c_2)$ However, in order to set up mathematical models for flow in the tissues that according to the former content can be considered as a porous medium, we introduce porosity, $0 < \epsilon < 1$ And hydraulic permeability (or Darcy permeability), K_D , That assumed the constant numerical values. About the free fluid we have $\epsilon = 1$.
Assuming that the blood plasma fills the empty space of porous media, for describes the flow in the tissue, several model searches that explain in below:
Darcy model represents a linear relationship between flow velocity and pressure gradient across the porous media that express as follows:

$$\vec{V} = -\frac{K}{\mu}\nabla p \tag{1.13}$$

that K is the permeability tensor, $\vec{V}$ the velocity vector, μ the dynamic viscosity ∇p is the gradient pressure. This model despite the useful has several significant deficiencies. In this model, the effects of a boundary or the inertia forces, ignored on the heat transfer and fluid flow through porous media [15]. To this end, a number of changed models presented. One of this changed models which is calculated for the effects of inertia, is the Darcy-Fotchheimer model:

$$\nabla p = -\frac{\mu}{K}V + c_F K^{-\frac{1}{2}}\rho|V|V \tag{1.14}$$

that c_F is a dimensioless parameter related to interial effects. Now for calculation of solid boundaries effects, Brinkman model is used. This model provides allow the use of boundary conditions non-slip in across the wall. Brinkman Model is defined as follows:

$$\nabla p = -\frac{\mu}{K}V + \mu_e \nabla^2 V \tag{1.15}$$

that μ_e is the visosity of the porous madium. μ_e Most of the isotropic porous media equal to μ . Vafai and Tien have presented a generalized model for flow transfer through porous media that to the related different effects is calculation. This model defined as follows:

$$\frac{\rho_f}{\varepsilon}\left[\frac{\partial < v >}{\partial t} + < (v \cdot \nabla)v > \right] =$$
$$-\nabla < p >^f + \frac{\mu}{\varepsilon}\nabla^2 < v > - \frac{\mu}{K} < V > - \frac{\rho_f F_\varepsilon}{k^{\frac{1}{2}}}[< V > \cdot < V >]J \tag{1.16}$$

that F is the dimensionless inertia coefficient, ρ_f the fluid vdensity, $\nabla < p >^f$ the average pressure and J a unit vector oriented along the velocity V Defined. the symbol <> represents the local volume average of a quantity associated with the fluid.

Mass transfer across the blood arterial wall

There are three major challenges in modeling the mass transfer in the arteries that include: detailed description of the geometry of the arterial, provided the appropriate set of equations and select the conditions border. In this regard, several geometric model of the arterial wall provided that these models based on the description of the arterial wall and basic assumptions. Prosi divided these models into three types as follows:

wall-free model

The simplest model is non-porous wall. This Model for blood flow in the lumen used when the effects arterial wall by an appropriate set of boundary conditions are computed. In this way, the solution that achieved through this model independent from the moving processes around arterial. Usually gives the amount of boundary conditions like filtration rate. In this model, Due to the simplicity needs to the few parameters. To Example: propagation in the plasma, the total mass transfer coefficient wall according to the intended solute and velocity filtration.

However, this model cannot state any information about the concentration in the wall. This model majority uses in the Fields of hemodynamic study and role of parameters Hemodynamic in start and formation of atherosclerosis. Also used in study of different solutes dynamic such as oxygen, low-density lipoproteins andalbumin.

Homogeneous-wall model

The second type of mass transfer model, are homogeneous wall models. In these models, the arterial wall is interfered, however, its complex heterogeneous structure approximated by a simple homogeneous layer. Feature of walls usually approximate values based on the assumptions that the arterial walls of porous media are homogeneous. These models shows the perfect match between the complexity of the initial data and the accuracy of the results and used in cases where the distribution of concentration in throughout the arterial wall not primary. In fact, for study hemodynamic and its localizing role, homogeneous wall model can be used as a tool to study the reaction between hemodynamic parameters and arterial wall. For example: stress-related endothelial permeability.

Investigation the multi-layer model

Multi-layered model for calculating the features and special specs each porous layer from the homogeneous wall model is useful. Using the multi-layer model and solve an appropriate set from governing equations and boundary conditions can be conclude an accurate description of the dynamics and distribution of macromolecules across the arterial wall. In fact, this model provides realistic descriptions about the anatomy of the artery. Although needed a greater number of parameters to determine the Transfer processes in each layer. To determine properties of tissue exist more restrictions, especially the tissue of the human body. For this reason, many efforts dedicated to specified parameters. The number of studies was presented on the assumption that layers of the arterial wall has a porous structure and the physical properties that can identify with using the pore case [16].
In continue proceeding to the full description of multi-layer model and pore case.
.In the wall-free model, fluid dynamics and mass transfer in the lumen of the arterial, described by using the Navier-Stokes equations and advection-diffusion equations Which are defined as follows:

$$\rho(\frac{\partial U}{\partial t} + (U \cdot \nabla)U) = -\nabla P + \mu \Delta U + \rho b$$

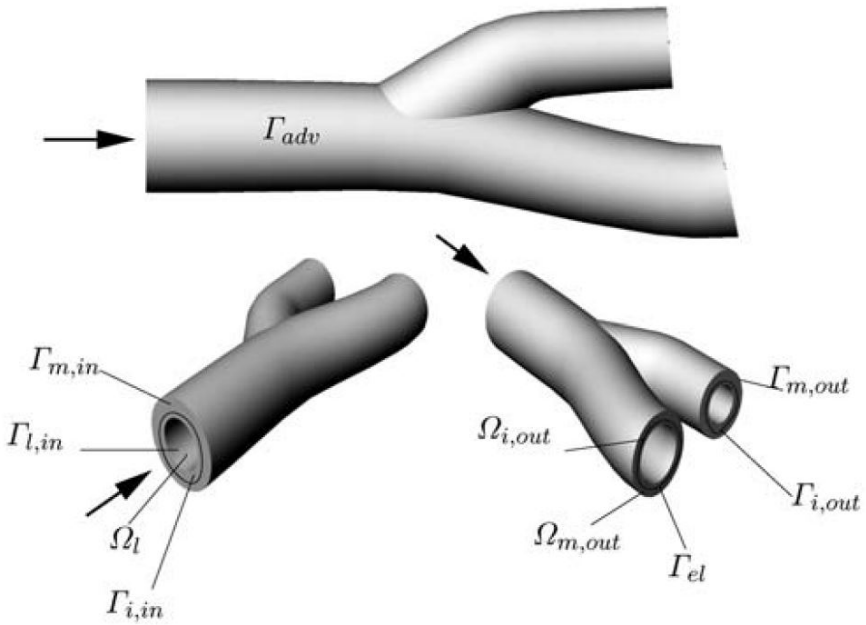
and

$$\frac{d}{dt}\int_{\Omega(t)} c(x,t)d(t) = \int_{\Omega(t)} \left(\frac{\partial c}{\partial t} + \nabla \cdot (cu)\right) d(x) \tag{1.17}$$

In the boundary between the lumen and arterial wall suitable condition for volume flux (J_v) and mass flux (J_s) assumed as follows:

$$\begin{aligned} u_l.n_l &= J_v \qquad on \quad \Gamma \\ (-D_l \nabla c_l + u_l.c_l).n_l &= J_s \qquad on \quad \Gamma \end{aligned} \tag{1.18}$$

In the homogeneous wall and multi-layer wall model need to a comparative appropriate boundary condition between the equations in different environments. These conditions provided by using the Kedem-Katchalsky equations and their application that the following explained for multi-layer model.
To launch the first multi-layer model consider all thickness of layers vessel (endothelial, intima, internal elastic layer, media and adventitia) that to reduce complexity results numerical problems, consider some of simplification. One the approach has been proposed and discussed, is consider the thin layers as the membrane, thus a problem that we here address consists three region pairs: the lumen, intima and media that respectively separated by endothelial and internal elastic layer. An introduction of the domain shown in the following figure:



To launch equations for model, indicate physically values related to lumen, intima and media by indexes l, i, m and also used the indexes end and iel for endothelial and internal elastic layer.

Dynamic models of fluids

To understand this subject that which model, Darcy model or Brinkman model, appropriate for our aim concentrates on the conditions between lumen and wall that corresponds with endothelium layer defined by Γ_{end} . In the environs of lomenal consider the interface as a non-slip surface

that only allows to normal flows.With assumption that the wall indicated by below parameters: diameter of pores (D_p) , porosity if wall (ϵ) , channel thickness (H) , permeability of wall (K_i) and δ that is the thickness of boundary layer ($\delta = 1 \quad \mu m$) , we observes that thickness of boundary layer half the thickness of the layer of Endothelium $\cong 2 \quad \mu m$, so in the multi-layer model, endothelium and IEL according to their very little thickness considered as a membrane. As a result, Darcy-Brinkman model is not fully compatible with multi-layer model and to suitable use can use in resolve details in the scale that is smaller than the thickness of endothelial or internal elastic layer. So coupled Navier-Stokes Darcy in our case is more appropriate.
Now, the system includes Navier-Stokes Darcy equations (using the symbols introduced in the previous) are as follows:

$$J_{v,end} = L_{p,end}(p_l - p_i) - L_{p,end}\sigma_d RT(c_l - c_i) \quad on \quad \Gamma_{end} \tag{1.19}$$

$$J_{v,iel} = L_{p,iel}(p_i - p_m) - L_{p,iel}\sigma_d RT(c_i - c_m) \quad on \quad \Gamma_{iel} \tag{1.20}$$

Then, according to the equations that described in below, the velocity values and pressure in the lumen, intima and media $(u_l, p_l, u_i, p_i, u_m, p_m)$ obtained.

$$\begin{aligned}
&(a)\ \frac{\partial u_l}{\partial t} + \Big(u_l \cdot \nabla\Big) u_l - div\frac{\sigma_l}{\rho} = 0 && in\ \ \Omega_l,\ \ t > 0\\
&(b)\ div\, u_l = 0 && in\ \ \Omega_l,\ \ t > 0\\
&(c)\ u_l = u_{l,in} && on\,\Gamma_{l,in},\ \ t > 0\\
&(d)\ \sigma_l n_l = p_{out} n_l && on\ \Gamma_{l,out},\ \ t > 0\\
&(e)\ u_l \times n_l = 0, \quad u_l \cdot n_l = u_i \cdot n_l && on\,\Gamma,\ \ t > 0\\
&(f)\ u_l = u_0 \quad with \quad div\, u_0 = 0 && in\,\Omega_l,
\end{aligned} \tag{1.21}$$

Where σ_l is the Cauchy stress tensor. The conditions 1.20_e state that Γ is a non-slip surface and allows for filtering in the normal direction.

$$\begin{aligned}
&(a)\ u_i + \frac{K_i}{\mu_i}\nabla p_i = 0 && in\ \Omega_i,\ \ t > 0,\\
&(b)\ div\, u_i = 0 && in\ \Omega_i,\ \ t > 0\\
&(c)\ u_i \cdot n_i = 0 && on\Gamma_{i,in} \cup \Gamma_{i,out},\ \ t > 0,\\
&(d)\ u_i \cdot n_i = -J_{v,end} && on\Gamma_{end},\ \ t > 0,\\
&(e)\ u_i \cdot n_i = J_{v,iel}, && on\Gamma_{iel},\ \ t > 0,
\end{aligned} \tag{1.22}$$

The equation 1.21_a is the Darcy filtration law with constant and Darcy permeable scalar (K_i) . Equation (1.21_b) calculates the mass concentration. Boundary conditions (1.21_c) applied the filtering of velocitys if view abuts them in the initial and final sections of the wall. Conditions ($1.21_{d,e}$) determined the filtered values speed in the Endothelium and internal elastic layer

according to Kedem Katalsky equations.

$$\begin{aligned}
&(a)\, u_m + \frac{K_m}{\mu_m}\nabla p_m = 0 && in\ \Omega_m,\ \ t>0,\\
&(b)\, div\, u_m = 0 && in\ \Omega_m,\ \ t>0\\
&(c)\, u_m \cdot n_m = 0 && on\Gamma_{m,in}\cup\Gamma_{m,out},\ \ t>0,\\
&(d)\, u_m \cdot n_m = -J_{v,end} && on\Gamma_{iel},\ \ t>0,\\
&(e)\, p_m = p_{adv}, && on\Gamma_{adv},\ \ t>0,
\end{aligned} \tag{1.23}$$

Equations ($1.22_{a,b}$) and conditions ($1.22_{c,d}$) are similar to equations in the intima. Condition (1.22_e) proves the amount of pressure on the adventita for a known value.

Dynamic models of solute

First of all be noted that due to the phenomenon of friction on molecular motion, the velocities of real movement in the wall is lower than the obtained filtration rate according to their equations. Now we rewriting the equation (2) from the Kedem Katchalsky equations with an average concentration in the membrane:

$$\begin{aligned}
J_{s,end} = \rho_{end}(c_l - c_i) + L_{p,end}(1-\sigma_f)(p_l - p_i)f(c_l,c_i) -\\
L_{p,end}(1-\sigma_f)\sigma_d RTf(c_l,c_i)(c_l - c_i)
\end{aligned} \tag{1.24}$$

$$\begin{aligned}
J_{s,iel} = \rho_{iel}(c_i - c_m) + L_{p,iel}(1-\sigma_f)(p_i - p_m)f(c_i,c_m) -\\
L_{p,iel}(1-\sigma_f)\sigma_d RTf(c_i,c_m)(c_i - c_m)
\end{aligned} \tag{1.25}$$

Using these definitions, according to the following equations and the problems that discusses for dynamics solute, we obtains the concentration values in the lumen, intima and media (c_l, c_i, c_m)

$$\begin{aligned}
&\frac{\partial c_l}{\partial t} + div\,(-D_l\nabla c_l + u_l c_l) = f_l\ , t>0,\ \ c_l(0) = c_{l,0} && in\Omega_l,\\
&(a)\, c_l = c_{l,in} && on\ \Gamma_{l,in},\ \ t>0,\\
&(b)\, \nabla c_l \cdot n_l = 0 && on\ \Gamma_{l,out},\ \ t>0\\
&(c)\ -D_l\nabla c_l \cdot n_l + u_l \cdot n_l c_l = J_{s,end} && on\Gamma_{end},\ \ t>0,
\end{aligned} \tag{1.26}$$

Condition 1.25_c according to the Kedem-Katchalsky equations determine the solute flux in the across of Γ_{end}

$$\begin{aligned}
&\frac{\partial c_l}{\partial t} + div\,(-D_i\nabla c_i + \frac{\gamma_i}{\epsilon_i}u_i c_i) + r_i c_i = f_i\ , t>0,\ \ c_i(0) = c_{i,0} && in\Omega_i,\\
&(a)\, \nabla c_i \cdot n_i = 0 && on\ \Gamma_{i,in}\cup\Gamma_{i,out},\ \ t>0,\\
&(b)\ -D_i\nabla c_i \cdot n_i + \frac{\gamma_i}{\epsilon_i}u_i \cdot n_i c_i = -J_{s,end} && on\ \Gamma_{end},\ \ t>0\\
&(c)\ -D_i\nabla c_i \cdot n_i + u_l \cdot n_l c_l \frac{\gamma_i}{\epsilon_i}u_i \cdot n_i c_i = J_{s,iel} && on\Gamma_{iel},\ \ t>0,
\end{aligned} \tag{1.27}$$

We introduce the term of $r_i c_i$ for use of chemicals material by constitutive tissues of intima. Condition 1.26_a applies an null diffusion flux in the first and the last Situation of intima. Conditions $1.26_{b,c}$ applies an obtained flux by Kedem-Katchalsky equation on the Γ_{end} and Γ_{iel}

$$\begin{aligned}
&\frac{\partial c_m}{\partial t} + div\,(-D_m \nabla c_m + \frac{\gamma_m}{\epsilon_m} u_m c_m) + r_m c_m = f_i\ , t > 0,\ \ c_i(0) = c_{i,0} && in \Omega_i,\\
&(a)\, c_m = c_{adventitia}\ \ or\ \ \nabla c_m \cdot n_m = 0 && on\ \Gamma_{adv},\ \ t > 0,\\
&(b)\, \nabla c_m \cdot n_m = 0 && on\ \Gamma_{m,in} \cup \Gamma_{m,out},\ \ t > 0\\
&(c)\ -D_m \nabla c_m \cdot n_m + \frac{\gamma_m}{\epsilon_m} u_m \cdot n_m c_m = -J_{s,iel} && on \Gamma_{iel},\ \ t > 0,
\end{aligned} \tag{1.28}$$

Equations and boundary conditions for the media are similar to those that established for intima.
These equations and problems that according to them we acquired the velocity, density and pressure in the layers of the lumen, intima and media, provides a multi-layer model for the mass transfer in across of vessel wall.

Pore theory

In order to achieve appropriate mathematical models for lumens, fluid transmission and solute dynamic requires appropriating parameters. These parameters describe the displacement characteristics of the assumed domains such as lumen, endothelial, intima, internal elastic layer, and media. Many parameters cannot be directly obtained by experimental measurement. In this section with using pore theory obtain a set of parameters for different types of wall layers.
Pore theory based on the assumption that the layers of the wall have the porous structure, which physical features can be identified by their geometry structure.

Mass transfer in porous intima and porous media

the healthy subendothelial endothelial of intima and media typically includes an extracellular matrix randomly distributed of collagen and protein. Transfer processes in the arteries wall layer occurs only in liquid phase. Fiber matrix is determined by a number of parameters that include: the thickness of the wall layer H the fibre radius r_f the total length of the fibres l_f within the unit volume, so The fraction of matrix voids volume of fiber result as follows:

$$\epsilon_f = 1 - \pi\, r_f^2\, l_f \tag{1.29}$$

Darcy permeability K_{Df} of porous tissue is defined as follows:

$$K_{Df} = \frac{r_f^2 \epsilon_f^2}{4G(1-\epsilon_f)^2} \tag{1.30}$$

that G is the Kozeny constant. Limited permeability of an intended solid with an average molecular radius r_{mol} in the extracellular matrix calculated by the following equation:

$$D_f = D \cdot \ exp\Bigg(- \Big(1 - \epsilon_f \Big)^{\frac{1}{2}} \Big(1 + \frac{r_{mol}}{e_f}\Big)\Bigg) \tag{1.31}$$

that D is the solute diffusivity in water and D_f is limited diffusion in the fiber matrix. the hindrance coefficient for convection transfer in matrix fiber can be calculated as follows:

$$\gamma_f = 2 - \phi_f \tag{1.32}$$

That ϕ_f indicates the relationship between available space for solute relative the available space for the water that is defined as follows:

$$\phi_f = \ exp\Bigg[- (1 - \epsilon_f)\Big(\frac{2r_{mol}}{r_f} + \frac{r_{mol}^2}{r_f^2}\Big)\Bigg] \tag{1.33}$$

As explained in the before media formed by the smooth muscle cell layer. Also in thickened intimal, this type cells has been observed. The effect of smooth muscle cells in the model included by additional volume fraction ϵ_{SMC} that reduces the total porosity of arterial wall:

$$\epsilon_{eff} = \epsilon_f(1 - \epsilon_{SMC})$$

Despite the smooth muscle cells, transmission parameters (diffusivity, Darcy permeability and lag coefficient) become to the effective parameters. Without the presence of smooth muscle cells the calculated parameters from the above equation shows effective parameters.

Transfer within endothelial and internal elastic layer

Endothelium and internal elastic layer considered as a selective permeable membrane. In addition, they assumed by layers with constant thickness. Exchange Of water and solute in the width of endothelial done through the pores that stands between endothelial. Pores can be inside healthy endothelial cells divide that modeled as cylindrical pore and leaky junctions, and estimates as pores with circular cross section in around leaky cells. Internal elastic layer consist the fenestral pores that through them takes place the transfer between intima and media. A fenestral pore can be approximated as cylindrical pores. Molecules that their sizes are smaller than the size of exists pores in the width of porous membrane, can be described transports of them with use advection-diffusion-reaction equations. The only effect of the porous membrane on the transfer is reducing the space available for the solution. Transfer large molecules limited through these membranes by the pore structure. It is assumed that molecule for enterce into a pore must passes without striking to its edge. This limitation is due to the reflection and sieving the large molecules in membrane surface in relation to the small pores. During the transfer from pores, molecules collide with the pores wall. These interactions between the pores and pores wall causes that energy of molecules that as a result of a limited transfer are from pores reduced [17].

Now proceeds to describe of pores that consists the cylindrical pores and the pores with circular cross section.

Cylindrical pores

Hydraulic conductivity of the cylindrical pores is defined as follows:

$$L_p = \frac{\rho_{pore}\,\pi\,R^4}{8\mu_{\,L}} \tag{1.34}$$

that ρ_{pore} is the average density if the pores, R is the radius and L the length of the pore. Limited diffusion coefficient in the pore is defined as follows:

$$D_p = D\ F(\alpha) \tag{1.35}$$

that $\alpha = \frac{r_{mol}}{R}$ is the ratio between the molecule radius and the radius of the pores and obtained a following

$$F(\alpha) = [2(1-\alpha)^2 - (1-\alpha)^4][1 - 2.1\alpha + 2.09\alpha^3 - 0.95\alpha^5] \tag{1.36}$$

and permeability pores calculated by the following equation

$$\rho_p = \phi D_P / L \tag{1.37}$$

that $\phi = (1-\alpha)^2$ assumes the pore reduction in width of cross-section that for the solute exist. The osmatic reflection coefficient obtains by following equation

$$\sigma_{d,p} = (1-\phi)^2 \tag{1.38}$$

and solvent drag reflection coefficient obtains by following equation

$$\sigma_{f,p} = \frac{16}{3}\alpha^2 - \frac{20}{3}\alpha^3 + \frac{7}{3}\alpha^4 \tag{1.39}$$

Pores with circular cross sections

A pore with circular cross section with the same way consider like an infinitely long slot with constant depth $2b$. So hydraulic conductivity of circular pore obtained by the following equation

$$L_s = \frac{b^2}{3\mu L} \tag{1.40}$$

that the limited permeability coefficient in pore defined as follow

$$D_S = DF_s(\alpha_s) \tag{1.41}$$

that $\alpha_s = \dfrac{r_{mol}}{b}$ is the ratio Molecular radius to the half size of width of the pore. Limited permeability pore function is as follows

$$F_s(\alpha_s) = (1-\alpha_s)(1 - 1.004\alpha_s + 0.418\alpha_s^3 - 0.169\alpha_s^5) \tag{1.42}$$

and permeability pores in the same previous way obtained with the following equation:

$$\rho_s = \phi_s D_s / L \tag{1.43}$$

that $\phi_s = 1 - \alpha_s$ osmatic reflection coefficient for pore with circular cross section equal with

$$\sigma_{d,s} = (1 - \phi_s)^2 = \alpha_s^2 \tag{1.44}$$

and the solvent drag coefficients equals to

$$\sigma_{f,s} = 1 - \left(1 - \frac{3}{2}\alpha_s^2 + \frac{1}{2}\alpha_{s^3}\right)\left(1 - \frac{1}{3}\alpha_s^2\right) \tag{1.45}$$

Connection of transmission parameters of the leaky clefts $L_{p,l_j}, \rho_{l_j}, \phi_{l_j}, \sigma_{f,l_j}, \sigma_{d,l_j}$ and normal junctions $L_{p,n_j}, \rho_{n_j}, \sigma_{f,n_j}, \sigma_{d,n_j}$ according to the as described equations in the above are calculated. The values of hydraulic conductivity, permeability and endothelial reflection coefficients of normal junctions and leaky clefts are as follows:

$$L_{p,end} = L_{p,n_j} + L_{p,l_j}\, \epsilon_{l_j} \tag{1.46}$$

$$\rho_{end} = \rho_{n_j} + \rho_{l_j}\, \epsilon_{l_j}\, \phi_{l_j} \tag{1.47}$$

$$\sigma = \frac{L_{p,n_j}\, \sigma_{n_j}\, \epsilon_{l_j} + L_{p,l_j}\, \sigma_{l_j}}{L_{p,e}} \tag{1.48}$$

2 Main Results

In this paper we presented the kinds of mathematical models for study of the molecules transfer like: oxygen, LDL or drug through the artery wall. From these models, the multi-layer wall model described the arterial anatomy with more accurate and provided the accurate comments for macromolecules distribution in the across of the arterial wall. Also to express the average velocity in the tissues; we used the models like Darcy-Brickman model and $\cdots$ that any one of these models has the substantial defects. From these models, model that showed by Vafai and Tien, with considering the Inercy force and effects of boundaries, average velocity in the tissues was described with accurate and completely details.

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