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Geometric investigation of blood flow and increasing effect angle of graft vessels on the vortical structures

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Abstract

Theories that in them geometrical shape and fluid mechanical forces are effect goes back to the mid-century. Earlier Denis Doorly and Spencer Sherwin in 2003 proceeded to investigate the important and efficient factors in the blood flow among the vessels wall in the vascular biology concepts. This paper examines the geometric characteristics of the vessel and its effect on blood flow as a Newtonian flow using Navier-Stokes equations and after studying the geometry of vessels and transplant notes to coordination between mechanical and physiologically properties of blood flow in the vessels and shows that in Anastomoses with Reynolds number 125 increase the bond angle produce a stronger vortical structures and secondary flows. It is prognosticated that strength of the vortical at higher angles graft followed growth and development of Atherosclerosis of disease. Therefor graft with lower angle decreasing the possibility of Atherosclerosis disease after graft.

Keywords: Blood flow, vessel geometry, Navier-Stokes equation, Atherosclerosis, Vortical.

1 Introduction

Atherosclerosis artery disease usually occurs in the regions where vessels are bent and bent close. Changes in shape of vessels according to medical pathology of blood vessels directly effects on the blood flow and exists the interaction between the dynamics of blood and vascular biology. Numerical Simulation and experimental observations have indicated that blood homodynamic have important role in the accession and extend disease Atherosclerosis [1,2]. The development and progression of this disease depends to important factors such as blood viscosity [3], shear stress [4] and apply pressure gradient on vessel wall [5]. Other important factors that provide homodynamic conditions for the development Atherosclerosis were the geometry of vessel and

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deflexion and Branch[6]. Wall grafts and areas inside the curves are areas prone to this disease. Recently, many studies and many simulations to describe blood flow in arteries and veins with a curved branch it has been done. Van De Vosse and colleagues [7] investigated perennial flow in 90-degree bend. Dwyer and assignment [8] numerically evaluated Flow passing in a 180 degree bend and found that in high Womersleys, less flow under the influence of geometry. For treatment of Atherosclerosis, aorta-coronary bypass surgery a surgery is common. Results indicate that about 30 to 50 percent of these surgeries fails [9,10] and the growth of vascular cells in the graft area and heel and toe parts are failure factors of these surgeries[11] which eventually led to the development of Atherosclerosis. In 1989 Inzoli and him assignments investigated the flow dynamic in the aorta-coronary bypass [12]. Therefor study of transplants, geometric nature them and results of the simulation can be effective real actions to better perform the graft and prevention the development of Atherosclerosis. Since each flow heavily influenced by the geometry of the flow channel, in the first we following examines how this relationship in vessels. We study the blood flow and its primary features as a fluid Newtonian incompressible. The modeled flow appropriate for vessels with a diameter $d > 1$ mm and in the vessels with a smaller diameter may be useful non-Newtonian flow. Assume that vessels wall is strong and fix. System equations of describing the flow model may be resolved for specific values and dense and clear geometry parameters and flow. Although there are changes in the parameters that cause leave the steady state system, but in general all solutions produces for arbitrary parameters in a closed set. Because the viscous component of probed equation, the Navier-Stokes equation, makes all the solutions be in a closed set. Starting with identifying parameters without dimension that named similar parameters and speak about their physical importance and then reviews basic relations of differential geometry of three-dimensional curves that offers exact definition of vorticity and bending flow and examined flow in vessels curve. Then we are going to see blood flow in Anastomose. The effect of increasing angle in grafts investigated and eventually considered example of the actual geometry of the body and speak about the Presented properties.

2 Dimensionless parameters

In inside body flow model is calculation and similar parameters related shall coincide after study the flow. In the tubes curved Reynolds and Dean numbers are important parameters. Although for unstable state flow, Womersley number and Reduced Velocity also effective.

Definition 2.1. Reynolds number(R_eD) :

In an internal flow with average segmental velocity U in a tube with specified diameter$D$as follows:

$$Re_D = \frac{\rho U D}{\mu}$$

Whereμ is Newtonian fluid viscosity and extension of$D$ in R_e indicated that used reference abscissa is diameter of Vessel. In fact, sometimes there may be other factors used that diameter is important measure of abscissa for the blood mechanic in a vessel. Reynolds number usually discusses in the Navier-Stokes dimensionless equations. Navier-Stokes equation for an incompressible fluid with velocity$U$ and pressure$P$ In the presence of foreign force b expressed as follows:

$$\rho(\frac{\partial U}{\partial t} + (U \cdot \nabla)U) = -\nabla P + \mu \Delta U + \rho b$$

The Reynolds number obtained using the scaling and conversion this equation to the dimensionless state. In the physical view the Reynolds number interpreted the ratio of inertia forces to Viscosity forces and since the flux inertia related to significance of inertia force can be Argued that while in a flow, have a large Reynolds number the inertia forces more than the Viscosity forces and vice versa.

Definition 2.2. Dean number De:

It is defined as follows:

$$De = \frac{\rho U D}{\mu}(4\sqrt{\frac{D}{R_c}}) = 4\sqrt{\frac{D}{R_c}} Re_D$$

Where, R_c is radius of curvature,D is the diameter of the tube and $R_e D$ is the Reynolds number. The Dean number can also be State physically as a proportion between the forces of inertia, centripetal acceleration and friction forces to as follows:

$$De = 4\sqrt{\frac{D}{R_c}} Re_D = 4\frac{\sqrt{\rho \overline{R_C} \frac{U^2}{R_c} \times \rho U^2}}{\mu \frac{U}{D}} = \frac{\sqrt{centripetal\ force \times intertiol\ forces}}{viscous\ forces}$$

In the above definition $\overline{R_C} = \frac{R}{c}/D$. The first study about the Newtonian fluid flow in a curved channel is done by Dean. He showed that the effect of centrifugal force of curvature lead to the creation of secondary flows. He introduced the Dean number as a good measure for this flow.

Definition 2.3. Womersley number, Reduced Velocity/ and strohal number:
When examines an unstable flow commonly used the Womersley number:

$$W_0 = \frac{D}{2}\sqrt{\frac{2\pi}{\nu T}}$$

Where $\nu = \frac{\mu}{\rho}$ and T is the fundamental Period of the swing. The interested problem Womersley was the oscillatory flow in a straight tube [13]. Therefor the number Womersley concerned to the exact solution of a Newtonian fluid in a straight tube with the periodic pressure difference.

Womersley number can be physically Interpret as diameter ratio tube to the slow growth of the boundary layer at a time T. An alternative without dimension from the time scales that usually used in the parts like downhill flow in body is the Strohal number that Displayed by S_t:

$$S_t = \frac{D}{UT}$$

And its reverse that will be remembered as Reduced Velocity as follows:

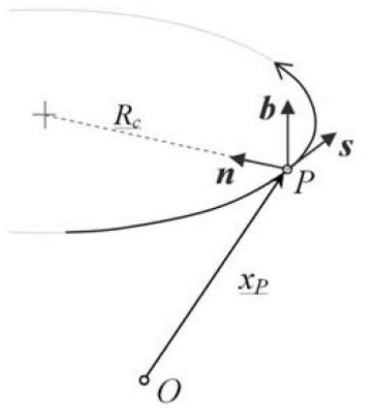
$$U_{red} = \frac{UT}{D}$$

It can be physically considered ratio of the distance traveled by the mean of flow to the diameter of the tube. Reduced Velocity and Womersley number can be related together by the Reynolds number:

$$U_{red} = \frac{\pi}{2}\frac{Re_D}{W_0^2}$$

3 Geometry of curves and tubes

The vessels in his way pass from a curved path and winding and with the repeated branching. Normally, the flow channel (Arterial lumen) is the circle. The vessels in his way pass from a curved path and winding and with the repeated branching. Normally, the flow channel (Arterial lumen) is the circle. Let us assume that$P$ is a point that a curve traverses in the three-dimensional space. As the shown in the figure 1, situation of point$P$ can be indicated based on the curvature abscissa s, $P(s(t))$, or based on its position vector X_P.



The unit vector tangent on the curve at the point$P$ that displayed with the$S$, obtained as follows:

$$S = \frac{dX_p}{d\mathtt{s}} = \dot{X}_P/v$$

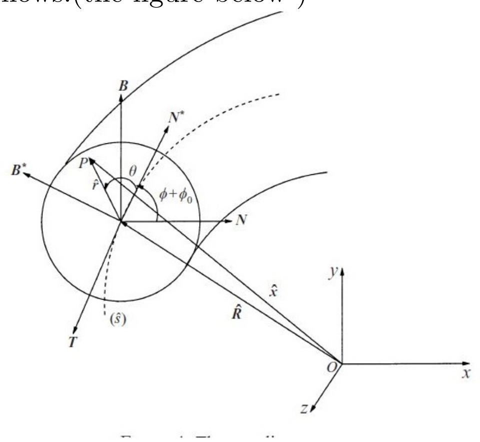
Where$V$ is the velocity at the point P along the curve:

$$v = \frac{d\mathtt{s}}{dt} = \mathtt{s}^{\cdot} = \|x_p\|$$

In the any point P we can make Frenet frame by use the unit orthogonal vector include$S$, the main normal vector$N$ and second normal Bthat interrelated together by following Fernet-Sert formulas:

$$\begin{aligned}
&\frac{dS}{d\mathtt{s}} = \mathtt{k}n = \frac{1}{R_c}n;\\
&\frac{dn}{d\mathtt{s}} = -\mathtt{k}S + \tau b\\
&S \times n = b,
\end{aligned}$$

where curvature k is reverse the curvature radiusR_c. In a coil tube, based on the curvature abscissa s at the central segment of the tube and position (r, θ) in normal page, i.e. the plane perpendicular to the central tangent,can be define a new coordinate system. Germano [14] in 1982-1989 received a periodic coordinate system with uses from the position vector$R(s)$ where each vector$x$ displayed as follows:(the figure below)



$$x = R(s) + r\cos(\theta + \phi(s) + \phi_0)N(s) + r\sin(\theta + \phi(s) + \phi_0)B(s)$$

Where(r, θ) are polar Coordinate in the segment surface of tube, and$\phi_0 = \frac{\pi}{2}$, $\phi(s) = -\int_0^s \tau(s')ds'$. Due to the fact that the vessels are typically in the form of a core tube the presence in the circulatory system of the flow and may be flow way in them associated with swirl, the coordinate system of Germano appropriate for describe the flow in this kind of vessels.

Definition 3.1. vorticity:
The primary Variable of Vorticity has abound usage in describe the geometry of the flow in

curves and thus blood flow. Blood in its path traverse the branching and winding pathways and accosted to some changes in the amount and velocity Direction. that leads to the production of vorticity. Vorticity is defined as the curl of the velocity:

$$w = \nabla \times u$$

Place:

$$q^2 = u \cdot u$$

We have the following relation:

$$\frac{1}{2}\nabla(u \cdot u) = u \times (\nabla \times u) + (u \cdot \nabla)u$$

Linear departure balance equation for a incompressible Newtonian fluid with instates relation high into the Navier-Stokes equation obtained as follows:

$$\rho\frac{DU}{Dt} = -\nabla P + \frac{1}{Re}\Delta U$$

If the vorticity is non-zero, flow is so-called rotatory . Tangent lines of aim velocity and aim vorticity, respectively flow lines and vorticity lines are called, placed in a level with the constant total pressure that is called the Bernoulli surface. vorticity properties by three laws Helmholtz says as follows:
First law: Power of a vorticity tube is fixed in its abscissa .
Second law: A vorticity tube cannot end in a fluid. It should be expanded to the borders of fluid or fabricated a closed route.
Third law: The absence of external rotational forces of fluid which initially had no rotation, no rotation will Remains.
With the applied cerl of the momentum equation 2 and applying the following criteria:

$$\nabla \cdot u = 0 = \nabla \cdot w$$

We gain an equation for vorticity transfer to the below form :

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u \cdot \nabla w = w \cdot \nabla u + \frac{1}{Re}\nabla^2 w$$

The above equation shows that if ignore the transfer by diffusion Viscosity, if at first all of the flow W=0, then it will remain the same (Third Law Helmholtz). Effect of distribution of vorticity in the flow depends largely on the amount obtained by Circulating number that determined as follows:

$$\Gamma = \int_A w.ndA$$

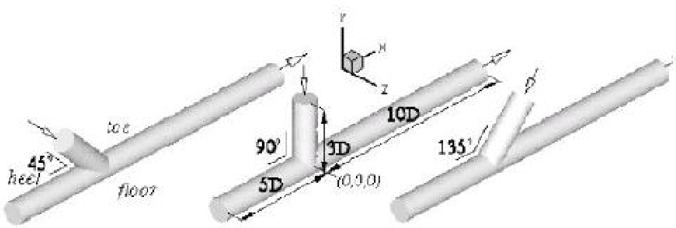
With the integration of the vorticity equation and uses the Stokes theorem can be shown that in a non-viscous flow, Circulating number remains a fixed amount that it called strengthens of the vortex tube . But in a viscous flow that in above obtained transfer equation, changes of circulation numbers in duration of time will be as follows:

$$\frac{d\Gamma}{dt} = \nu \int_A \nabla^2 w.ndA$$

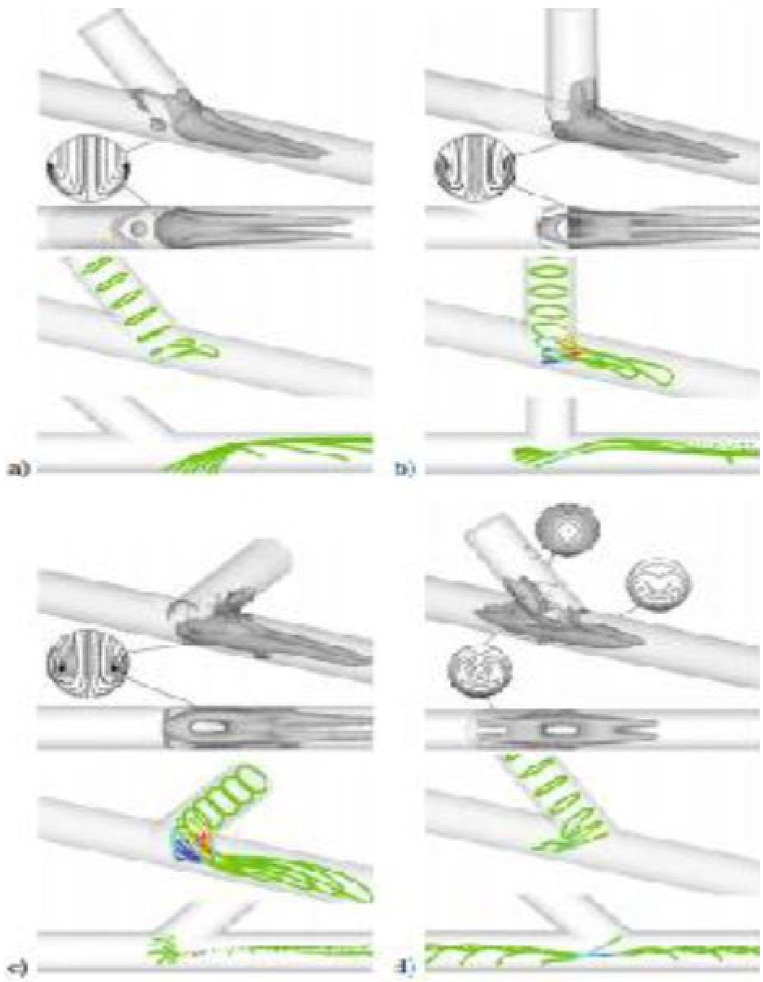
vortical and turning velocity components often occur in arteries and may be formed by rapid change flow and produced curves and branches in duration of the vessel. To understand the movements of vortical Jeong and Hussain [15] suggested that a vortex corresponds to an area of connective where the second component of eigenvalues is negative. This condition of the subject's adherence that to a minimum Local press, two eigenvalues of the Hussain matrix is positive. With this condition that in formation of Hussain pressure from gradient of motion equations, overlooked either Unstable pressure conditions (that can Produced minimize local pressure) and adhesion.
D. Doorly and Spencer Sherwin [16] showed that this can be related to the local kinematics flow and showed that the value of directly related to strength of torsion. So, although is not a complete index of visualization and the torsional strength, but offers a very useful measure of the structure and strength of the vorticity tube.

4 Anastomoses and branches

Vascular Anastomosis is meant to create a link between blood vessels which may be is between the artery with an artery, vein by vein or artery by Vein that based on the three types, normal, abnormal and pathological and caused by the treatment. As that shown in the Figure 3, a basic figure consists of three vessels that all of them from the diagonal D. The passing minor vessel have the 3D lengths and hosts vein graft displayed by two section with lengths 5D and 10D. According to the contract an area where the graft intersection with host in the end of distal as and proximal intersection as and lower part of host that is a graft intersection as a known. Distal and Proximal respectively defined as the maximum and minimum distance with graft intersection. Also the hosts proximal and distal interpret the flow direction in the condition in a link closed. The area closed located in the host proximal.



Discussed flow under the premise of stable flow in the vessels with Reynolds number$R_e = 125$ regulation based on the mean velocity and vessel diameter. This Reynolds number is compatible with the coronary arteries conditions and environmentally vessels. In figure 4 by using the scale and ring of transfer particles, which are colored, we have shown a coherent structure for Reynolds number 125. In this figure in the part a to c seen three modes which not exist the flows close to the source and graft angle changes between 45 to 90 and 135 degrees. In all states of figure we see that produces two Dean vortexes and seen that have the shape like the vortex in the single branch; though the angle of the bend have significant effect on the vortex strength. Underneath of figure indicates how the transfer particles after effect on the substrate of graft. According to the transfer ring before graft can be seen when ring arrived to the substrate of graft, in state that graft angle is greater, have more deviation than the attendance in the host distal, which can be directly related to stretch in axial direction from the bottom vessel and the establishment of Dean vortexes . In the blocked area with Anastomosis 45 degrees appears a time vortex structure that vorticity is strong sufficient. When we increase the angle of the branch, the effect of particles for produce a vortex structure is strong sufficient and not only in the blocked area, but on both sides of the graft leads to the ring structures. This process is similar to the effect vortex loop on a solid surface curve.



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5 Results

Reynolds number is a leading global parameter that in determining Stability of flow is very effective. In the graft the Dean number is used dominant parameter to describe the flow. With According to a conducted survey, the graft curvature in graft coronary is most susceptible area for incidence vortexes and with increasing graft angle increased vortexes strength. The continued rise in power Whirlpool, which leads to incidence Atherosclerosis disease and therefore causes ineffective the graft, is under review and results will be measured. So the best angle for graft in this study considered the lower angle i.e. 45 and consequently, selection a graft angle of 45 it makes reduce possibility failure of arterial graft surgery. Due to that today bypass graft with angle between 20 to 40 degree doing, relative proximity of proposed model to a real graft model viewable.

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