FUZZY LINEAR PROGRAMMING FOR DETERMINING OPTIMUM ORDER QUANTITY:
USING FUZZY ELECTRE METHOD

Rashed Sahraeian¹*, Ali Rajabali Kani²
Industrial Engineering of Shahed University-Tehran-Iran, sahraeian@shahed.ac.ir
Industrial Engineering of Shahed University-Tehran-Iran, Rajabalikani@shahed.ac.ir

Abstract
The supplier selection problem is an important case in the supply chain management. Different tools are used for the solution of which. Why it is possible to create competitive privileges for the organization through selecting a suitable set of suppliers. This research aims to study the supplier selection problem in fuzzy status in multiple sourcing conditions. We apply linguistic variables in trapezoidal fuzzy number. For arranging the best suppliers a decision making algorithm called Fuzzy ELECTRE is suggested. Eleven steps are expressed to assess suppliers distinction in uncertainty conditions, and finally a fuzzy linear programming is presented to determining the optimum order quantity of each suppliers. The results show that by proposed method we can calculate optimum order quantity in uncertainty conditions.

Keywords: Supplier Selection, Quantity, Fuzzy ELECTRE, Linear Programming.

1. Introduction
This For success of company, selecting the suitable set of supplier is more important and vital. Because in most industries, costs of primary substances and standard parts contain the main part of cost products that selecting suitable suppliers can decreased the price of purchasing considerably and increase the ability of competition. The approach of selecting suitable suppliers in supply chain management is a way which it can use for increase the competition advantage of supply chain. The supplier selection problem is expressed as a multiple criteria decision making problem, because of variety of the necessary criteria. In this problem various constraints as budget, capacity, quality, and etc are exist. According to the criteria and constraints, it should select the best supplier and determine the optimum order quantity to be able do best purchase with the least cost of possible. The complexity of this problem is increased by meeting uncertainty that fuzzy set theory can overcome it. The literature review shows that studying this subject in multiple sourcing conditions is so important, so in this research as Guneri et al.(2009) model[8], it is presented a fuzzy programming model difference with, first, ELECTRE method is presented instead of TOPSIS method in which distinguish the arrangement of alternatives and superior alternative. In addition to calculating effective matrix, it is computed distinction of each alternatives, too which by through this they can be compared. Second, objective function of linear programming model isn’t single and in addition to be multi-objective, it is presented in vagueness conditions. Because calculated scores for each of alternatives through the suggested algorithm are fuzzy numbers. Ultimately, it is calculated the optimum order quantity of each suppliers through this model which included budget, capacity of supplier, and least of acceptable quality constraints. The paper is organized as follows: first literature review is represented then in section 3, proposed method is presented which will consider multiple sourcing and uncertainty conditions. In section 4, the proposed models are validated by a numerical example, and in last section, conclusion and the future research are discussed.
2. Literature review
Chen et al. (2006) presented a fuzzy decision making method about the supplier selection problem in supply chain system [7]. In that research linguistic variables were used for determining the rate and weight of factors and then closeness coefficients of alternatives based on distant form fuzzy positive and fuzzy negative ideal solutions are calculated. Wang et al. (2009) introduced a fuzzy model in discount situation for selecting suppliers [18]. Lee et al. (2009) proposed a model for selecting suppliers in fuzzy situation which benefit, price, risk, and opportunities were considered in it. In that paper Fuzzy Analytical Hierarchy Process (FAHP) was used [12]. Berilacqua et al. (2006) suggested new method which is used HOQ approach from QFD for supplier selection process and they used it in production industries for efficiency test [5]. Gray-based decision making was used for supplier selection problem by Li et al. (2007) [14]. Boran et al. (2009) presented a fuzzy multiple criteria fuzzy model with TOPSIS method for supplier selection problem. In that research TOPSIS method with suggested intuitionistic fuzzy set has combined for suitable selecting supplier by group decision making [6]. Hiao and Rittscher, (2007) introduced a multi-objective model on stochastic demand conditions for supplier selection problem. In that paper flexibility for demand quantity and its uncertainty was extended on stochastic conditions [9]. Lee and Yang, (2009) used neural network approach for forecasting the suggested prices in supplier selection process [13]. A weighted additive fuzzy multi-objective model for supplier selection problem under drop price break conditions in a supply chain was presented by Ghodsypour et al. (2007) [1]. Wan Lung Ng, (2008) presented a linear programming for multi-objective supplier selection problem [16]. Amin & Razimi, (2009) suggested an integrated fuzzy model for supplier management in an internet services (ISP) by use of fuzzy set theory and QFD [2]. Sanayei et al. (2008) proposed an integrated group decision making for supplier selection and order allocation by use of linear programming and multi-attribute utility theory [17]. Comparing between proposed model in this paper and other models are illustrated in Table 1.

Table 1. Comparing the introduced models

<table>
<thead>
<tr>
<th>References</th>
<th>Decision making techniques</th>
<th>Mathematical programming</th>
<th>Objective</th>
<th>Sourcing</th>
</tr>
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<tbody>
<tr>
<td>Karpak et al. (2001)[11]</td>
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<td>Hong et al. (2005)[10]</td>
<td>-</td>
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<tr>
<td>Chen et al. (2006)[7]</td>
<td>Fuzzy Topsis</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
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<td>Sanayei et al. (2006)[17]</td>
<td>Multi-attribute Utility Theory (MAUT)</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Ng (2008)[16]</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Lee (2009)[12]</td>
<td>Fuzzy Analytic Hierarchy Process (FAHP)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Amin &amp; Razimi (2009)[2]</td>
<td>Quality Function Development</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Guneri et al. (2009)[8]</td>
<td>Fuzzy Topsis</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>This Paper</td>
<td>Fuzzy ELECTRE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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</tbody>
</table>

3. New proposed method
In ELECTRE method, in addition to use of concordance concept of alternatives, concept of discordance is used too. Namely in this method all the alternatives are evaluating by use of comparison of discordance and useless alternatives are omitted [3]. In this research the algorithm is presented on the basis of fuzzy data to be able to calculate the coefficient superiorities in last step in addition to determining the arrangement of alternatives and superior alternative. In this method fuzzy numbers are defined as the trapezoidal fuzzy numbers. Therefore in this model as Chen et al. (2006) method [7] linguistic variables shown in Figs. 1 and 2 are used to determine the weight of criteria and alternative rating. In the next section, suggested algorithm is presented which linguistic variables are used for overcoming on uncertainty.
In this paper, new Fuzzy ELECTRE algorithm is as following:

First to third steps of this algorithm are as Chen et al. (2006) steps [7] which have presented in their research. Forth to eleventh steps are the same as ELECTRE steps which Asgharpour, (2007) [3] presented in his book, but here it is used fuzzy set theory. The twelfth step is the last proposed step that fuzzy distinction of each alternatives are determined, and by using the arranging of fuzzy numbers method can compare the fuzzy distinctions and arrange them.

Step 1- Constructing Fuzzy decision matrix.
Step 2- Weighted normalized Fuzzy decision matrix is constructed as applied method in Chen’s paper[7].
Step 3- Constructing the concordance set: It is used arranging fuzzy number method for comparing each pair alternatives. Three criteria is expressed which must be applied respectively in this method.

First criteria (Walled surface):
Azar and Faraji, (2007) defined the walled surface between crisp number \( K=0 \) and fuzzy number \( \tilde{M} \) as follows [4], and the calculating manner of the walled surface is shown in Figure. 3.

\[
S_k(\tilde{M},0) = \frac{a_3 + a_4}{2}
\]

\[
S_L(\tilde{M},0) = \frac{a_1 + a_2}{2}
\]

\[
S(\tilde{M},0) = \frac{1}{4} [S_L(\tilde{M},K) + S_k(\tilde{M},K)] = \frac{a_1 + a_2 + a_3 + a_4}{4}
\]

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**Figure. 1. Linguistic variables for weight of criteria.**

**Figure. 2. Linguistic variables for rating of alternatives.**

**Figure. 3. Calculating \( S(\tilde{M},K) \) for a trapezoidal fuzzy number.**
Second criteria (Mod):
After arranging the fuzzy numbers with first criteria then which are in a group, can be arranged by mod criteria [4].

Third criteria (Scope):
If by using two previous criteria, there are numbers in a group, the scope of them must be calculated which the number has the greatest scope is the greatest [4].

Concordance set (set is all criteria in which alternative K is superior than alternative L) can be constructed by use of above criteria.

\[ C_{KL} = \{ j, S(\bar{\nu}_j,0) \geq S(\tilde{\nu}_j,0) \} \]  \hspace{1cm} (4)

**Step 4- Constructing the discordance set:** This set is set of criteria in which alternative K has no superiority on alternative L.

\[ D_{KL} = \{ j, S(\bar{\nu}_j,0) < S(\tilde{\nu}_j,0) \} \]  \hspace{1cm} (5)

**Step 5- Constructing the concordance matrix:** Elements of this matrix are set of criteria weights which alternative K is superiority on alternative L [3].

\[ C_{KL} = \sum_{j \in C_{KL}} w_j \]  \hspace{1cm} (6)

**Step 6- Constructing the discordance matrix:** Calculating the distance between two trapezoidal fuzzy numbers approach is sued for constructing this matrix. Figure 4 shows how the distance between two trapezoidal fuzzy numbers is calculated.

![Figure 4: Calculating manner between two trapezoidal fuzzy.](image)

\[ S(\tilde{\nu}_i, \tilde{\nu}_j) = \frac{1}{2} \left[ S_L(\tilde{\nu}_i, \tilde{\nu}_j) + S_R(\tilde{\nu}_i, \tilde{\nu}_j) \right] \]  \hspace{1cm} (7)

\[ S_L(\tilde{\nu}_i, \tilde{\nu}_j) = S_L(\bar{\nu}_j,0) - S_i(\bar{\nu}_j,0) \]  \hspace{1cm} (8)

\[ S_R(\tilde{\nu}_i, \tilde{\nu}_j) = S_R(\bar{\nu}_j,0) - S_i(\bar{\nu}_j,0) \]  \hspace{1cm} (9)

So the distance between two trapezoidal fuzzy numbers \( \tilde{\nu}_i \) and \( \tilde{\nu}_j \) is expressed as:

\[ S(\tilde{\nu}_i, \tilde{\nu}_j) = \frac{1}{2} \left[ \left( a_i + a_i - b_j b_j - \frac{2}{2} \right) + \left( a_i + a_i + a_i - b_j b_j - \frac{2}{2} \right) \right] = \frac{(a_i + a_i + a_i + a_i) - (b_j b_j + b_j + b_j + b_j)}{4} \]  \hspace{1cm} (10)

Therefore the elements of discordance matrix are computed as:

\[ d_{KL} = \frac{\max_{j \in D_{KL}} \left| S(\tilde{\nu}_j, \tilde{\nu}_j) \right|}{\sum_{j \in D_{KL}} \left| S(\tilde{\nu}_j, \tilde{\nu}_j) \right|} \]  \hspace{1cm} (11)

**Step 7- Calculating mean of all elements of concordance and discordance matrixes:**

\[ \bar{C} = \frac{\sum_{k=1}^{m} \sum_{l=1}^{m} C_{KL}}{m(m-1)} \]  \hspace{1cm} (12)

\[ \bar{N}C = \frac{\sum_{k=1}^{m} \sum_{l=1}^{m} NC_{KL}}{m(m-1)} \]  \hspace{1cm} (13)

**Step 8- Constructing effective concordance matrix(F):** The elements of this matrix are 1 and zero.

\[ f_{KL} = 1, \text{ if } \tilde{C}_{KL} \geq \bar{C} \]  \hspace{1cm} (14)

\[ f_{KL} = 0, \text{ if } \tilde{C}_{KL} < \bar{C} \]  \hspace{1cm} (15)
Step 9 - Constructing effective discordance matrix (G) as:

\[ g_{KL} = 1 \quad \text{if} \quad \tilde{N}C_{KL} \leq \tilde{N}C \]  

\[ g_{KL} = 0 \quad \text{if} \quad \tilde{N}C_{KL} > \tilde{N}C \]  

Step 10 - Constructing the final effective matrix (H): The elements of this matrix is obtained by multiplying each elements of matrix F to the same elements of matrix G.

\[ h_{kl} = f_{kl} \cdot g_{kl} \]  

Step 11 - Calculating superiority coefficients of each alternatives: At first it must be multiplied each elements of matrix H to the same elements of matrix C, then coefficients are computed by summing the elements of each rows of obtained matrix. Because concordance matrix is constructed by fuzzy numbers, so computed coefficients are fuzzy numbers, therefore they must be compared by arranging fuzzy numbers method.

In phase 2, a fuzzy programming model are proposed which the objective function coefficients are superiority coefficients of suppliers. In next section this method is validated by a numerical example.

4. Validation

In this research, it is used the numerical example which was applied by Guneri et al. (2009). Because of determining the accuracy of proposed Fuzzy ELECTRE algorithm and comparing the quality of obtained solution with the results of Guneri et al. (2009) model is aim if this validation.

Example: A textile company will select the suitable suppliers to purchase yarn for producing new product. A group decision makers includes three decision makers to select the most suitable suppliers from 4 candidate company. Five criteria of supplier selection are as:

- Relationship closeness (C1),
- Reputation and position in industry (C2),
- Performance history (C3),
- Conflict resolution (C4),
- Delivery capacity (C5) [8].

First phase:

Step 1 - The decision matrix is constructed as follows: Three decision makers determine the importance weight of each criteria by use of linguistic variables shown in Figure. 1. The results of this weights are shown in Table 2 [8]. Then Three decision makers ascertain the rating of alternatives by use of Figure. 2 that the results of rating are shown in Table 3 [8]. Table 2 and 3 are converted to fuzzy number and decision matrix is constructed as Table 4. The weight of each criteria is shown in it.

| Table 2. Weight of criteria from group decision makers. |
|-------------|-------------|-------------|-------------|
|             | D1          | D2          | D3          |
| C1          | H           | H           | H           |
| C2          | H           | H           | VH          |
| C3          | H           | H           | H           |
| C4          | VH          | VH          | H           |
| C5          | VH          | VH          | VH          |
Step 2 - Weighted normalized fuzzy decision matrix is constructed as in Table 5.

Table 5. Normalized fuzzy decision matrix.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Suppliers</th>
<th>Decision makers</th>
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<tbody>
<tr>
<td>C1</td>
<td>A1</td>
<td>VG</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>MG</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>MG</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VG</td>
</tr>
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</table>

Table 4. Fuzzy decision matrix

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Suppliers</th>
<th>Decision makers</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(7.8,9,3,10)</td>
<td>(6.7,3,3,9)</td>
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<tr>
<td></td>
<td>(6.7,3,10)</td>
<td>(7.8,3,9,10)</td>
</tr>
<tr>
<td></td>
<td>(7.8,3,9,10)</td>
<td>(6.7,3,7,3,9)</td>
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<td>(7.8,3,9,10)</td>
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<td></td>
<td>(7.8,3,9,10)</td>
<td>(6.7,3,7,3,9)</td>
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Step 3, 4 - Concordance and discordance set is construct as mentioned methods.

Step 5 - Concordance matrix(C) is constructed as in Table 6.

Table 6. Concordance matrix(C).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Suppliers</th>
<th>Decision makers</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-</td>
<td>(2.8,3,3,4,3,8)</td>
</tr>
<tr>
<td></td>
<td>(8.9,1,1)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(2.2,3,5,2,6,7,2,9)</td>
<td>(2.9,3,4,3,6,3,9)</td>
</tr>
<tr>
<td></td>
<td>(1.5,7,1,3,8,7,2)</td>
<td>(2.1,2,5,2,6,2,9)</td>
</tr>
</tbody>
</table>

that $c_{12} = w_1 + w_2 + w_3 + w_4 = (2.8,3,3,4,3,8)$

Step 6- Discordance matrix is constructed as:

$D = \begin{bmatrix}
-0.38 & 1 & 1 \\
1 & -1 & 1 \\
0.86 & 0 & -0.37 \\
0.76 & 0.21 & 1 \\
\end{bmatrix}

that $d_{12} = \frac{\max(\{-0.05\})}{\max(0.01,0.04,0.043,0.14,-0.05)} = 0.05 = 0.38$
Step 7: The mean of elements of concordance and discordance matrixes is calculated as:
$$C\overline{C} = (1.8, 2.1, 2.2, 4), \quad \overline{C} = 0.63$$

Step 8, 9, 10 - Constructing matrix F, G and H are constructed as:

$$F = \begin{bmatrix} - & 1 & 0 & 1 \\ 0 & - & 0 & 0 \\ 1 & 1 & 1 & - \\ 0 & 1 & 0 & - \end{bmatrix}$$
$$G = \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 1 & - & 1 \\ 0 & 1 & 0 & - \end{bmatrix}$$
$$H = \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 1 & - & 1 \\ 0 & 1 & 0 & - \end{bmatrix}$$

Step 11 - Superiority coefficients of alternatives are computed as follows:

$$H.C = \begin{bmatrix} - & 2.8, 3.3, 4.3, 8 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 2.9, 3.4, 3.6, 3.9 & 0 & 2.9, 3.4, 3.6, 3.9 \\ 0 & 2.1, 2.5, 2.6, 2.9 & 0 & 0 \end{bmatrix}$$

Second phase:

According Zhang et al. (2003) model [21], fuzzy programming model is expressed as:

$$\text{maximize } \sum_{i=1}^{4} C_i x_i$$

The membership function of these coefficients are defined as:

$$\mu_i(x) = \begin{cases} 0 & x < 2.8 \\ \frac{x - 2.8}{3.3 - 2.8} & 2.8 \leq x < 3.3 \\ 1 & 3.3 \leq x < 3.4 \\ \frac{x - 3.8}{3.4 - 3.8} & 3.4 \leq x < 3.8 \\ 0 & x > 3.8 \end{cases}$$

Then multi-objective model is constructed and constraints are added (Constraints and parameters are the same as Guneri et al.2009 model). $x_i$ is order quantity for $i$th supplier. Total demand in this model is equal 5000. $q_1=6$, $q_2=3$ $q_3=1$, and $q_4=8$ are defect quality rate of each suppliers and company’s maximum acceptable defect quality rate is equal 4. $p_1=8.9$, $p_2=9.3$ $p_3=9.5$, and $p_4=9.4$ are unit price
of each suppliers and company’s maximum acceptable unit price for purchasing this orders is equal 9.2. \(c_1=2500, c_2=3500, c_3=2000,\) and \(c_4=3000\) are capacity of each suppliers.

Maximize \(= \sum w_i \{2.8x_1 + 5.8x_3 + 2.1x_4, 3.3x_1 + 6.8x_3 + 2.5x_4 \}
\[
\begin{align*}
&\text{w}_3\{3.4x_1 + 7.2x_3 + 2.6x_4, \text{w}_4\{3.8x_1 + 7.8x_3 + 2.9x_4 \}} \\
\end{align*}
\]

Subjected to:

\(x_1 + x_2 + x_3 + x_4 = 5000\)
\(6x_1 + 3x_2 + x_3 + 8x_4 \leq 20000\)
\(8.9x_1 + 9.3x_2 + 9.5x_3 + 9.4x_4 \leq 46000\)
\(x_1 \leq 2500\)
\(x_2 \leq 3500\)
\(x_3 \leq 2000\)
\(x_4 \leq 3000\)
\(x_i \geq 0, \quad i = 1, 2, 3, 4\)

Phrase 21 is demand constraint, phrase 22 is quality constraint, and phrase 23 is budget constraint. Phrase 24,25,26, and 27 guarantee that the order quantity of ith supplier will not be more that their capacities.

In this example it is assumed that \(w_i=1\), so this problem is solved by LINGO solver. The optimum order quantity of each suppliers are computed as:

\(OFV=91480, \quad X_1=2500, \quad X_2=200, \quad X_3=2000, \quad X_4=300\)

It is possible to obtain desired optimum solution by changing the importance weights of objective functions. Comparing the results of each model (Guner et al.2009 and proposed model) shows that the quality of answers are the same. So it is concluded that first, proposed Fuzzy ELECTRE method is valid for arranging alternatives and selecting the best supplier, and second, although there are different method in multiple criteria decision making , but ultimately there is no difference in correct and suitable use of them can make effective improvement in solving the supplier selection problem. The advantage of proposed model is that the parameters of objective function of this model are fuzzy numbers, and it is able to use this model for determining the optimum order quantity of each suppliers in uncertainty conditions which alternatives superiority are vague.

5. Conclusion

As we know, selecting the suitable set of suppliers in a supply chain can decrease the purchase costs considerably. So, in this research the supplier selection problem studied in uncertainty and vagueness conditions. In this research Fuzzy ELECTRE logarithm was proposed to Obtain the best selection, and the supplier selection problem studied in multiple sourcing situation and a fuzzy programming model was presented to achieve the optimum order quantity of each suppliers which their superiorities of supplier are vague. As a future research, other parameters of this model can be considered as fuzzy numbers.

References