The Effect of Subcarrier-Based Channel Estimation on the BER Bound of Turbo Coded OFDM/SDMA System

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***Abstract*—In this paper, we study the performance of the channel estimation in a Turbo coded, orthogonal frequency division multiplexing (OFDM) with multiple antennas at the receiver. We survey the accuracy of the channel estimation for different number of users and different number of pilot subcarriers. Moreover, since there is not a closed form solution to the BER of the Turbo coded system for different number of constellation size. We develop an equation to obtain the BER bound of the Turbo coded OFDM/SDMA system with M-ary quadrature amplitude modulation (MQAM) in the Rayleigh flat fading channel as a function of the signal constellation and characteristic of the Turbo code with a curve fitting and then propose a method for selecting switching threshold in adaptive modulation case.**

***Index Terms*—channel estimation, OFDM/SDMA systems, Turbo coding, BER bound, adaptive modulation.**

I. INTRODUCTION

OFDM has been adopted for the physical layer of many popular communication systems because of the high spectral efficiency and robustness against fading environment. We can improve the performance of this system by using the Turbo code [1] in combination with multiple antennas at the receiver. Using multiple antennas extremely increase the capacity of the wireless channel. In [2], the downlink OFDM/SDMA systems have been considered, the authors in [3] present an adaptive resource management algorithm for multiuser transmission with OFDM signaling in uplink mode, when both transmitter and receivers exploit multiple antennas. Since the usage of multiple antennas for user nodes are infeasible in most of applications; we consider a single antenna for each user. Receiver exploits multiple antennas. In the other hand, advent of Turbo codes greatly enhances the research in the area of wireless communication. Therefore, in this article, we use an OFDM/SDMA system in combination with Turbo coding. By considering the fact that, the performance of different SDMA techniques [4] and Turbo coding [5] depend on channel estimation. We study the performance of a Turbo coded OFDM/SDMA system with channel estimation. We can suppose that channel is pseudo stationary; therefore, pilot symbol assisted estimation is sufficient. In [6] authors use of the time division multiplexing (TDM)/FDM pilots for the channel estimation in the fast fading channel and in [7] the two dimensional channel estimation for OFDM system is considered. Authors in [8-9] study the channel estimation in the multiuser case. In [8] the OFDM/SDMA system at the downlink with ZF beamforming is considered. It is found that the equidistant placement of pilot symbols minimizes the mean squared error (MSE) of the channel estimation [10] and maximizes the capacity [11].

Pilot design should compromise between the accuracy and consuming of the resources; therefore, we should scatter the pilot symbols in the frequency, time, and spatial domain. Thus, interpolation in frequency and time domain and spatial is inevitable. It is necessary to obtain the optimum number of pilot symbols for channel estimation.

The BER is a key parameter in analyzing the performance of the coded systems. We need the BER for obtaining switching thresholds, in adaptive modulation case. Therefore our criteria for the performance of pilot assisted estimation, is the BER of the system. Indeed we want to know that by what amount of pilot subcarrier the BER had small change. We study this value for different number of user and different iteration at the Turbo decoder. Since the BER of the Turbo coded system is key parameter in scrutinizing their performance we want to obtain the formula for this parameter. On the other hand, an LDPC error correcting code has received considerable attention in multicarrier OFDM communication systems. An upper bound to the performance of the Turbo code has been obtained in [12]. But, there is not an equation for the BER of Turbo coding schemes for OFDM/SDMA system in a fading channel. Lately, the authors in [13] obtained an approximated BER for the LDPC coded system with MQAM modulation which is a function of some constant parameters that can be found through curve fitting, and to the constellation size and characteristic of the Turbo code (number of iteration).

With due attention to the fact that the Turbo and LDPC codes have almost the same performance, we can use the formula obtained for LDPC coded system in the OFDM/SDMA system with Turbo coding.

978-0-7695-3960-7/10 $26.00 © 2010 IEEE
DOI 10.1109/ICSAP.2010.82

In second section, we introduce the system model; also, equations for the minimum mean square error (MMSE) and MMSE-successive interference cancellation (SIC) in an OFDM/SDMA system are addressed. The method for channel estimation is introduced in section three. In section four, we develop a formula for the BER bound of the Turbo coded OFDM system. The simulation results are presented in section five; finally, we conclude the paper in section six.

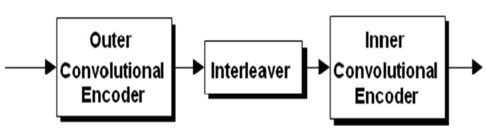
Notations: boldface capital letters refer to the matrix and boldface letters represent vectors. The operator $(\cdot)^H$ denotes Hermitian. Finally, square brackets indicate frequency-domain variables.

II. System Model and Detection Algorithm

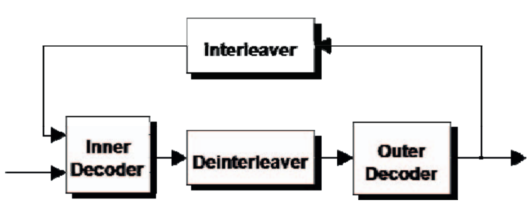
A. System Model

In Fig.1, the OFDM/SDMA system with post-FFT processing and Turbo coding is portrayed. We employ a serially concatenated convolutional code as a Turbo coding scheme as shown in Fig. 2. The signal of each user after coding and modulation inserted to *N*=64 subcarrier of OFDM and then is transmitted by the specific antenna of each user. *A* antennas were used for the BS. It is assumed that the cyclic prefix (CP) is long enough to suppress the inter-symbol interference (ISI), Therefore the frequency domain received signal is equal to:

$$\underbrace{\begin{bmatrix} y_1[n] \\ \vdots \\ y_A[n] \end{bmatrix}}_{\mathbf{y}[n]} = \underbrace{\begin{bmatrix} \overbrace{h_1^1[n]}^{H^1[n]} & \cdots & \overbrace{h_1^U[n]}^{H^U[n]} \\ \vdots & \ddots & \vdots \\ h_A^1[n] & \cdots & h_A^U[n] \end{bmatrix}}_{\mathbf{H}[n]} . \underbrace{\begin{bmatrix} x^1[n] \\ \vdots \\ x^U[n] \end{bmatrix}}_{\mathbf{x}[n]} + \underbrace{\begin{bmatrix} n_1[n] \\ \vdots \\ n_A[n] \end{bmatrix}}_{\mathbf{n}[n]} \quad (1)$$



2-a) Encoder



2-b) Decoder

Fig. 2. Block diagram of serially concatenated Turbo encoder and decoder.

In this equation **y**[n] is the (*A*-dimensional) received signal and **x**[n] (*U*-dimensional) transmitted signal. **n**[n] is the (*A*-*dimensional*) additive noise at the receiver and it is assumed that **n**[n]~ CN (0, I), **H**[n] ($A \times U$ - dimensional) is the frequency-domain channel matrix at the n^{th} tone, and $\mathbf{H_k}$[n] denote the k^{th} column of **H**[n].
The impulse response of the time varying radio channel can be shown by the discrete time FIR filter.

$$h(\tau;t) = \sum_l \alpha_l(t) e^{-j2\pi f_c \tau_l(t)} \delta(\tau - \tau_l(t)) \quad (2)$$

Since for the wireless LAN applications, the channel is pseudo stationary and does not change during transmission of one packet the equation 2 does not depend on time and is equal to:

$$h(t) = \sum_l \alpha_l e^{-j2\pi f_c \tau_l} \delta(\tau - \tau_l) \quad (3)$$

Therefore the channel frequency response is equal to:

$$\mathbf{H} = \underset{N-\text{point}}{DFT}\{h(t)\} = [h[1],...,h[\text{N}]] \quad (4)$$

In this equation $h[\text{n}]$ is the frequency response of the channel for the n^{th} symbol. Without loss of generality, this notation is like $h_a^u[n]$ in equation (1) that we obtain channel response for the u^{th} user and the a^{th} Antenna.

B. Multiuser Detection Algorithm

In the multiuser detection block, we have to obtain soft estimate of the transmitted symbols $x^1[\text{n}], x^2[\text{n}],..., x^U[\text{n}]$, which are denoted by $\overline{x}^1[\text{n}], \overline{x}^2[\text{n}],..., \overline{x}^U[\text{n}]$, from the received signals **y**[n] for all subcarriers *n*. these estimations are obtained by multiplying the (U × A) –dimensional Antenna weight matrix F[n] by received signal **y**[n]. Then, at the demodulator we have to obtain a soft estimation of transmitted bits. This soft information is then decoded by a Turbo decoder. In this section, we briefly review of MMSE, MMSE-SIC algorithms and their antenna weight matrixes **F**[n], which is utilizable for application in multiple antennas reception assisted by OFDM/SDMA-based communication systems.

- *MMSE* Algorithm

In the MMSE linear algorithm, the (U × A) –dimensional antenna weight matrix F[n] is given [14]:

$$F[n] = \left(\overline{H}^H[n].\overline{H}[n] + \sigma_n^2 I\right)^{-1}.\overline{H}^H[n] \quad (5)$$

where $\overline{\mathbf{H}}[n]$ is the estimate of channel coefficients matrix and σ_n^2 is the AWGN noise variance. Thus, estimation of transmitted signal is equal to $\overline{\text{x}}[\text{n}] = \mathbf{F}[\text{n}].\mathbf{y}[\text{n}]$.

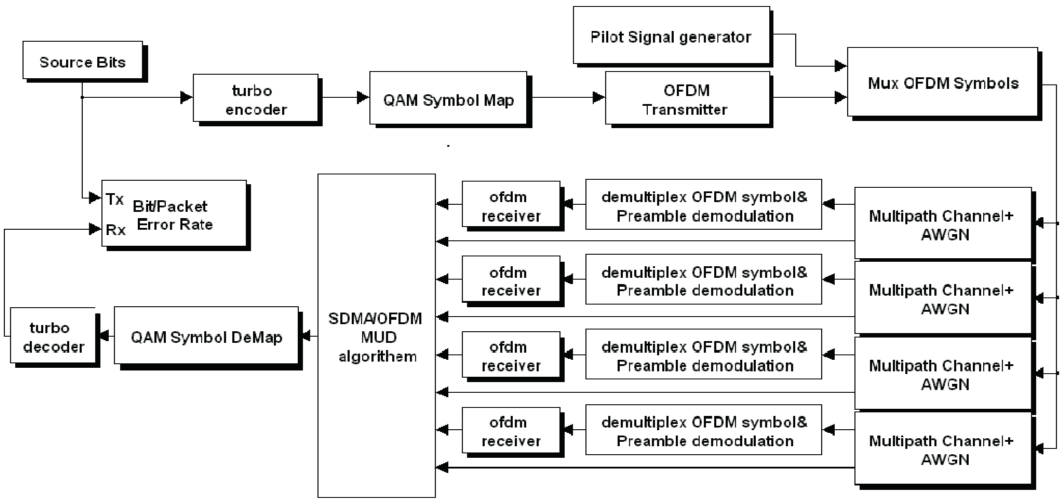


Fig.1 Block diagram of the OFDM/SDMA system with a post-FFT processing and Turbo coding.

- *SIC* Algorithm

The procedure of the detection in this algorithm is as follow [15]. In each iteration, we detect the signal of one user with highest SNR or SINR that is outputted from MMSE detector, which stated in the previous section. The iteration is done *U*-times sequentially, in order to detect the signal of each user. The following equation explains this algorithm explicitly.

$$\begin{bmatrix} y_1^u[n] \\ \vdots \\ y_A^u[n] \end{bmatrix} = \begin{bmatrix} y_1^{u-1}[n] \\ \vdots \\ y_A^{u-1}[n] \end{bmatrix} - \begin{bmatrix} \bar{h}_1^{U-1}[n] \\ \vdots \\ \bar{h}_A^{U-1}[n] \end{bmatrix} .\hat{x}^{U-1}[n] \qquad (6)$$

Now, new soft estimation is:

$$\bar{\mathbf{x}}^u[n] = \underbrace{[f_1^u[n], f_2^u[n]..f_A^u[n]]}_{\mathbf{F}^u[n]} . [y_1^u[n], y_2^u[n]..y_A^u[n]]^T \qquad (7)$$

$\bar{\mathbf{x}}^u[n]$, that is obtained from the previous equation for each user and each subcarrier will input to the demodulator of the u^{th} user. The soft demodulated bit goes to the Turbo decoder block. These information are considered as a priori information for the inner decoder. This decoder and the outer decoder exchange the extrinsic information back and forth for the number of iteration. The output of the outer decoder gives us the value of BER for different number of iteration. Block diagram of Turbo encoder and the Turbo decoder is shown in Fig. 2.

III. Channel Estimation Method

The full channel estimation is studied in Section III.*A* and interpolated channel estimation is studied in Section III.B.

A. Full Channel Estimation

In the system with the data transmission application most of the bandwidth dedicated to the data. By considering the above mentioned matter and this fact that the channel is constant during one packet transmission, we must obtain a fast and simple method for the channel estimation in *k* subcarriers. Here, we use two training sequences C_1, C_2 for the channel estimation. For each training sequences we sent a sequences of symbol $x_k \in \{1,-1\}$ in *k* subcarriers. After receiving $Y_{1,k}, Y_{2,k}$ we obtain the value of $\hat{H}_k$ by the following equation:

$$\hat{H}_k = \frac{1}{2}\left(\frac{Y_{1,k} + Y_{2,k}}{X_k}\right) = H_k + \frac{1}{2}\left(\frac{N_{1,k} + N_{2,k}}{X_k}\right) \qquad (8)$$

$\hat{H}_k$ is the value of channel estimation when we have pilot on all subcarriers and $Y_{n,k} = H_k . X_{n,k} + N_{n,k}$. This method is shown in fig. 3.a.

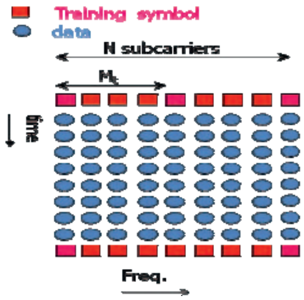
B. Interpolated Channel Estimation

In this case we did not send pilot sequences for all subcarriers but we send pilots for some subcarrier and for other subcarriers we obtain the channel estimation by linear interpolation. In the linear interpolation method the channel estimation at the subcarrier between two pilots, $\hat{H}_{LS,K}$ and $\hat{H}_{LS,K+1}$ is given by:

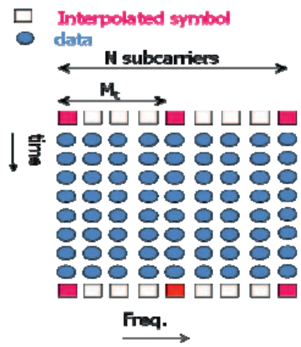
$$\hat{H}_{KM_t+t} = \hat{H}_{LS,K} + (\hat{H}_{LS,K+1} - \hat{H}_{LS,K})(t / M_t) \qquad 0 \le t \le M_t \quad (9)$$

Where, M_t is a distance between the pilot symbols and $\hat{H}_{LS,n}$ is the n^{th} subcarrier that has a pilot signal. Altogether $M = N/M_t$ subcarriers have a pilot signal which N is total number of subcarrier.

The fig. 3.b shows this method. In this article, we send the pilot for second and fourth subcarrier which M_t is equaled to two and four, respectively. We compare the result with FULL channel estimation; we want to know that how much this method effect to the performance of the system. We address these results in section V.



3.a full channel estimation



3.b channel estimation with interpolation

Fig. 3 Channel estimation with pilot signals

IV. BER Bound

As it mentioned before there is not a closed form expression for the BER of Turbo coded OFDM/SDMA system. Therefore, we obtain the relation between the BER and the SNR by simulation. The serially concatenated convolutional Turbo coding with rate 0.5 is used. We want to exploit an equation from this simulation with different constellation size in order to obtain switching threshold for adaptive modulation with constant BER. For obtaining the BER, the equation proposed in [13] is applied. This equation was approximated for LDPC coding with M-QAM modulation and we exploit that for Turbo coding scheme.

$$p = \exp(-\frac{(a\gamma_r - b)^2}{(M-1)^2} - 0.5) \tag{10}$$

This equation is a function of some constant parameter (*a,* and *b*) which can be found through curve fitting, and a constellation size (M). Simulation results have been considered in the next section. For obtaining the formula to the BER-bound, we assume a constant code rate and select an iteration that without reduction in performance has a less complexity. The channel estimation method is also selected in this way. We then find *a* and *b* coefficients for these cases.

V. Simulation Results

To examine the performance of the proposed Turbo coded OFDM/SDMA system, we employ an OFDM simulation model [16] at the TGn-B fading channel. It consists of one or two user terminals each equipped with a single antenna and a BS equipped with four-element antenna array to exploit spatial diversity. For each user, the random source data bits first are encoded by a Turbo code with rate 0.5 and then mapped to M-QAM symbols. Then OFDM transmitter modulates them on correspondence sub-carriers by *N*= 64 point FFT. We eliminate the ISI, by adding CP which contains the last 16 tones of the data OFDM symbol.

Here, we first addressed the simulation results for the case of one user then the result has obtained for the case of two users. In Fig 4.a for the case of full channel knowledge, we observe that for the different constellation size, the performance is almost the same for two and three iteration at the Turbo decoder. Therefore, in order to reduce the complexity of the decoding, we can use a two iteration decoder, without reduction at the performance of the system for full channel estimation.

Fig 5.b shows that with reduction at the number of pilot subcarriers for 16QAM modulation at one iteration, the change at the value of the BER is very significant. Therefore, if we did not use iterative decoder (for example if we use convolutional code), BER value have severe variation with reduction at the number of pilot subcarrier.

Adaptive modulation improves the performance of the system. Our target is decreasing the BER as much as possible and achieving to the maximum rate at the constant BER. Moreover, for the 64QAM modulation the difference between the two and three iteration performance at low BER raises by diminishing the value of BER for the cases of fourth channel estimation therefore, for obtaining the switching threshold of the adaptive modulation, we select the third iteration for 64QAM cases and for other constellation we use the switching threshold related to two iteration.

As we see in the figure 4 the difference between the full and half channel estimation far all of the constellation sizes are very low therefore we can use half estimation instead of full channel estimation with negligible loss at the performance. In all of the above simulation we used MMSE detection at the case of one user. The value of *a, b* at the case of two iteration for BPSk, QPSK, 16QAM, and three iteration for 64QAM modulation is shown in table I. In Fig. 6, we simulate the average BER for a system with two users and four iterations at Turbo decoder as a function of the received SNR per symbol and per antenna with different modulation for MMSE-SIC detection and compare this result with approximated formula through curve fitting. The results show that the approximated equation with parameter in Table II has very little difference with simulation results. Considering Tables I, II we see that the BER bound is very accurate for the case of one and two user.

VI. CONCLUSIONS

In this paper, it is shown that, we can use half of the subcarrier for channel estimation without loss at the performance, in comparison to the full channel estimation. Moreover, we developed an equation to obtain the BER of an OFDM/SDMA system with Turbo coding in a fading channel. The formula had good accuracy for the single and multiuser cases; by expanding the above approximated formula for the other cases we can obtain a closed form formula for general Turbo coded cases. Furthermore, we obtained switching thresholds for a multiuser cases in an adaptive modulated system with constant Turbo coding rate and small constant (BER=10^{-5}), by variation of modulation (constellation size) related to the changing of the channel condition for the cases of MMSE-SIC

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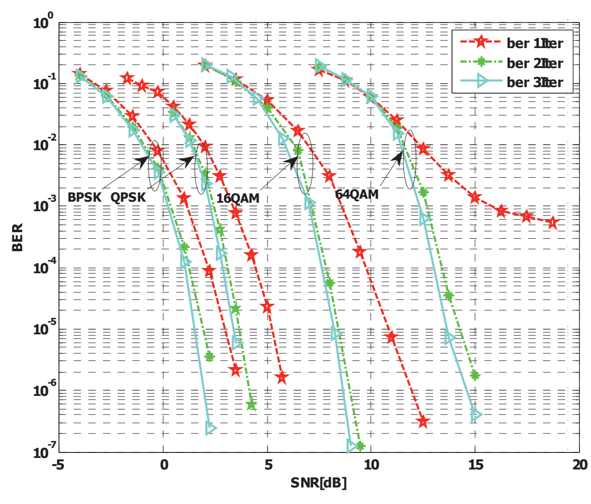
Table I: *a* and *b* coefficient for OFDM system and a rate ½ Turbo encoder for a single user with MMSE detection at NLOS-TGn-B channel with different channel estimation.

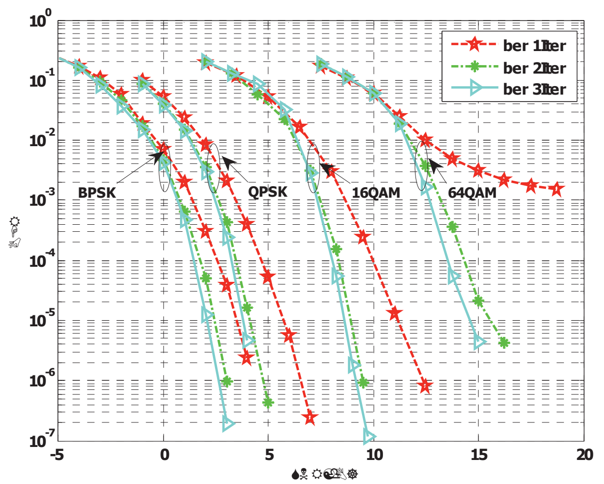
Modulation scheme	M	full		half		fourth	
		a	b	a	b	a	b
NO TX	-	-	-	-	-	-	-
BPSK	2	-2	0.6	-1.3	0.9	-1.2	0.7
QPSK	4	-4	0.6	-3.2	1.7	-1.9	3.2
16QAM	16	-6	6.5	-5	9	-2.7	17
64QAM	64	-8	19	-6.5	30	-3.7	64

Table II: Switching threshold at BER=10^{-5} and *a* and *b* coefficient for OFDM/SDMA system and Rate ½ Turbo encoder with 4 iteration, two user and four antenna at LOS-TGn-B channel.

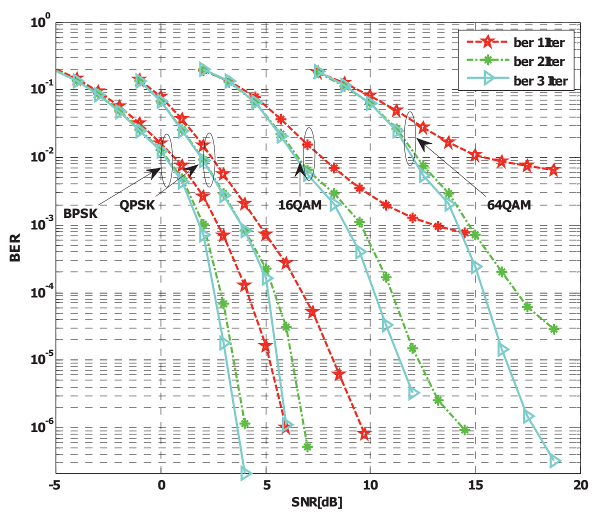
M	a	b	SNR Thresholds
NO TX	-	-	-
2	-2.1	1	0.5
4	-4.5	2	2.5
16	-5.5	7	9
64	-6.5	44	14



4. a) full

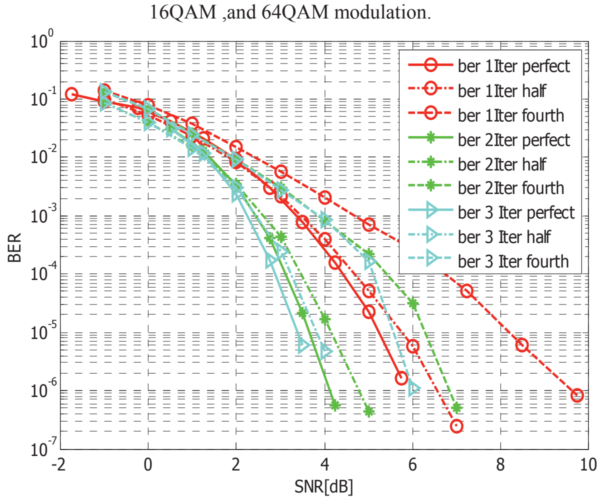


4. b) half

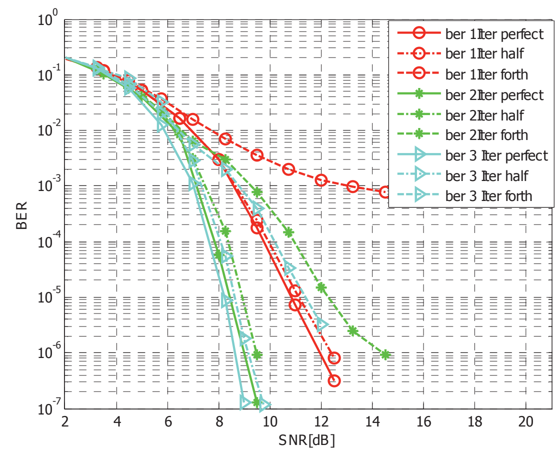


4. c) fourth

Fig. 4 the Bit Error Rate of a single user four antenna with MMSE detection at NLOS-TGn-B channel with channel estimation for BPSK, QPSK, 16QAM ,and 64QAM modulation.



5. a) QPSK



5. b) 16QAM

Fig. 5. The Bit Error Rate of a single user 4 antenna with MMSE detection at NLOS-TGn-B channel with different channel estimation.

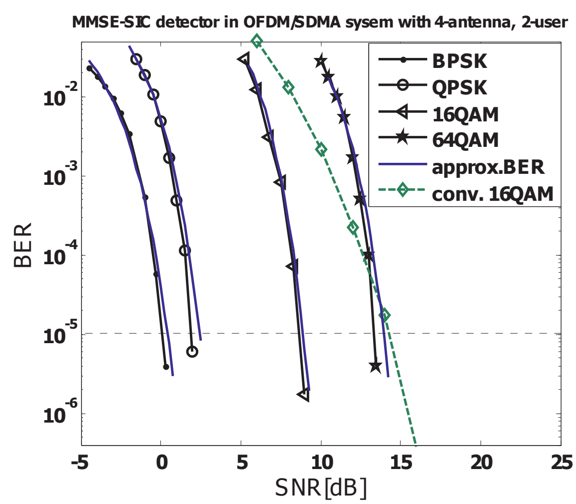


Fig.6 Simulated and approximated BER of different constellation size modulation for the MMSE-SIC and rate ½ Turbo code for four iterations at an OFDM/SDMA system at LOS-TGn-B channel.