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Novel approach to adjust the step size for closed-loop power control in wireless cellular code division multiple access systems under flat fading

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Abstract: In this article, we study the power control (PC) process in wireless cellular code division-multiple access systems under flat fading and propose a novel approach to find an optimum step size for closed-loop power control algorithms. In this approach, an optimum step size will be computed from a proposed function. This function depends on system parameters such as, the number of co-channel users, processing gain, the period of PC, Doppler frequency, channel attenuation and the order of diversity. Based on this computation, the mobile station (MS) adjusts its transmit power optimally to decrease interference for other co-channel users. Simulation results for different sets of system parameters show that the proposed algorithm decreases the bit error rate, the outage probability at the base station (BS), and increases the battery life of the MS compared with other values of the step size. The performance of the proposed algorithm is compared with the fixed-step-size power control algorithm and superiority of its performance is confirmed by simulation results. Moreover, the upper and lower bounds of the outage probability and the received signal-to-interference ratio for the proposed algorithm at the BS will be calculated.

1 Introduction

The primary goal of cellular radio systems is to provide reliable communication for users irrespective of their location and situation. Third generation of wireless systems are mostly based on the direct sequence-code division multiple access (DS-CDMA) technique, which effectively uses the radio spectrum by means of universal frequency reuse [1]. Since in practice, the signals used in DS-CDMA systems are not completely orthogonal, all users experience co-channel interference from all other users in the system. Thus, the co-channel interference is the dominating factor in CDMA systems and the interference level limits the capacity of such systems [2]. There are several techniques to reduce the co-channel interference. One of them is the power control (PC) method in which the output power of each transmitter is adjusted to reduce the co-channel interference [3]. The necessity of PC in the uplink channel (mobile-to-base) is more serious than that of downlink (base-to-mobile). Because in the uplink channel, the near-far distance problem is inherent, so the multiple access interference becomes a serious problem. Hence, we only consider the uplink PC problem in CDMA systems. To compensate for channel attenuation, mobile station (MS) receives a known signal, that is, the pilot signal, from the base station (BS) in the downlink channel, and adjusts its transmit power proportional to the inverse of the measured value. Since the pilot signal is transmitted at a constant power, the variations of its strength convey sufficient information about the downlink channel attenuation. This is called open-loop power control (OLPC) [4]. The centre frequencies that are allocated to the uplink and downlink transmissions are usually widely separated. As a result, the correlation between the uplink and downlink attenuation variations will be very low. Therefore it is better to use the transmission power updates of the MS, which are based on feedback information from the received signal-to-interference ratio (SIR) at the BS; this forms a closed-loop system for PC [5]. This closed-loop power control (CLPC) (or inner-loop PC) keeps the received uplink signal power level at a specified target value; and if a fast PC is employed, the effects of multipath fading can be compensated [5]. Moreover, the targeted SIR can be varied, because the SIR requirement for a given bit error rate (BER) varies because of change of channel conditions. This is the role of the outer-loop power control (OULPC) [5–7].

PC algorithms can be centralised or distributed [8]. A centralised power control (CPC) scheme uses complete information of each user [8], and control measurements are taken for all the users. On the other hand, a distributed power control (DPC) scheme uses only the local information about each user to make a decision [9]. Depending on the type of algorithm, the power control command (PCC) can be in the form of decision feedback (DF) or information feedback (IF) [10]. In the algorithms based on DF, only one bit is needed for the PCC [10], but in algorithms based on IF, several bits are needed. Moreover, to minimise the loop delay, the PCC bits are sent without any error correction provisions. Hence, the probability of receiving an erroneous command can be relatively high (up to 10% [11]). Also in [12] it is shown that the transmitting PCC based on IF in several bits, takes a considerable bandwidth of the downlink channel; and changes in the transmitted power level will also lead to abrupt increase or decrease of interference for other co-channel users. In [13], the effect of incorrect PCC on DF and IF is studied, and it is shown that using the IF method, any error in the PCC leads to unexpected adjustment of the transmit power and this will reduce the stability of the PC process. In the DF method, PCC is sent using one bit in the downlink channel. A bit takes the values of zero or one that has no significant effect in adjustment of the transmit power, and if received incorrectly, the resulting inaccurate transmit power setting will be corrected in the following accurate commands [14]. Variable and adaptive values for step size in CLPC are investigated in [15] and [16], respectively. In [17] an optimisation method to adjust system parameters is presented. In [18], the effect of loop delay on the PC process is investigated and a time-delay compensator (TDC) is proposed to mitigate this effect. The effective parameters in the PC process are presented in [19] and a new PC algorithm for fast measurement of the received SIR in the BS is proposed in [20]. The optimal PC is examined in [21]. The effect of multipath fading on the PC process is studied in [22] and [23], in which, antenna diversity is used to mitigate this effect.

This paper is organised as follows. The PC system model is briefly described in Section 2. Section 3 presents the observations based on simulation results for the system model of Section 2. In Section 4, PC system parameters are stated and investigated, and an analytical function for the SIR standard deviation will be proposed. In Section 5, an optimum step-size function is analytically computed from the proposed function in Section 4. Also, the performance of the proposed PC algorithm will be compared with the fixed-step-size power control (FSPC) algorithm and superiority of its performance is confirmed by simulation results. The upper and lower bounds of the measured outage probability and the received SIR at the BS are calculated in Section 6. Finally, Section 7 concludes the paper.

2 PC system model

First, we study the uplink PC system under flat-fading channel. Fig. 1 shows the PC system model in which N mobile co-channel users are uniformly distributed in a cell. Each user generates its data at r_1 bits per second (b/s), which takes on the values $(-1, 1)$. The generated data are then modulated using binary phase shift keying (BPSK) and its spectrum is spread using the corresponding spreading sequence. It is assumed that the sequence of chips is known to the transmitter and the receiver, and the data are spread with a rate of r_2 chips per second (chips/s) as illustrated in Fig. 1. The spreading waveform is transmitted in the uplink channel after adjusting its power (which is done once in each PC period). The transmitted signal of the ith user, at time t is given by

$$x_i(t) = \sum_{m=0}^{\infty}\left(b_i[m]\sqrt{p_i\left(\left\lfloor\frac{m}{v}\right\rfloor\right)} \times \sum_{j=0}^{P_G-1}(c_i[mP_G + j]A(t-(mP_G + j)T_C))\right) \quad (1)$$

where $b_i[m]$ is the mth data symbol related to the ith user for the BPSK modulation. P_G is the processing gain, which equals to (T_s/T_c), T_s is the data symbol period, T_c is the chip period of the spreading sequence, $c_i[mP_G + j]$ is the spreading sequence corresponding to the ith user in the $(mP_G + j)$th chip, $A(t)$ is the signature waveform, v is the number of data symbols in each PC group (in this paper, the PC group is defined for an accurate calculation of the received power in the BS, and each PC group includes several data symbols, which are shown by v, and their powers are the same and constant during the PC process. Using the PC group, SIR measurement is done via more data symbols and thus there is no need to adjust the transmitting power for each transmitted symbol by the BS,

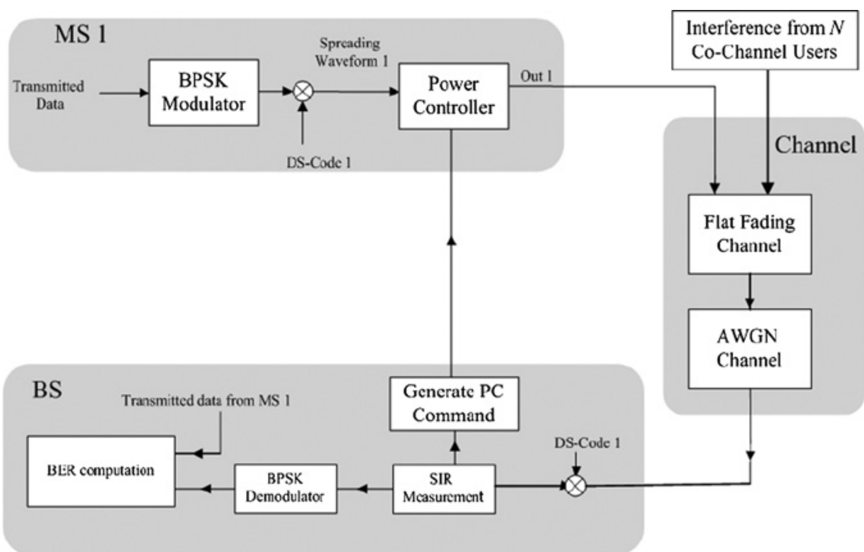

Fig. 1 *CDMA closed-loop power control system model*

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$p_i(\lfloor \frac{m}{v} \rfloor)$ is the transmitted power of the ith user in the $\lfloor \frac{m}{v} \rfloor$th PC group and $\lfloor \arg \rfloor$ is the floor function which rounds the value of arg to the nearest integer less than or equal to arg (in the following for simplicity, k is used, that is, $k = \lfloor \frac{m}{v} \rfloor$).

The transmitted signals of all users will experience channel attenuation that is the most important parameter of the radio channel which is shown by h and calculated as [18]

$$h = h_p h_s h_m \tag{2}$$

where h_p represents the path loss, which models the large-scale distance-dependent attenuation of signal power [7]). h_s represents the shadowing loss, which models the medium-scale attenuation caused by reflection, refraction and diffraction of signal from obstacles such as the buildings, trees, and rocks. These result in relatively slow variations of the mean signal power known as slow fading, and are modelled as a log-normal random variable [24]. h_m represents multipath fading or fast fading attenuation, which is the rapid fluctuation in the received signal power caused by the constructive and destructive addition of signals that propagate through different paths having different path delays from the transmitter to the receiver. In urban environments, the number of significant signal paths is typically much larger than rural areas. This affects the multipath spread, which is the time between the first occurrence of the transmitted signal at the receiver and the last significant reflection of the same signal at the receiver [25]. If the multipath spread is smaller than the inverse of the bandwidth of the information-bearing signal, that is, smaller than the duration of a transmitted symbol, then the fading is said to be frequency non-selective or flat fading [26]. Multipath fading is usually modelled as a filtered complex Gaussian process, resulting in a Rayleigh-distributed envelope and a classical Doppler spectrum. This model is applicable in no-line-of-sight (NLOS) situations. Jakes in [27] proposes a model for a Rayleigh fading channel with the required spectral properties based on a sum of sinusoids. In this model which is used in this paper, the channel gain is given by [27]

$$g = \frac{1}{h} = (A_p d^{-\rho})(10^{\xi/10})(A_f) \tag{3}$$

where A_p is a coefficient which depends on the transmitted signal wavelength and the environment (rural, suburban and urban), d is the distance between the transmitter and receiver, ρ is the path loss exponent with typical values ranging from 2 (free-space propagation) to 5 (dense urban areas) [27], ξ is a mean zero Gaussian random variable with standard deviation between 3 and 8 dB [24], and A_f is assumed to be an independent exponentially distributed random variable (In a Rayleigh fading environment, the received signal envelop, $\sqrt{A_f}$ has Rayleigh distribution and its power, A_f, has exponential distribution [25]). In other words, the received power at a BS (from a MS) is an exponentially distributed random variable.

Taking into account users' movement, transmitted signals in uplink and downlink experience all types of fading. Transmitted signals are also polluted by additive white Gaussian noise (AWGN). The received signals in a BS are a sum of co-channel signals. The received signal is despread by the corresponding spreading sequence. The power of the despread signal related to each user is measured in the BS, that is, the sum of the transmitted power and other users' interference plus noise power. In CDMA mobile systems, co-channel users' interference power is much greater than the noise power, and thus we use interference power instead of interference plus noise power in the following analysis. Therefore the ratio of the transmitted signal power to interference power is called SIR. For accurate measurement, in each PC group, SIR is measured for each received data symbol; and at the end of the PC group, the average of the measured SIRs defines an SIR value in the related PC group. Therefore it is necessary to perform the measurement over more data symbols in each PC group. Increase in the accuracy of the measurement using more data symbols because of more dispreading signals will increase the time of measurement, which will lead to a PC speed reduction. When the speed of measurement is less than the fading rate (fading rate is the product of Doppler frequency (f_D) and PC period (T_p), that is, $f_D T_p$ [18]), then the transmitted PC commands of the BS to the MS will be outdated and the MS will not be able to adjust its transmit power. Thus, it seems that there is a tradeoff between the accuracy and the speed of measurement. The ith user's received SIR in the kth PC group is calculated as follows

$$\gamma_i(k) = \frac{\sum_{n=1}^{B} P_i((k-1)B+n)}{\sum_{j=1, j\neq i}^{N} \sum_{n=1}^{B} P_j((k-1)B+n)} \tag{4}$$

where B is the number of the processed data symbols in each PC group, $P_i((k-1)B+n)$ and $P_j((k-1)B+n)$ are the measured power of the ith and the jth user for the $((k-1)B+n)$th data symbol at the BS, respectively.

PC techniques reduce the fluctuations of the measured SIR around its target value at the BS. These fluctuations are called error and should be minimised. Here, we introduce the SIR variance that is a criterion for this type of error. Assume that the ith user's SIR (estimated) variance is defined as

$$\sigma_i^2 = \frac{1}{K} \sum_{k=1}^{K} (\gamma_i(k) - \gamma_i^T)^2 \tag{5}$$

where γ_i^T is the SIR target related to the ith user and K is the number of adjustments of the transmit power in the PC process. The SIR standard deviation in decibels (dB) concluded from (5) is written as

$$\sigma_{i,\mathrm{dB}} = 10 \log \left(\frac{1}{K} \sum_{k=1}^{K} (\gamma_i(k) - \gamma_i^T)^2 \right) \tag{6}$$

The ith user's PCC for DF algorithm in kth PC group is calculated as

$$u_{tx,i}(k) = \mathrm{sign}(\gamma_i(k) - \gamma_i^T) \tag{7}$$

where

$$\mathrm{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \tag{8}$$

The calculated PCC in each PC group is sent to the MS through the downlink (feedback) channel and then the MS adjusts its transmitting power as

$$p_i(k+1) = p_i(k) + \delta u_{tx,i}(k) \quad (9)$$

where δ is the step size of PC that can be constant or variable for FSPC and variable step-size power control (VSPC), respectively.

Despread signals are demodulated by the BPSK demodulator and compared with the generated data by the user at the BS as illustrated in Fig. 1. In this way, the BER against measured SIR could be computed.

3 Observations based on simulation results

In this section, for better evaluation of the PC system performance, we simulate the mentioned system model to obtain the SIR standard deviation for different values of system parameters such as the step size, the order of diversity and the fading rate. Order of diversity is the number of antennas that are used in BS [22]. We can construct an analytical model based on these observations. The parameters that are used in these simulations are chosen from a real system selected from the third generation (3G) category of CDMA systems. Here, we use system parameters near those of the universal mobile telecommunications system (UMTS) [28]. The parameters and initial assumptions for our simulations are as follows.

The number of users that are uniformly distributed over a co-channel cell is 80. Each user generates data at a constant rate of 135 kb/s The generated data are modulated using BPSK and then, using corresponding spreading sequence, it is spread with a rate of 11.52 *M*chip/s (therefore the processing gain will be 19.3 dB). Spread signal is transmitted in the uplink channel after adjusting its power level (each PC group contains 100 data symbols, which is constant during the simulations. Note that, a data symbol equals a bit because BPSK modulation is used, so in the following bit is used instead of data symbol). Here, the SIR target is supposed to be about 6 dB for every user. Ideal handovers are assumed in the sense that each user is connected to the BS with the largest link gain at all times. The flat fading channel is assumed and simulations are done for two fading rates (i.e. $f_D T_p = 0.01$ and 0.033) [18]. Also, the transmitting signals are assumed to be corrupted by a zero mean AWGN with standard deviation of 6 dB [18] and the working area is urban. Simulations are performed with and without diversity. The order of diversity is assumed to be two and the maximal ratio combining (MRC) algorithm is employed [29]. The BS measures SIR using (4) in each PC group, generates PCC using (7) and sends it to the MS in the downlink channel and then the MS adjusts its transmit power level using (9). Also, the BS measures the SIR standard deviation by (6). Simulations are done for different values of step size and the SIR standard deviations are calculated for each value. Fig. 2 shows variations of the SIR standard deviation against step size for the mentioned fading rates with and without diversity technique. Note that, increasing the fading rate will increase the SIR standard deviation and using the diversity technique, the SIR standard deviation will be decreased. Also, from Fig. 2, it seems that the standard deviation against the step size has convex manner, that is, increasing or decreasing the step size from the optimum value (to be discussed in the next section), will increase the SIR standard deviation. In the sequel, we try to construct a model based on these observations and other models reported in literature such as [7, 17, 30–32].

4 PC system parameters

In this section, first, one of the most important VSPC algorithms is studied and then the PC system parameters will be briefly introduced and investigated. In order to solve the problems of the studied VSPC algorithm based on the investigated system parameters, a new analytical function for computation of SIR standard deviation against the mentioned system parameters and the simulation results from Section 2 is proposed and in the next section the step size of PC process will be calculated.

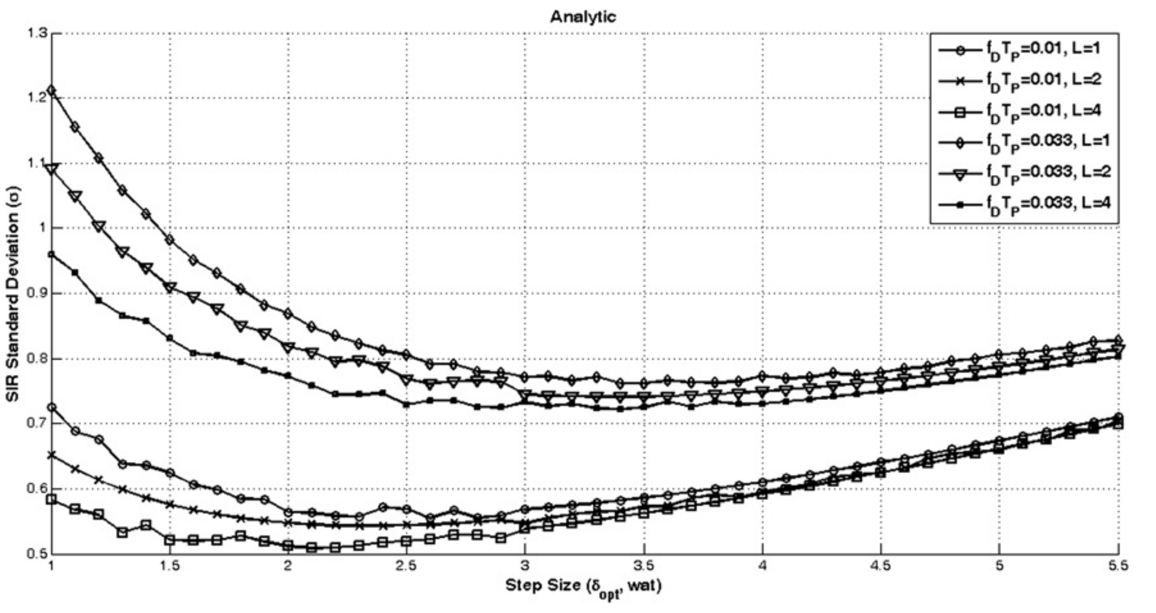

Fig. 2 *SIR standard deviation against step size achieved by simulations for N = 80, L = 1, 2 and 4, $f_D T_P = 0.01$ and 0.033*

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4.1 Radical step-size power control (RSPC) algorithm

RSPC is a VSPC-type algorithm [16], and is described in Fig. 3. In this algorithm, the coefficients α and β are used by the MS, and initialised by zero. When the measured SIR in the BS at the first PC group is less than the target SIR value, the BS sends a PCC to MS to increase its power level and then at the MS side, α is increased one unit. The transmitted power is given by

$$\begin{aligned} p_i(k+1) &= p_i(k) + \sqrt{\alpha}\delta \\ &= p_i(k) + \delta \end{aligned} \tag{10}$$

In the next iteration (i.e. the next PC group), if the received SIR in the BS is less than the targeted SIR, the PCC is sent to MS to increase its power again, the MS increases α, by one unit and it equals to two, the transmitting power is

$$\begin{aligned} p_i(k+2) &= p_i(k+1) + \sqrt{2}\delta \\ &= p_i(k) + (1+\sqrt{2})\delta \end{aligned} \tag{11}$$

At the Nth iteration, the transmitting power by the MS is written as

$$\begin{aligned} p_i(k+n) &= p_i(k+n-1) + \sqrt{N}\delta \\ &= p_i(k) + (1+\sqrt{2}+\cdots+\sqrt{N})\delta \\ &= p_i(k) + \delta\sum_{n=1}^{N}\sqrt{n} \end{aligned} \tag{12}$$

The maximum transmitting power by the MS should be less than or equal to the maximum allowable transmitting power ($p_i^{\max}$). Using (12), we obtain

$$\sum_{n=1}^{N}\sqrt{n} \le \frac{p_i^{\max} - p_i(k)}{\delta} \tag{13}$$

If the received SIR at the BS is more than the target value, the BS sends a new PCC to MS to decrease its transmit power. At first, the values of α and β will be zero and one, respectively, then the transmitting power by the MS is adjusted as

$$p_i(k+1) = p_i(k) - \sqrt{\beta}\delta = p_i(k) - \delta \tag{14}$$

If the MS receives the successive decreasing PCCs, the transmit power by the MS in Nth iteration is written as

$$\begin{aligned} p_i(k+n) &= p_i(k+n-1) - \sqrt{N}\delta \\ &= p_i(k) - (1+\sqrt{2}+\cdots+\sqrt{N})\delta \\ &= p_i(k) - \delta\sum_{n=1}^{N}\sqrt{n} \end{aligned} \tag{15}$$

The minimum transmitting power should be greater than or equal to the minimum allowable transmit power ($p_i^{\min}$). Using (15), we will have

$$\sum_{n=1}^{N}\sqrt{n} \le \frac{p_i(k) - p_i^{\min}}{\delta} \tag{16}$$

This algorithm has higher speed of convergence, lower probability of bit error and higher robustness against the loop delay compared with the FSPC algorithm. But, in the RSPC algorithm, the SIR fluctuations around the target SIR value are more than the FSPC algorithm [16], which is undesirable. In order to solve this problem, after describing the system parameters in the following, a new analytical function for finding the optimum step size is derived.

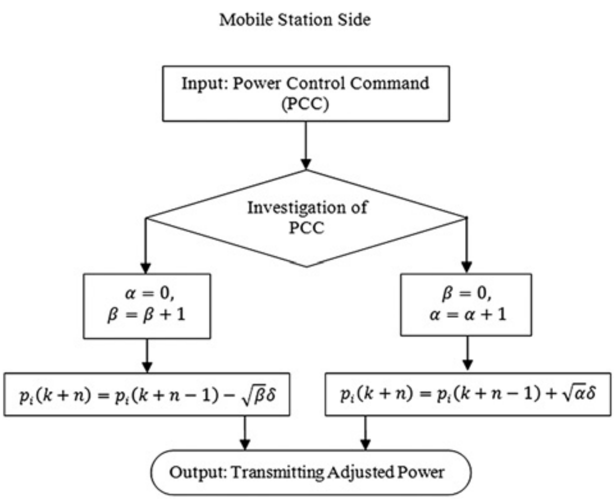

Fig. 3 *RSPC algorithm*

4.2 System parameters

The system parameters that affect the PC process performance are:

1. *Number of co-channel users:* an increase in the number of co-channel users (N) will increase the co-channel interference power. In [30] and [31], it is shown that the SIR standard deviation is proportional to N^{θ} in which, θ depends on cell size, channel conditions, transmission wavelength and its value is between zero and one ($0 < \theta < 1$). In this paper, θ is assumed to be equal to 0.5.
2. *Processing gain:* to avoid the problems of frequency and time synchronisation, we can use signals that are 'almost orthogonal.' A code is assigned to each user and transmission is done using the entire frequency band. The receiver extracts the desired signal by cross-correlating the received signal and the code. This spread-spectrum technique is referred to as DS-CDMA. By spreading and despreading the signal, the link becomes more robust against fading and interference. The performance gain obtained by spreading is referred to as the processing gain (P_G). In a system with data rate R and channel bandwidth W, P_G is given by the ratio (W/R) [32]. An increase in the processing gain (P_G) of the CDMA system, will result in an increase in the number and length of the spreading sequences and also a decrease in the co-channel interference and the SIR standard deviation [31]. The relation between the SIR standard deviation and processing gain is ($\sigma \propto P_G^{-\varphi}$), where φ depends on the kind of the sequence used (for example in the direct sequence spread spectrum systems, $\varphi = 0.5$ [32]).
3. *Power update rate:* In order to overcome fast fading, power update rate of the MS should be faster than the fading rate. Transmitting power of each user during the PC

period is constant. Therefore the reduction of the PC period (T_p) leads to an increase in the power update rate. For the sake of reliability, power update rate should be several times greater than the fading rate; so, the variations in the link attenuation can be tracked. Note that the update rate cannot be arbitrarily high because of the inherent delay imposed by the SIR measurement process. (Both the measuring and signalling operations in cellular systems take much time, which results in delay in the system. The power-update rate depends on the measurement period of the receive SIR at the MS and a higher power-update rate will require a shorter measurement period.) Since the available feedback channel bandwidth for PC signalling is limited in practice, there is a tradeoff between the accuracy of the SIR measurement and power-update rate.

4. *Doppler frequency:* in wireless cellular communication systems, MS may have different velocities with respect to the BS. Thus, the received signal frequency in the downlink channel by the MS is different from that of the signal transmitted by the BS. In other words, transmitted signals from the BS and the MS experience Doppler spread, which is shown by f_D). Numerous techniques for estimating the Doppler spread and the velocity at the receiver (here MS) are proposed, which most of them can be categorised into two classes: techniques based on the level crossing rate (LCR) [25] and [33] and techniques based on the covariance of the channel estimates [34]. The performance of these techniques has been extensively studied in [35]. All these techniques have been proven to be efficient and robust against the variations in the propagation medium provided that the signal to noise ratio (SNR) should be high enough. The Doppler frequency has a direct relation with the MS speed, and any increase in the Doppler frequency will increase the SIR standard deviation [14]. The product of Doppler frequency (f_D) and PC period (T_p) is called the fading rate $f_D T_p$ [14]. In this paper, the flat fading channel is which the coherence bandwidth of the channel is larger than the bandwidth of the signal. Therefore all frequency components of the signal will experience the same magnitude of fading. Hence, the standard deviation (σ) is proportional to $(f_D T_p)^{0.5}$ [14]. We can conclude that an increase in fading rate will increase the SIR standard deviation.

5. *Channel attenuation:* an increase in the channel attenuation (introduced in Section 2 and shown by h) will increase the SIR standard deviation at the BS [18].

6. *Diversity:* in wireless cellular systems, in many situations, there may not be a line-of-sight between the transmitter and the receiver. Therefore transmitted signals are received at the receiver from different paths having different delays and this causes abrupt increase and decrease in the received signal power. This problem is more troublesome in uplink channel because of the low transmitting power of the MS. It results in two important problems related to deep fading. First, the PC ability to track deep fading is limited because of imperfect knowledge of parameters in real systems. Second, if the PC algorithm is improved (e.g. using VSPC algorithms) to better track deep fading, a user who experiences deep fading will raise its transmit power significantly and will affect the SIR experienced by other users. This could lead to instability problems, because other users will also raise their power. The problem that we want to solve here is how to combat deep fading, so that the PC process can better track the channel. Combating deep fading by PC alone is not only difficult, but also results in instability problem as described above. Therefore we need to solve the problem using a different approach. A well-known method to reduce the effect of deep fading is to use antenna diversity techniques in the BS, which is used in this paper. Several antennas are used in the receiver and the distance between each pair of antennas would be at least several times greater than the transmitted signal wavelength, so that the signal transmission paths would stay independent from each other. Then, if the probability of a received signal from an antenna falls below the threshold, equals P_r, the probability that all the received signals by L antennas simultaneously falls below the threshold value, equals P_r^L, which is considerably smaller than P_r. The received signals by all antennas are combined using the MRC method [29], in which a specific weight is assigned to each antenna as

$$w_l = \frac{\sqrt{\gamma_l}}{\sum_{l=1}^{L} \sqrt{\gamma_l}} \tag{17}$$

where w_l is the weighting factor related to the lth antenna and γ_l is the received SIR of the lth antenna and the output of the diversity combiner, γ_{div}, is calculated as

$$\gamma_{div} = \sum_{l=1}^{L} w_l \gamma_l \tag{18}$$

The SIR standard deviation is proportional to $L^{-\lambda}$, where for the MRC algorithm, λ equals to 0.5 [36].

4.3 SIR standard deviation as a function of system parameters

In order to compute the SIR standard deviation, the relation between the system parameters and the SIR standard deviation can be classified as

$$\begin{cases} \sigma \propto N^{0.5}, & [31] \\ \sigma \propto P_G^{-0.5}, & [32] \\ \sigma \propto (f_D T_p)^{0.5}, & [14] \\ \sigma \propto h, & [18] \\ \sigma \propto L^{-0.5}, & [36] \end{cases} \tag{19}$$

where $f_D T_p$ is the fading rate, P_G is the processing gain, L is the number of antennas, N is the number of co-channel users and h is the channel attenuation. Note that we assume flat fading channel, so $\sigma \propto (f_D T_p)^{0.5}$ is satisfied.

Without the PC process, and using the above relations, the SIR standard deviation can be estimated as

$$\sigma \simeq h\left(\frac{f_D T_p N}{P_c L}\right)^{0.5} \tag{20}$$

We will show validity of the above proposed equation in later. The mentioned parameters are applicable in all wireless cellular CDMA systems. But, for the system based on the PC process, there are other parameters, which have important roles to adjust the desired SIR standard deviation that as stated below.

Step size: The step size that is shown by δ in (9), is the most important factor for adjustment of the uplink transmitting power of the MS. Choosing a large step size causes abrupt increase or decrease of the MS-transmitting power. Abrupt increase of the transmit power of the MS increases co-channel interference in the uplink channel for other co-channel users and this decreases the SIR for them.

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As a result, users increase their transmit power, and this leads to an increase in the co-channel interference. As a result, the SIR standard deviation increases the PC system instability and users' outage probability [7]. Choosing a small step size reduces the speed of PC convergence towards the SIR target. Therefore in fast fading, the PC algorithm will not be able to track channel attenuation. Also, when the received command by the MS is in decreasing mode (i.e. the MS should decrease its transmit power), by the sake of using a small step size, transmit power will decrease with a slow pace; then, co-channel interference will decrease slowly [7] and [17]. Selecting a too large or too small step size for the PC process reduces the capacity and performance of the wireless cellular system because of increase of the SIR standard deviation [7, 17, 20]. Based on this intuition and simulation results in Section 3, we can conclude that the SIR standard deviation varies as a convex function of the step size. Therefore a proper function to describe variations of the SIR standard deviation as a function of the step size can be proposed as follows

$$\sigma = \frac{a_1}{\delta + a_2} + a_3\delta \tag{21}$$

where σ is the SIR standard deviation, δ is the step size, a_1, a_2 and a_3 are the SIR standard deviation adjustment coefficients that are determined by other system parameters. The above equation shows that increasing or decreasing the step size, will increase the SIR standard deviation. Also, for $\delta = 0$ the SIR standard deviation (σ) equals to a_1/a_2 in which the PC technique does not have been used (i.e. transmit power is constant and $p_i(k+1) = p_i(k)$, which is concluded from (9)).

Loop delay: the loop delay refers to the overall loop delay in CLPC. It greatly affects the performance of a PC algorithm. This delay is a combination of delays owing to the SIR measurement process, transmission of the SIR information over the radio channel, processing of the SIR information to calculate and adjust the transmit power, and propagation time after which the new transmission power affects the next SIR measurement. Therefore the power update could be based on the outdated information of SIR. This may cause instability in the PC algorithms, leads to large variations in the interference powers at the receivers and diminish the capacity. Hence, the probability of receiving an erroneous command can be relatively high. In this paper, we suppose that the effect of loop delay can be compensated using TDC proposed in [18].

The SIR standard deviation adjustment coefficients of (21) are computed as in the following steps:

1. If the PC process is not used then in (21), δ equals to zero and the SIR standard deviation will be a_1/a_2. Also, using (20), the a_1/a_2 is derived as

$$\frac{a_1}{a_2} = h\left(\frac{f_D T_p N}{P_G L}\right)^{0.5} \tag{22}$$

using (22), we suppose that

$$\begin{cases} a_1 = \sqrt{f_D T_p N} \\ a_2 = \dfrac{\sqrt{P_G L}}{h} \end{cases} \tag{23}$$

2. Take the derivative of (21) with respect to step size (δ)

$$\frac{\partial\sigma}{\partial\delta} = -\frac{a_1}{(\delta + a_2)^2} + a_3 \tag{24}$$

3. In order to obtain the optimum step size, $\frac{\partial\sigma}{\partial\delta} = 0$, then we have

$$\partial^2 + 2a_2\delta + a_2^2 - \frac{a_1}{a_3} = 0 \tag{25}$$

and

$$\begin{cases} \delta_1 = -a_2 + \sqrt{a_1/a_3} \\ \delta_2 = -a_2 - \sqrt{a_1/a_3} \end{cases} \tag{26}$$

4. In a real PC system, a_1, a_2 and a_3 are positive; therefore δ_2 is negative and could not be used as a value for the step size, hence δ_1 should be positive, then

$$a_3 < \frac{a_1}{a_2^2} \tag{27}$$

5. Using Steps 1 and 4, a_3 is calculated as

$$a_3 < \frac{\sqrt{f_D T_p N}h^2}{P_G L} \tag{28}$$

6. a_3 can be approximated as in [37]

$$a_3 \cong 0.1\frac{\sqrt{f_D T_p N}h^2}{P_G L} \tag{29}$$

7. Standard deviation should have real values, and so using (23) and (29), the SIR standard deviation is obtained as

$$\sigma(\delta) = \frac{\sqrt{f_D T_p N}}{\delta + (\sqrt{P_G L}/h)} + 0.1\frac{\sqrt{f_D T_p N}h^2}{P_G L}\delta \tag{30}$$

If $N > 30$, $P_G > 50$ and $h > 10$ also for given system parameters of section 3, $\sqrt{f_D T_p N}h^2 \cong LP_G$, then we have

$$\sigma(\delta) \cong \frac{\sqrt{f_D T_p N}}{\delta + (\sqrt{P_G L}/h)} + 0.1\delta \tag{31}$$

Fig. 4 shows the analytic results of (31) for parameters of Section 3. Simulations have been repeated also for another fading rate (i.e. $f_D T_p = 0.067$) with and without diversity and then compared with the results of (31) as shown in Fig. 4.

Figs. 2 and 4 show that increasing the fading rate will increase the SIR standard deviation and the optimum step size (to be discussed in the next section). These figures also show that by using diversity techniques, the SIR standard deviation and the optimum step size will be decreased. Comparing Fig. 2 (simulation results) with Fig. 4 (analytic results) confirms the accuracy of the proposed SIR standard deviation function (i.e. see (31)).

5 Optimum step-size computation for PC

In the previous section, effective parameters for the PC process were studied and a function was proposed to express their relation to the SIR standard deviation. Finally,

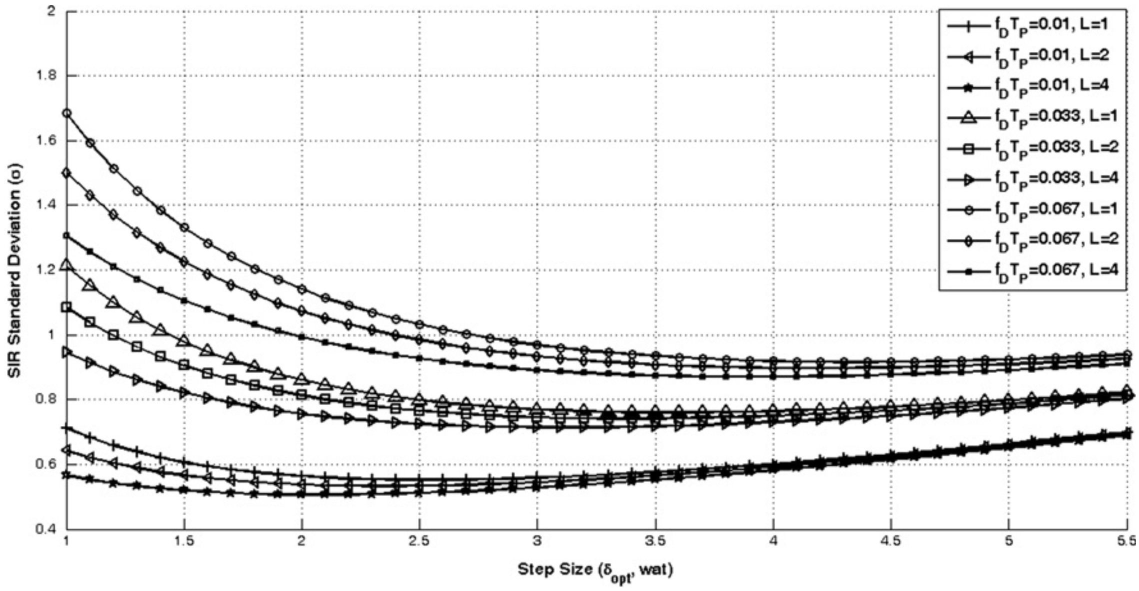

Fig. 4 *SIR standard deviation against step size achieved by analytical results from (31) for $N = 80$, $L = 1$, 2, and 4, $f_D T_P = 0.01$, 0.033 and 0.067*

the accuracy of (31) for the parameters used in the simulations (which was shown in Fig. 2) is confirmed according to Fig. 4.

In this section, we take the derivative of (30) with respect to step size. Therefore the optimum step size is given by

$$\delta_{\text{opt}} \cong -\sqrt{LP_{\text{G}}}/h + 3\sqrt[4]{Nf_{\text{D}}T_{\text{p}}} \tag{32}$$

Using DF command, transmit power of the ith user for the $(k+1)$th PC group is adjusted as

$$p_i(k+1) = p_i(k) + \delta_{\text{opt},i}(k)u_{tx,i}(k) \tag{33}$$

In which $p_i(k)$ is the transmit power of the ith user at the kth PC group and $\delta_{\text{opt}}(k)$ is the optimum step-size value for the kth PC group. In order to verify (32), first, we assume the system parameters according to Table 1, and Section 3, then simulations are repeated for the optimum step-size value (calculated by (32)) and two other values of the step size, that is, $\boldsymbol{\delta}^{(1)}$ and $\boldsymbol{\delta}^{(2)}$. Simulations for the computation of BER against SIR have been done according to system parameters presented in Table 1. Simulation results based on **(a-1)**, **(a-2)** and **(a-3)** in Table 1 are shown in Fig. 5. It can be seen that for the conditions in **(a-1)**, the optimum step size ($\delta_{\text{opt}} = 2.37$) results in a less BER than the two other sizes, which are 2 and 2.8. For the conditions in **(a-2)**, the optimum step size ($\delta_{\text{opt}} = 2.18$) leads to the lower BER than that of 1.8 and 2.6. Results for other conditions in Fig. 6 confirm the validity of (the proposed) (32).

Taking the simulation results of Fig. 2 into account, we can see that an increase in the fading rate, besides increasing the SIR standard deviation, increases the optimum step-size value, which can be implied also from (32).

In the PC process, provided that the MS adjusts its transmit power along with the optimum step size value, both the SIR standard deviation and the received BER in the BS will be decreased as shown in Figs. 2, 4, 5 and 6. In order to evaluate the performance of the proposed algorithm, simulations are repeated for the proposed algorithm, the FSPC algorithm [10] and the RSPC algorithm [16] (which is discussed in Section 4.1) for system parameters based on Tables 1–3. In opposite of our proposed algorithm and RSPC algorithm, using FSPC algorithm, the MS adjusts its transmitting power by a fixed value of step size. The main reason for choosing the FSPC algorithm is its application in the UMTS and IS-95 networks. Also, the reason for choosing the RSPC algorithm as a criterion for comparison is its better performance compared with the FSPC algorithm. The simulation results in Fig. 7 show and confirm better performance of the proposed algorithm compared with the FSPC and RSPC algorithms. Fig. 8 analytically shows the optimum step-size based on the number of co-channel users. It is shown that increasing the number of co-channel users increases the optimum step-size value. Also, the optimum step-size against the fading rate and the order of diversity are analytically depicted in Fig. 9, which shows that either increasing the fading rate or decreasing the order of diversity, will increase the optimum step-size value.

Table 1 Closed loop power control system parameters for our simulations

Modes	Parameters							
	$f_D T_p$	L	N	P_G	h	δ_{opt}	$\delta^{(1)}$	$\delta^{(2)}$
a-1	0.01	1	80	85(19.3 dB)	20	2.37	2	2.8
a-2	0.01	2	80	85(19.3 dB)	20	2.18	1.8	2.6
a-3	0.01	4	80	85(19.3 dB)	20	1.91	1.5	2.3
b-1	0.033	1	80	85(19.3 dB)	20	3.36	2.9	3.9
b-2	0.033	2	80	85(19.3 dB)	20	3.17	2.8	3.6
b-3	0.033	4	80	85(19.3 dB)	20	2.9	2.5	3.3
c-1	0.067	1	80	85(19.3 dB)	20	4.1	3.7	4.5
c-2	0.067	2	80	85(19.3 dB)	20	3.91	3.5	4.3
c-3	0.067	4	80	85(19.3 dB)	20	3.64	3.1	4

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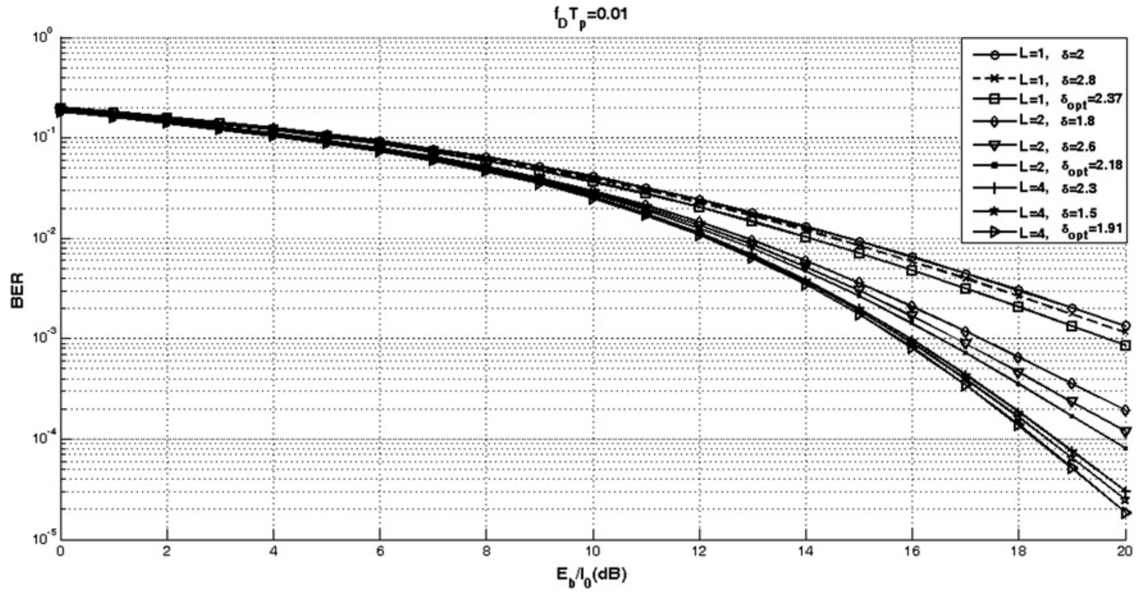

Fig. 5 *BER against E_b/I_0 for $N = 80$, $f_D T_P = 0.01$ and $L = 1$, 2 and 4*

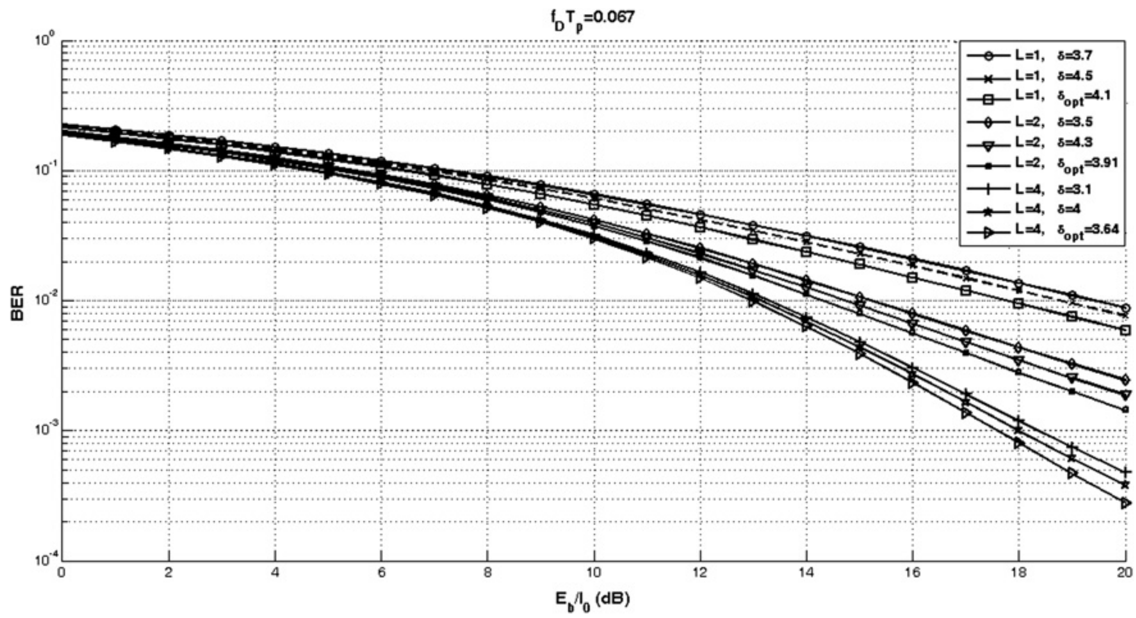

Fig. 6 *BER against E_b/I_0 for $N = 80$, $f_D T_P = 0.067$ and $L = 1$, 2 and 4*

6 Outage probability

In this section, we compute an upper and lower bounds for outage probability for the proposed method. If the ith MS is served by the bth BS, then the uplink measured SIR for the ith MS at the kth PC group is calculated as

$$\gamma_{bi}(k) = \frac{g_{bi}(k)p_i(k)}{(\sum_{j=1,j\neq i}^{N} g_{bj}(k)p_j(k)) + \eta_b} P_G \quad (34)$$

where $p_i(k)$, $g_{bi}(k)$ and η_b are the transmitted power of user i, link gain between MS i and BS b at kth PC group (which is an

Table 2 CLPC system parameters for the FSPC algorithm [10]

Modes	Parameters					
	$f_D T_p$	L	N	P_G	h	δ_{fixed}
d-1	0.033	1	80	85(19.3 dB)	20	2
d-2	0.033	2	80	85(19.3 dB)	20	2
d-3	0.033	4	80	85(19.3 dB)	20	2

Table 3 CLPC system parameters for the RSPC algorithm [16]

Modes	Parameters					
	$f_D T_p$	L	N	P_G	h	δ_{init}
d-1	0.033	1	80	85(19.3 dB)	20	1
d-2	0.033	2	80	85(19.3 dB)	20	1
d-3	0.033	4	80	85(19.3 dB)	20	1

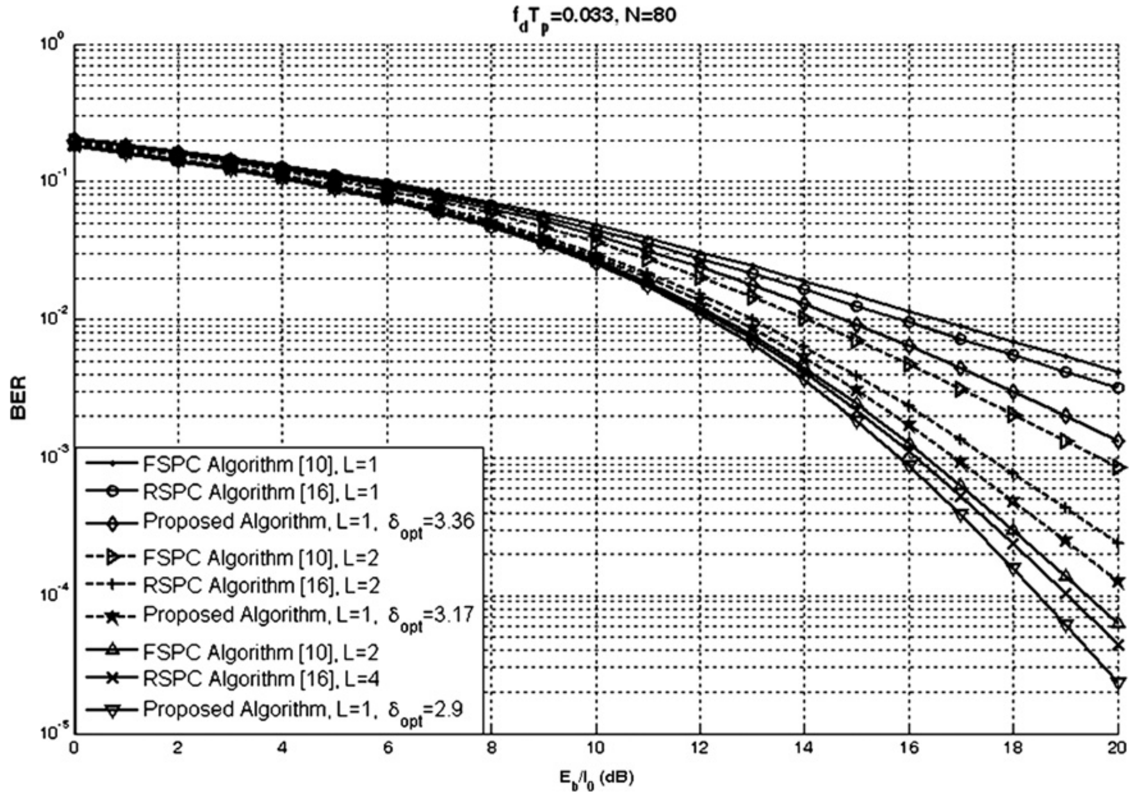

Fig. 7 *BER against E_b/I_0 for $N = 80$, $f_D T_P = 0.033$ and $L = 1$, 2 and 4 for the FSPC algorithm [10] the, RSPC algorithm [16] and the proposed algorithm*

exponentially distributed random variable) and the power of noise that is AWGN, respectively, N is the number of users that are distributed in the environment and P_G is the processing gain.

Note that in a Rayleigh fading channel γ_{bi} is a random variable with a complex distribution, since it is the ratio of an exponential random variable to a sum of exponential random variables with different means [38]. Here, we assume that the requested quality of service (QoS) is provided when SIR exceeds a given threshold (γ^{th}). The outage probability of the ith user at the kth PC group is given by

$$\begin{aligned} O_i(k) &= \text{Prob}\{\gamma_{bi}(k) \le \gamma_i^{\text{th}}\} \\ &= \text{Prob}\left\{\left(\frac{g_{bi}(k)p_i(k)}{\sum_{j=1,j\ne i}^{N} g_{bj}(k)p_j(k) + \eta_b} P_G\right) \le \gamma_i^{\text{th}}\right\} \\ &= \text{Prob}\left\{g_{bi}(k)p_i(k) \le \frac{\gamma_i^{\text{th}}}{P_G}\left(\sum_{j=1,j\ne 1}^{N} g_{bj}(k)p_j(k) + \eta_b\right)\right\} \end{aligned} \tag{35}$$

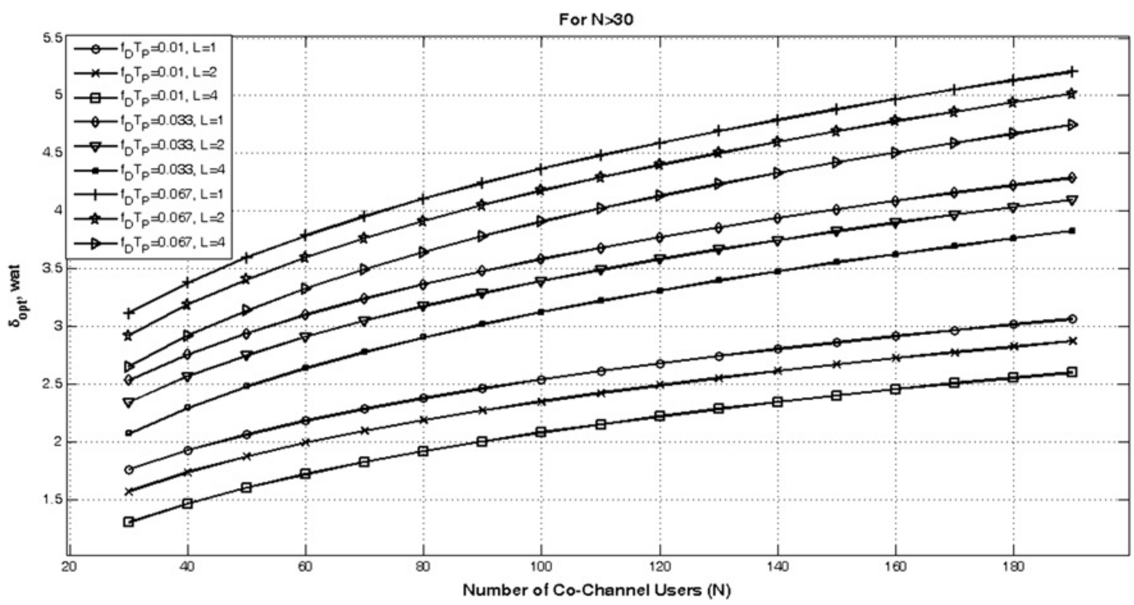

Fig. 8 *Optimum value of step size against number of co-channel users for $L = 1$, 2 and 4 and $f_D T_P = 0.01$, 0.033 and 0.067*

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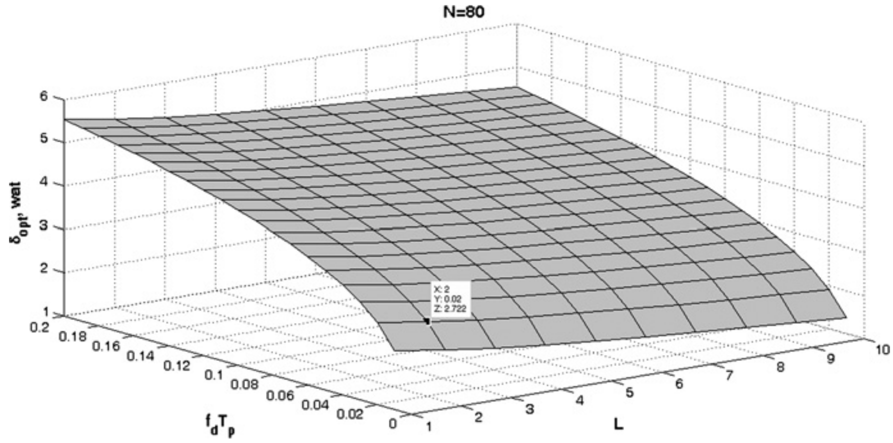

Fig. 9 *Optimum value of step size against the order of diversity and fading rate for N = 80*

Lemma 1: Suppose $z_1, z_2, \ldots, z_n$ are independent exponentially distributed random variables with mean $(E(z_i) = 1/\gamma_i)$, and c is a constant, then

$$\mathrm{Prob}\left(z_1 \le \sum_{i=2}^{n} z_i + c\right) = 1 - \left[\mathrm{e}^{\gamma_1 c} \prod_{i=2}^{n}\left(1 + \frac{\gamma_1}{\gamma_i}\right)\right]^{-1} \quad (36)$$

Proof: See Appendix 1.

Since $g_{b1}(k)p_1(k), g_{b2}(k)p_2(k), \ldots, g_{bN}(k)p_N(k)$ are independent exponentially distributed random variables with mean $(E(g_{bi}(k)p_i(k)) = \bar{g}_{bi}(k)\bar{p}_i(k))$ [12], and η_b is the noise power, which is constant, using (36), the measured outage probability is calculated as

$$O_i(k) = 1 - \left[\mathrm{e}^{(\eta_b \gamma_i^{\mathrm{th}}/\bar{g}_{bi}(k)\bar{p}_i(k)P_G)} \prod_{j \ne i}\left(1 + \frac{\gamma_i^{\mathrm{th}}\bar{g}_{bj}(k)\bar{p}_j(k)}{\bar{g}_{bi}(k)\bar{p}_i(k)P_G)}\right)\right]^{-1} \quad (37)$$

Lemma 2: Suppose $z_1, z_2, \ldots, z_n \ge 0$ and $c \ge 0$ is a constant, then

$$1 + c + \sum_{i=1}^{n} z_i \le \mathrm{e}^c \prod_{i=1}^{n}(1 + z_i) \le \mathrm{e}^{c + \sum_{i=1}^{n} z_i} \quad (38)$$

Proof: See Appendix 2.

Using the right-hand side of inequality in (31), the upper bound of the outage probability is given by

$$O_i(k) \le 1 - \frac{1}{\mathrm{e}^{[(\eta_b \gamma_i^{\mathrm{th}}/\bar{g}_{bi}(k)\bar{p}_i(k)P_G) + \sum_{j \ne i}(\gamma_i^{\mathrm{th}}\bar{g}_{bi}(k)\bar{p}_j(k)/P_G\bar{g}_{bi}(k)\bar{p}_i(k))]}} \quad (39)$$

The certainty-equivalent margin (CEM) was defined in [38] as

$$\mathrm{CEM}(k) = \frac{\hat{\gamma}_{bi}(k)}{\gamma_i^{\mathrm{th}}} \quad (40)$$

where

$$\hat{\gamma}_{bi}(k) = \frac{E(g_{bi}(k)p_i(k))}{E(\sum_{j=1, j \ne i}^{N} g_{bj}(k)p_i(k)) + \eta_b} P_G = \frac{\bar{g}_{bi}(k)\bar{p}_i(k))}{(\sum_{j=1, j \ne i}^{N} \bar{g}_{bj}(k)\bar{p}_j(k)) + \eta_b} P_G \quad (41)$$

CEM represents a margin of error for the average received SIR at the BS. It means that increasing CEM results in an increase in the received SIR at the BS compared with the threshold SIR and this, results in smaller outage probability and improves system performance. Then, using (40), (41) is rewritten as

$$O_i(k) \le 1 - \mathrm{e}^{-1/\mathrm{CEM}(k)} \quad (42)$$

In a similar way, using the left-hand inequality in (39), the lower bound of outage probability is given by (see (43))

and using (40), (43) is rewritten as

$$O_i(k) \ge \frac{1}{1 + \mathrm{CEM}(k)} \quad (44)$$

Using (42) and (44), the upper and lower bounds of outage probability against CEM are written as

$$\frac{1}{1 + \mathrm{CEM}(k)} \le O_i(k) \le 1 - \mathrm{e}^{-1/\mathrm{CEM}(k)} \quad (45)$$

Fig. 10 shows the outage probability against CEM and the fact that the lower and upper bounds are very close for large CEM (i.e. smaller outage probability). Also, a large value of CEM is an indicator of greater SIR at the BS. The variation range of the SIR at the kth PC group is given by

$$\gamma_{bi}^{\min(k)} < \gamma_{bi}(k) < \gamma_{bi}^{\max(k)} \quad (46)$$

$$O_i(k) \ge 1 - \frac{1}{1 + (\eta_b \gamma_i^{\mathrm{th}}/\bar{g}_{bi}(k)\bar{p}_i(k)P_G) + \sum_{j \ne i}(\gamma_i^{\mathrm{th}}\bar{g}_{bj}(k)\bar{p}_j(k)/P_G\bar{g}_{bi}(k)\bar{p}_i(k))} \quad (43)$$

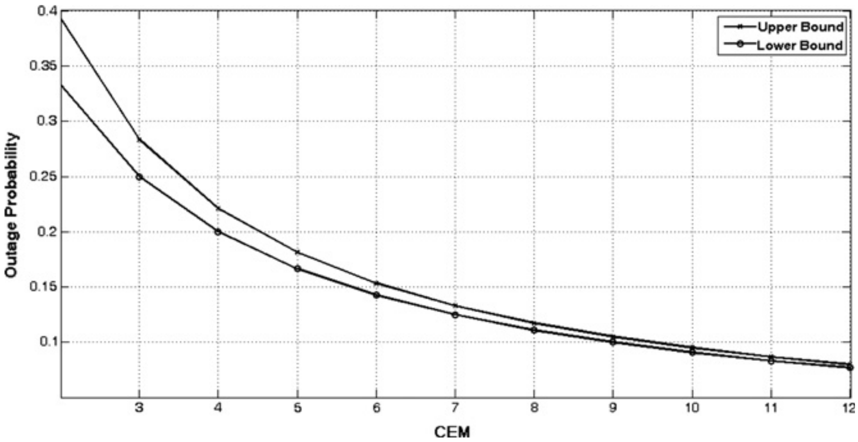

Fig. 10 *Upper and lower bounds of outage probability O as a function of CEM*

where the right-hand inequality in (46) is calculated as

$$\gamma_{bi}^{\max(k)} = \frac{(\max_k g_{bi}(k))(p_i(k-1)+\delta_{\mathrm{opt},i}(k))}{\sum_{j=1,j\neq i}^{N}(\min_k g_{bj}(k))(p_j(k-1)+\delta_{\mathrm{opt},j}(k))+\eta_b}P_{\mathrm{G}}$$
$$= \frac{(\max_k g_{bi}(k))(p_i(1)+\sum_{m=2}^{k}\delta_{\mathrm{opt},i}(m))}{\sum_{j=1,j\neq i}^{N}(\min_k g_{bj}(k))(p_j(1)-\sum_{m=2}^{k}\delta_{\mathrm{opt},j}(m))+\eta_b}P_{\mathrm{G}} \tag{47}$$

$(p_i(1) - \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m))$ is the transmit power by the ith MS at the kth PC group in increasing step size. It should be less than the maximum transmit power by the MS to increase the battery life and decrease the health-related hazards and the interference generated for the other co-channel users. Also, for getting the maximum value of the received SIR, $(p_j(1) - \sum_{m=2}^{k} \delta_{\mathrm{opt},j}(m))$ is replaced by $p_j^{\min}$. Finally, the above equation is rewritten as

$$\gamma_{bi}^{\max(k)} = \frac{g_{bi}^{\max}\left(p_i^{\min}(1) + \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m)\right)}{\sum_{j=1,j\neq i}^{N} g_{bj}^{\min} p_j^{\min} + \eta_b} P_{\mathrm{G}} \tag{48}$$

where $p_i^{\min}(1)$ is the minimum value of transmit power at the first PC group. The left-hand inequality in (46) is calculated as

$$\gamma_{bi}^{\min}(k)$$
$$= \frac{(\min_k g_{bi}(k))(p_i(k-1) - \delta_{\mathrm{opt},i}(k))}{(\sum_{j=1,j\neq i}^{N}(\max_k g_{bj}(k))(p_j(k-1) + \delta_{\mathrm{opt},j}(k)) + \eta_b}P_{\mathrm{G}}$$
$$= \frac{(\min_k g_{bi}(k))(p_i(1) - \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m))}{(\sum_{j=1,j\neq i}^{N}(\max_k g_{bj}(k))(p_j(1) + \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m)) + \eta_b}P_{\mathrm{G}} \tag{49}$$

In the above equation, $p_i(1)$ should be greater than $\sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m)$.Otherwise, at the kth PC group, the ith MS will be turned off. To prevent this, $p_i(1)$ should be large enough, which is denoted by $p_i^{\max(1)}$. Also, for getting the minimum value of the received SIR

$$\left(p_j(1) + \sum_{m=2}^{k} \delta_{\mathrm{opt},j}(m)\right)$$

is replaced by $p_j^{\max}$. Finally, (49) is rewritten as

$$\gamma_{bi}^{\min(k)} = \frac{g_{bi}^{\min}(p_i^{\max}(1) - \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m))}{\sum_{j=1,j\neq i}^{N} g_{bj}^{\max} p_j^{\max} + \eta_b} P_{\mathrm{G}} \tag{50}$$

The upper and lower bounds of outage probability in (46) are rewritten as

$$\frac{1}{1+\mathrm{CEM}^{\max(k)}} \leq O_i(k) \leq 1 - \mathrm{e}^{-1/\mathrm{CEM}\min(k)} \tag{51}$$

Therefore using (39), (43), (48), (50) and (51), the upper and lower bounds of outage probability against $\sum_{m=2}^{K} \delta_{\mathrm{opt},i}(m)$ can be calculated as

$$\frac{1}{1 + ((p_i^{\min}(1) + \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m))/b_{\min})}$$
$$\leq O_i(k) \leq 1 - \mathrm{e}^{-\left(b_{\max}/p_i^{\max}(1) - \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m)\right)} \tag{52}$$

where

$$\begin{cases} b_{\min} = \dfrac{\gamma_i^{\mathrm{th}}(\sum_{j=1,j\neq i}^{N} g_{bj}^{\min} p_j^{\max} + \eta_b)}{g_{bi}^{\max} P_{\mathrm{G}}} \\ b_{\max} = \dfrac{\gamma_i^{\mathrm{th}}(\sum_{j=1,j\neq i}^{N} g_{bj}^{\max} p_j^{\max} + \eta_b)}{g_{bi}^{\min} P_{\mathrm{G}}} \end{cases} \tag{53}$$

At the receive side, diversity techniques may be used and the maximum output of the diversity combiner, $\gamma_{\mathrm{div}}^{\max}$ is calculated as

$$\gamma_{\mathrm{div}}^{\max} = \sum_{l=1}^{L} w_l \gamma_l^{\max}$$
$$= \sum_{l=1}^{L} w_l \left(p_i^{\min}(1) + \sum_{m=2}^{k} \delta_{\mathrm{opt},i}(m)\right) / b_{\min,l} \tag{54}$$

where $\gamma_l^{\max}$ is the maximum-received SIR of the lth antenna at the BS. The minimum output of the diversity combiner, $\gamma_{\mathrm{div}}^{\min}$,

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is calculated as

$$\gamma_{\text{div}}^{\min} = \sum_{l=1}^{L} w_l \gamma_l^{\min} = \sum_{l=1}^{L} w_l \left(p_i^{\max}(1) - \sum_{m=2}^{k} \delta_{\text{opt},i}(m) \right) / b_{\max,l} \quad (55)$$

The upper and lower bounds of outage probability against $\sum_{m=2}^{k} \delta_{\text{opt},i}(m)$ can be calculated as

$$\frac{1}{1 + \sum_{l=1}^{L} (w_l (p_i^{\min}(1) + \sum_{m=2}^{k} \delta_{\text{opt},i}(m))/b_{\min,l})} \le O_i(k) \le 1 - \mathrm{e}^{-\sum_{l=1}^{L}\left(b_{\max,l}/w_l\left(p_i^{\max}(1) - \sum_{m=2}^{k} \delta_{\text{opt},i}(m)\right)\right)} \quad (56)$$

The upper and lower bounds of outage probability against $\sum_{m=2}^{k} \delta_{\text{opt},i}(m)$ are depicted in Fig. 11. In this figure, the upper and lower bounds of outage probability for analytical results of (52) and (56) and simulation results of Section 3 are shown. The values of system parameters are assumed as in Tables 4 and 5 for (52) and (56), respectively. In Table 5, the order of diversity is supposed to be equal to 3 and for simplicity, weighting factors (w_l) are supposed to be fixed. Simulation results shown in Fig. 11 confirm that the range of variations of outage probability is limited between the calculated upper and lower bounds of the outage probability as in (52) and (56). The capacity of such system at the kth PC group for bandwidth of ω is calculated as

$$C(k) = \omega \ln(1 + \gamma_{bi}(k)) \quad (57)$$

The upper and lower bounds of the system capacity at the kth PC group is given by

$$C^{\min}(k) < C(k) < C^{\max}(k) \quad (58)$$

Using (48) and (50), the upper and lower bounds of system capacity at the kth PC group in (58) is rewritten as

$$\omega \ln(1 + \gamma_{bi}^{\min}(k)) < C(k) < \omega \ln(1 + \gamma_{bi}^{\max}(k)) \quad (59)$$

$$\omega \ln\left(1 + \frac{g_{bi}^{\min}(p_i^{\max}(1) - \sum_{m=2}^{k} \delta_{\text{opt},i}(m))}{\sum_{j=1, j\neq i}^{N} g_{bj}^{\max} p_j^{\max} + \eta_b} P_{\text{G}}\right) < C(k) < \omega \ln\left(1 + \frac{g_{bi}^{\max}(p_i^{\min}(1) + \sum_{m=2}^{k} \delta_{\text{opt},i}(m))}{\sum_{j=1, j\neq i}^{N} g_{bj}^{\min} p_j^{\max} + \eta_b} P_{\text{G}}\right) \quad (60)$$

and using diversity techniques, the upper and lower bounds of the system capacity at the kth PC group is calculated as

$$\omega \ln(1 + \gamma_{\text{div}}^{\min}(k)) < C(k) < \omega \ln(1 + \gamma_{\text{div}}^{\max}(k))$$

$$\omega \ln\left(1 + \sum_{l=1}^{L} w_l \left(p_i^{\max}(1) - \sum_{m=2}^{k} \delta_{\text{opt},i}(m)\right) / b_{\max,l}\right) < C(k) \quad (61)$$

$$< \omega \ln\left(1 + \sum_{l=1}^{L} w_l \left(p_i^{\min}(1) + \sum_{m=2}^{k} \delta_{\text{opt},i}(m)\right) / b_{\min,l}\right) \quad (62)$$

Table 4 System parameters for (analytical) (52)

$b_{\min}$	$b_{\max}$	$p_i^{\min}(1)$	$p_i^{\max}(1)$
0.01	0.7	0.1	8

Table 5 System parameters for (analytical) (56)

$b_{\min,1}$	$b_{\min,2}$	$b_{\min,3}$	$b_{\max,1}$	$b_{\max,2}$	$b_{\max,3}$	$p_i^{\min}(1)$	$p_i^{\max}(1)$	w_1	w_2	w_3
0.01	0.005	0.003	0.7	0.3	0.1	0.1	8	0.8	0.2	0.1

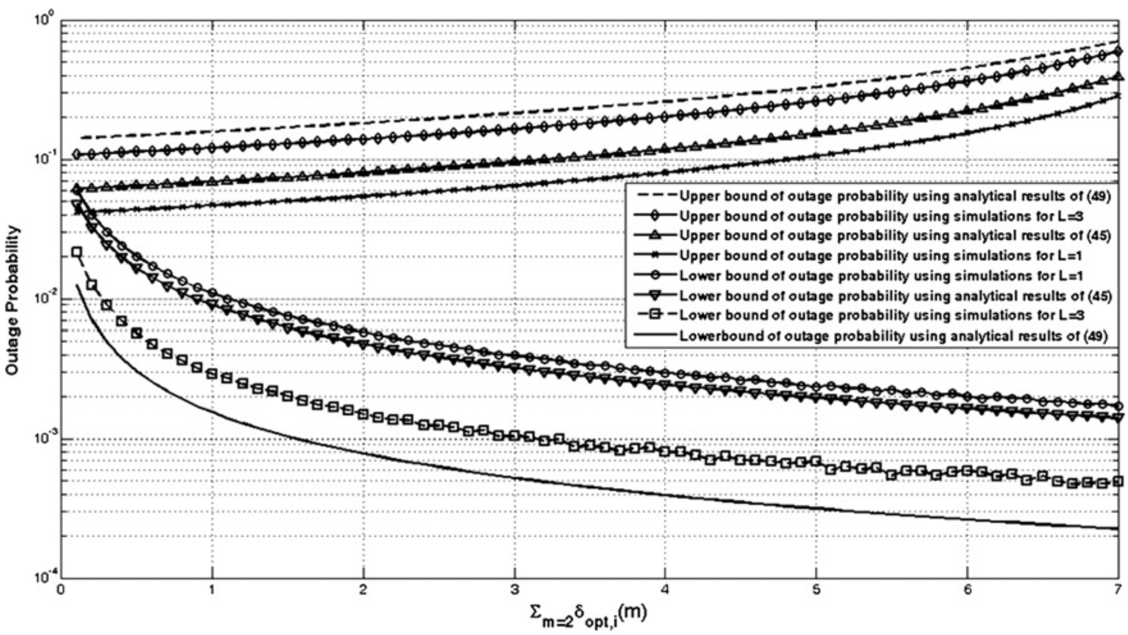

Fig. 11 *Upper and lower bounds of outage probability O as a function of $\sum_{m=2}^{k} \delta_{opt,i}(m)$ for analytical results of (52) and (56) based on system parameters on Table 5 and simulation results based on Section 3*

7 Conclusion

In this paper, we described the CLPC system model and investigated effect of system parameters on the SIR standard deviation using simulation studies. Also, we studied the CLPC system parameters and proposed a function to obtain the SIR standard deviation as a function of the mentioned parameters. Then by calculating the derivation of the proposed function with respect to step size, the optimum value of the step size was analytically computed and its accuracy was confirmed by simulations. Using the computed step size, the MS will adjust its transmitting power in each PC group optimally. It was also realised that the optimal adjustment of the transmit power, decreases the standard deviation and BER. Moreover, the effect of different fading rates and the number of co-channel users on the optimum value of step size were investigated. It is shown that each of these values increases the optimum value of the step-size. Moreover, performance of the proposed algorithm was compared with that of using fixed values for the step-size power control (FSPC) algorithm and its better performance was shown by simulations. Moreover, the upper and the lower bounds of the outage probability and the system capacity at the BS were calculated.

8 References

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9 Appendix 1: proof of Lemma 1

In this section, we prove (29). Suppose $z_1, z_2, \ldots, z_n$ are independent exponentially distributed random variables with mean $(E(z_i) = 1/\gamma_i)$, and c is a positive constant, then

$$\mathrm{Prob}\left(z_1 \le \sum_{i=2}^{n} z_i + c\right) = 1 - \left[\mathrm{e}^{\gamma_1 c} \prod_{i=2}^{n}\left(1 + \frac{\gamma_1}{\gamma_i}\right)\right]^{-1}$$

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To prove this, we note that

$$
\begin{aligned}
&\mathrm{Prob}\left(z_1 \le \sum_{i=2}^{n} z_i + c\right)\\
&= \int_0^\infty \cdots \int_0^\infty \int_0^{\sum_{i=2}^n z_i + c} f(z_1) f(z_2) \cdots f(z_n)\, \mathrm{d}z_1\, \mathrm{d}z_2 \ldots \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty \left(\int_0^{\sum_{i=2}^n z_i + c} f(z_1)\, dz_1 \right) f(z_2)\, \mathrm{d}z_1 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty \left(\int_0^{\sum_{i=2}^n z_i + c} \gamma_1 \mathrm{e}^{-\gamma_1 z_1}\, \mathrm{d}z_1 \right) f(z_2)\, \mathrm{d}z_2 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty (1 - \mathrm{e}^{-\gamma_1 \left(\sum_{i=2}^n z_i + c\right)}) \gamma_2 \mathrm{e}^{-\gamma_2 z_2}\, \mathrm{d}z_2 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty \gamma_2 (\mathrm{e}^{-\gamma_2 z_2} - \mathrm{e}^{-\gamma_1 \left(\sum_{i=2}^n z_i + c\right) - (\gamma_1 + \gamma_2) z_2})\, \mathrm{d}z_2 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty \left(1 - \frac{\gamma_2\, \mathrm{e}^{-\gamma_1 \left(\sum_{i=2}^n z_i + c\right)}}{\gamma_1 + \gamma_2} \right) f(z_3)\, \mathrm{d}z_3 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty \left(1 - \frac{\gamma_2 \gamma_3\, \mathrm{e}^{-\gamma_1 \left(\sum_{i=4}^n z_i + c\right)}}{(\gamma_1 + \gamma_2)(\gamma_1 + \gamma_3)} \right) f(z_4)\, \mathrm{d}z_4 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \int_0^\infty \cdots \left(\int_0^\infty \left(1 - \frac{\gamma_2 \gamma_3\, \mathrm{e}^{-\gamma_1 \left(\sum_{i=4}^n z_i + c\right)}}{(\gamma_1 + \gamma_2)(\gamma_1 + \gamma_3)} \right) f(z_4)\, \mathrm{d}z_4 \right)\\
&\quad \cdots f(z_n)\, \mathrm{d}z_n\\
&= \left(1 - \mathrm{e}^{-\gamma_1 c} \prod_{i=2}^{n} \left(\frac{\gamma_i}{\gamma_1 + \gamma_i} \right) \right)\\
&= 1 - \left[\mathrm{e}^{\gamma_1 c} \prod_{i=2}^{n} \left(1 + \frac{\gamma_1}{\gamma_i} \right) \right]^{-1} \qquad (63)
\end{aligned}
$$

□

10 Appendix 2: proof of Lemma 2

In this section, we prove (31). If $c, z_1, z_2, \ldots, z_n \ge 0$, then

$$1 + c + \sum_{i=1}^{n} z_i \le \mathrm{e}^c \prod_{i=1}^{n} (1 + z_i) \le \mathrm{e}^{c + \sum_{i=1}^n z_i}$$

To establish the left-hand inequality, we expand the middle expression as

$$
\begin{aligned}
\mathrm{e}^c \prod_{i=1}^{n} (1 + z_i) &= \mathrm{e}^c \left(1 + \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots \right)\\
&= \left(1 + c + \frac{c^2}{2!} + \cdots \right)\\
&\quad \times \left(1 + \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots \right)\\
&= \left(1 + \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots + c \right.\\
&\quad + c \sum_{i=1}^{n} z_i + c \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots + \frac{c^2}{2!}\\
&\quad \left. + \frac{c^2}{2!} \sum_{i=1}^{n} z_i + \frac{c^2}{2!} \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots \right)\\
&= \left(1 + c + \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots \right.\\
&\quad + c \sum_{i=1}^{n} z_i + c \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots + \frac{c^2}{2!}\\
&\quad \left. + \frac{c^2}{2!} \sum_{i=1}^{n} z_i + \frac{c^2}{2!} \sum_{i=1}^{n} \sum_{j>i}^{n} z_i z_j + \cdots \right) \qquad (64)
\end{aligned}
$$

The first, second and third terms are the left-hand side of the inequality that we want to establish; the third and other remaining terms are non-negative, since they consist of sum of products of z_i, which are non-negative. To establish the right-hand inequality, we will derive the equivalent inequality

$$\sum_{i=1}^{n} \log{(1 + z_i)^c} \le c + \sum_{i=1}^{n} z_l \qquad (65)$$

This follows from the simple inequality $\log{(1 + z)^c} \le c + z$ for $z \ge 0$. □