Achieving Simultaneous Spectrum Utilization and Revenue Improvements in Practical Wireless Spectrum Auctions

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***Abstract*— Spectrum auctions have been considered a promising approach to improve the efficiency of spectrum use. Spectrum reusability is also one of the important properties in spectrum auctions. To handle spectrum reusability, a buyer grouping procedure has been applied in many existing spectrum auction schemes. It is important to note that almost none of the proposed buyer grouping algorithms in the existing works has been specifically designed for spectrum allocation problem. However, buyer grouping in a practical spectrum auction mechanism has its own challenges such as heterogeneity and truthfulness. In this paper, first we illustrate the challenges of buyer grouping in a practical spectrum auction mechanism. Then we propose the novel algorithms for spectrum buyer grouping to solve these challenges. By extensive simulations, we show that our proposed algorithms can not only solve the challenges caused by radio spectrum properties but also provide good performance on various auction metrics.**

***Index Terms*—** Buyer Grouping, Heterogeneity, Spectrum Auction, Spectrum Reuse, Truthfulness

I. INTRODUCTION

Recently, radio spectrum is becoming more and more scarce due to the rapid growth of wireless technologies, applications and services. On the other hand, spectrum occupancy measurements in various countries have indicated that a significant amount of the licensed spectrum remains unused in many places much of the time [1, 2], so the traditional exclusive licensing spectrum policy leads to the low efficiency of spectrum utilization. The spectrum scarcity has increasingly led administrations to more efficiently manage spectrum through various mechanisms. Spectrum auctions are one of the best-known market-based solutions to improve the efficiency of spectrum use. In this solution, spectrum owners are allowed to lease their spectrums to secondary service providers. In return, the spectrum owners can get paid from secondary service providers. Spectrum auctions are fundamentally different from conventional auctions because of the spectrum's unique property of reusability. Unlike traditional goods, the spectrum can be spatially reused concurrently [3].

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To handle spectrum reusability, a buyer grouping procedure has been applied in many existing spectrum auction schemes, in which the buyer's interference conditions are modeled as conflict graph. Buyer grouping problem can be transformed into the problem of finding chromatic number or maximum independent set of a graph, which is NP-hard and there is no efficient algorithm till now. Several approximate algorithms have been proposed to tackle the spectrum reuse problem. It is important to note that almost none of the proposed algorithms for spectrum buyer grouping have been specifically designed for spectrum allocation. However, buyer grouping in a practical spectrum auction mechanism has its own challenges. In a practical auction environment, several aspects of heterogeneity such as heterogeneity in the availability of frequency spectrums for different buyers, heterogeneity in transmission range of frequencies, and heterogeneity in buyers' valuation for different frequency spectrums can exist. So it is required to consider these aspects of heterogeneity in the buyer grouping mechanisms. Also, the density of secondary users (buyers) can be different between various auctions. Considering the impact of the density on the performance of various buyer grouping algorithms, it is required to adapt the buyer grouping mechanism with respect to the density of buyers. On the other hand, since grouping affects on many of auction performance metrics such as spectrum utilization, buyer/seller satisfaction ratio and number of traded channels, and an improvement in one performance metric may reduce the other metrics, it is required to design an buyer grouping mechanism such that improve various performance metrics as possible. Finally, the buyer grouping mechanism should be designed in a way that maintains truthfulness which is essential to resist market manipulation and ensure auction fairness and efficiency.

The main motivation of this paper is to examine the aforementioned challenges of buyer grouping in a practical spectrum auction environment and propose the novel algorithms for spectrum buyer grouping to solve these challenges. By extensive simulations, we show that our proposed algorithms can not only solve the challenges caused by radio spectrum properties but also provide good performance on various auction metrics.

In summary, the main contributions of this paper are as follows:

- We examine the various challenges faced by the buyer grouping in a practical spectrum auction mechanism.
- We propose the novel algorithms for spectrum buyer grouping that solve the related challenges.
- We use channel dependent interference graphs to tackle with the problem of frequency-variant channel characteristics caused by spectrum frequency heterogeneity.
- We design our algorithms so that maintain truthfulness which is essential to resist market manipulation and ensure auction fairness and efficiency.
- Since buyer grouping affects on many of spectrum auction metrics such as spectrum utilization, buyer/seller satisfaction ratio and number of traded channels, we design the algorithms in such

a way that provide good performance on various auction metrics.

The rest of the paper is organized as follows. Preliminaries and problem description are reviewed in section II. The challenges of spectrum buyers' grouping in a practical auction environment are examined in section III. Proposed methods for grouping secondary users are presented in section VI. Simulation results are given in section V. We discuss related works in section VI. Finally, we draw conclusions and point out possible future work in section VII.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, we first formulate the problem of spectrum exchange between spectrum owners and spectrum demanders as a double auction. Then, we provide an overview of buyer grouping as a solution for spectrum reuse problem and introduce some common methods for it.

A. *Problem formulation*

Since the buyer grouping is one of the stages of the most existing spectrum auctions, we consider the scenario where N secondary service providers (called buyers) trying to purchase spectrum resources with K various types from M spectrum owners (called sellers) through a single-round double spectrum auction with one auctioneer, M sellers, and N buyers. We assume that each seller can contribute multiple channels with various spectrum types and each buyer can obtain multiple channels with various spectrum types. Also, the channels with the same spectrum type can potentially be reused by multiple non-conflicting buyers to achieve high spectrum efficiency. We also assume that the channels with the same spectrum type are homogeneous, but the channels with the different spectrum types are heterogeneous, so each buyer has different valuations for the channels with various spectrum types, but its valuations are the same for the channels with the same spectrum type. We allow buyers to express their preferences over each spectrum type separately. Therefore, the buyers' bids are spectrum type-specific. The notations in this paper are summarized in Table I.

In Table I, $A = (a_{ij})$ is a 2-dimensional (0,1)-matrix which represents the buyers' channel type availability such that $a_{ij} = 1$ means that channel type t_j is available for buyer w_i. Also, $C = (c_{ijk})$ is a 3-dimensional (0,1)-matrix which represents the conflict relationships between buyers for each type such that $c_{ijk} = 0$ means that buyers w_j and w_k don't interfere with each other in channel type t_i. Also, D^b and D^s are 2-dimensional (0,1)-matrices which represent the buyers' and the sellers' demands for each channel type belongs to T. For instance, $d_{ij}^b = 1$ means that the buyer w_i wants to buy a channel with type t_j.

Table I. Notations

Symbol	Meaning
M	Number of spectrum owners (sellers)
N	Number of secondary users (buyers)
K	Number of spectrum (channel) types
$S = \{s_1, s_2, \dots, s_M\}$	Set of sellers
$W = \{w_1, w_2, \dots, w_N\}$	Set of buyers
$T = \{t_1, t_2, \dots, t_K\}$	Set of spectrum types
$B_n^b = \{b_{n1}^b, b_{n2}^b, \dots, b_{nK}^b\}$	Bid vector of buyer n
B^b	Bid matrix of all buyers
$B_m^s = \{b_{m1}^s, b_{m2}^s, \dots, b_{mK}^s\}$	Bid vector of seller m
B^s	Bid matrix of all sellers
$A = (a_{ij})$	Availability matrix of the channel types
$C = (c_{ijk})$	Conflict matrix of the buyers
D^b	Demands matrix of the buyers
D^s	Demands matrix of the sellers

B. Spectrum reuse by grouping secondary users

Spectrum auctions are fundamentally different from conventional multiunit auctions because of the spectrum's unique property of reusability. Unlike traditional goods, the spectrum can be spatially reused concurrently such that two buyers that are far enough away from each other may reuse the same spectrum concurrently. To handle spectrum reusability, a buyer grouping procedure has been applied in many existing spectrum auction schemes, in which the buyer's interference conditions are modeled as conflict graph. Buyer grouping problem can be transformed into the problem of finding chromatic number or maximum independent set of a graph, which is NP-hard and there is no efficient algorithm till now. Several approximate algorithms have been proposed to tackle the spectrum reuse problem. Here we make brief introduction of four well-known channel allocation algorithms to form buyer groups [4, 5]:

- Max-IS: It assigns channels by finding the maximum independent set of the conflict graph. To form each independent set, it recursively picks a buyer (or node) with the minimal maximum independent set in the induced sub-graph in its neighborhood.
- Greedy-U: To form a group, it recursively chooses a node with the minimal degree in the current conflict graph, eliminates the chosen node and its neighbors, and updates the degree values of the remaining nodes.
- Greedy: It is the same as the Greedy-U except that it chooses the nodes based on its original degree value.
- Random: It randomly chooses a node and tries to allocate one channel while satisfying non-conflict constraints.

It is important to note that almost none of the proposed algorithms for buyer grouping, such as above-mentioned algorithms, have been specifically designed for spectrum allocation. However, buyer grouping in a practical spectrum auction mechanism has its own challenges. In the following

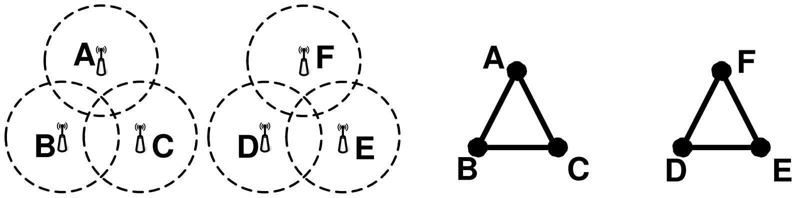

Fig. 1. Transmission ranges and related interference graph for frequency f_1

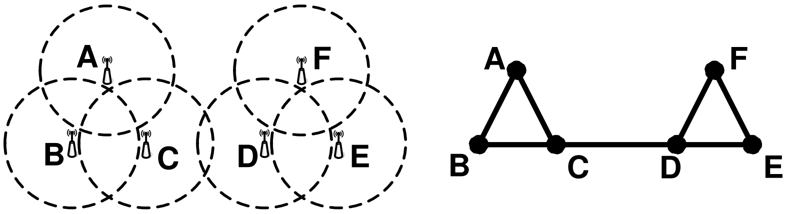

Fig. 2. Transmission ranges and related interference graph for frequency f_2 so that $f_2 << f_1$

section, we briefly explain the challenges of buyer grouping in a practical spectrum auction mechanism.

III. THE CHALLENGES OF SPECTRUM BUYERS' GROUPING FOR SPECTRUM ALLOCATION

In this section, we briefly explain the challenges of secondary users grouping for radio spectrum allocation in a practical spectrum auction mechanism.

A. Heterogeneity in spectrum availability

Spectrum availability may vary by location. It means that some channels provided by the primary users could be not available in the locations of some buyers, for example, because of geography limitations in the license of primary users or occupying of these channels in these locations by other users and so on. Since two not interfering buyers with any common available channels should not be grouped together it is required to consider this type of heterogeneity in grouping algorithm. The most existing works don't consider spectrum availability in buyer grouping.

B. Heterogeneity in transmission range of frequencies

It means that different frequencies have different path losses and therefore, different transmission ranges such that we have:

$$L \propto 10\, log\, f^2 \tag{4}$$

where L is the total path loss in decibel and f is the transmission frequency in mega-hertz [6]. Since the spectrums offered by spectrum owners may consist of a wide range of frequencies, so the interference relationships among spectrum buyers in different channels are non-identical. Fig. 1 and Fig. 2 show an example of such non-identical relationships for two different frequencies.

As shown in these figures, two neighbor nodes C and D do interfere with each other for frequency f_2

because of increasing in transmission ranges. So the interference graphs are different for frequencies f_1 and f_2. Therefore, grouping algorithm is required to consider non-identical interference graphs to group heterogeneous channels.

C. Heterogeneity in buyers' valuation for different frequency spectrums

It means that the valuations of a buyer for different frequency bands are different. For example, some secondary users only request the spectrums residing in lower-frequency bands due to the limitation of wireless devices. On the other hand, the valuations of every buyer are usually the same for the frequencies with similar propagation profiles. Also, spectrum buyers may express different preferences for different spectrum types. Therefore, buyer grouping is required to consider buyers' preferences for different frequency spectrums.

D. Dependency to the graph type

Several grouping algorithms have been proposed in the existing works to tackle the spectrum reuse problem. According to our studies, these algorithms have different performances on different types of graph, including sparse, medium density and dense graphs. Since grouping affects on many of auction metrics such as spectrum utilization, buyer/seller satisfaction ratio and number of traded channels, it is required to firstly determine the type of the corresponding interference graph and according to the results, select a suitable algorithm to group the spectrum buyers.

E. Trade-off between channel utilization and other auction metrics

To handle spectrum reusability, the most auction mechanisms use a buyer grouping procedure which finds the maximal independent set of buyers in the corresponding interference graph to increase the channel utilization. On the other hand, as the size of groups increases, the total number of groups decreases. Since these groups compete for a known number of channels, therefore, fewer buyer groups and consequently fewer sellers can win in the auction for this channel, especially if the number of buyer groups be less than the number of sellers. So the seller satisfaction ratio may decrease. In contrast to this problem, we offer a buyer group enhancement algorithm to increase both channel utilization and other auction metrics such as seller satisfaction.

F. Maintaining the truthfulness

The buyers grouping as one step of spectrum auction mechanism should be designed in a way that maintains truthfulness which is essential to resist market manipulation and ensure auction fairness and efficiency.

IV. PROPOSED METHODS FOR GROUPING SECONDARY USERS

As discussed in the previous section, buyer grouping in a practical spectrum auction mechanism has its own challenges. On the other hand, almost none of the proposed algorithms for buyer grouping have been specifically designed for spectrum allocation and so cannot solve the related challenges. In this section, we propose the novel algorithms for spectrum buyer grouping to solve these challenges. To meet the challenge of maintaining the truthfulness, all the proposed algorithms are designed in a way to be bid-independent to guarantee truthfulness in the auction [7] and performed by auctioneer. Also, to meet the challenge of heterogeneity in buyers' valuation for different frequency spectrums, we assume in our model that each buyer has different valuations for the channels with various spectrum types, but its valuations are the same for the channels with the same spectrum type. Consequently, we allow buyers to express their preferences over each spectrum type separately. In other words, the buyers' bids are spectrum type-specific.

A. Channel dependent conflict graph formation

Similar to many existing works, we model the buyer's interference conditions as conflict graph. However, most existing works use non-variant graph to group buyers. On the other hand, the propagation loss on a terrestrial line-of-sight path relative to the free-space loss is the sum of different contributions. Each of these contributions has its own characteristics as a function of frequency, path length and geographic location [8, 9]. Since channel characteristics are dependent on the frequency used, we can expect that the shape of the interference regions will be channel dependent [10]. To meet the challenge of frequency heterogeneity, we use channel dependent interference graph for proposed buyer grouping mechanism. Of course, since the channels belong to each spectrum type show similar propagation and quality characteristics, we assume that these channels are homogeneous and have a same interference graph. Also, we present and proof the following theorem for obtaining the conflict graph related to the channels belong to each spectrum type:

Theorem 1. *To group the buyers belong to each spectrum type $t_i \in T$, only buyer grouping using the interference graph related to the smallest channel in this spectrum type can guarantee interference safe.*□

Proof: First, we sort the channels belong to spectrum type t_i in non-decreasing order. Let h_0 be the smallest channel. We create the interference graph $G(t_i|h_0)$ related to the smallest channel h_0 and consider each two arbitrary nodes (buyers) w_j and w_k with no-edge between them. We denote $TR(w_j|h_l)$ as the transmission range of w_j using the channel h_l. According to (4) we have:

$$TR(w_j|h_l) \leq TR(w_j|h_0)\,,\ TR(w_k|h_l) \leq TR(w_k|h_0)\,,\ \forall h_l \in t_i \tag{5}$$

It means that if two nodes w_j and w_k do not interfere with each other in h_0, they do not interfere in any other channel. Therefore, if we group the buyers using $G(t_i|h_0)$, interference safe will be guaranteed.

Also, we show that the buyer grouping using any other interference graph such as $G(t_i|h_l)$ so that $h_l > h_0$ can cause interference between buyers. We create the interference graphs $G(t_i|h_l)$ and consider two nodes w_j and w_k with no-edge between them, so we should have:

$$TR(w_j|h_l) + TR(w_k|h_l) < d_{jk} \tag{6}$$

Where d_{jk} denotes the distance between nodes w_j and w_k.

From (5) and (6), the following condition can be established for any channel $h_s < h_l$:

$$TR(w_j|h_l) + TR(w_k|h_l) < d_{jk} < TR(w_j|h_s) + TR(w_k|h_s) \tag{7}$$

It means that two nodes w_j and w_k do interfere with each other in h_s. So our claim holds.

In conflict graph formation, at first, we obtain the set of candidate buyers Q_i for each channel type $t_i \in T$. As mentioned before, some channel types provided by the primary users could be not available in the locations of some buyers. To meet the challenge of heterogeneity in spectrum availability, we involve availability matrix A in our calculation. So the candidate buyers set $\mathrm{Q_i}$ includes the buyers which have a demand for the channel type $\mathrm{t_i}$ and also, a channel with type $\mathrm{t_i}$ is available for them:

$$Q_i = \{w_k | w_k \in W \wedge a_{ki} = 1 \wedge d_{ki}^b = 1\} \tag{8}$$

After obtaining Q_i, we create the interference graph $G(t_i|h_0^i)$ related to channel type t_i on the candidate buyers set Q_i according to the adjacency matrix *C* such that h_0^i is the smallest channel with type t_i belong to the buyers in Q_i. According to the *theorem 1*, for channels with type t_i, only buyer grouping using $G(t_i|h_0^i)$ can guarantee interference safe.

B. Adaptive buyer grouping according to the graph type analysis

Several grouping algorithms have been proposed in the existing works to tackle the spectrum reuse problem such as MAX-IS, Greedy, Greedy-U and Random. According to our studies, these algorithms have different performances on different types of graph, including sparse, medium density and dense graphs. To meet the challenge of dependency to the graph type, in our proposed buyer grouping mechanism, we first analyze the density of a conflict graph of buyers and according to the results, select a suitable algorithm for buyer grouping relating to this graph.

1) Graph type analysis

For find out the type of a graph, we use the graph density parameter which is the ratio of the number of edges in the graph versus the number of edges in a complete graph with the same number of vertices [11]. On the other words, the density of a graph *G= (V, E)* measures how many edges are in set *E* compared to the maximum possible number of edges between vertices in set *V*. Since a simple undirected graph can have at most $\binom{|V|}{2}$ edges, so its density is calculated as:

$$Graph\ density = |E|/\binom{|V|}{2} = 2\,|E|/|V|(|V|-1) \tag{9}$$

In (9), $|E|$ is the number of edges in the interference graph $G(t_i|h_0^i)$ related to channel type $\mathrm{t_i}$whichis equal to the half of the number of non-zero cells in above-mentioned matrix $C = (c_{ijk})$.

Also, $|V|$ is the number of vertices in the interference graph $G(t_i|h_0^i)$ related to channel type t_i which is equal to the size of the candidate buyers set Q_i, i.e. $|Q_i|$. So we can calculate the density of the conflict graph related to channel type t_i as:

$$d\left(G(t_i|h_0^i)\right) = |E|/\binom{|V|}{2} = \sum_{i,j,k}|c_{ijk}|/|Q_i|(|Q_i|-1) \tag{10}$$

We can classify graphs according to their density $d(G)$ as:

Sparse graph: a graph with only a few edges. It means the graph density is much less than "1"; i.e., $d(G) \leq \alpha, s.t. \alpha \ll 1$.

Dense graph: a graph in which the number of edges is close to the maximal number of edges. It means the graph density is close to "1"; i.e., $d(G) \geq \beta, s.t. \beta \approx 1$.

Medium-density: a graph in which the density is neither much less than nor close to "1"; i.e., $\alpha < d(G) < \beta$.

2) Adaptive buyer grouping (ABG)

When a graph is sparser, the size of groups becomes larger and yields high spectrum reuse. Unfortunately, as we see in the following, since the bid of a buyer group is the minimum bid of the buyers in the group, a larger group size yields smaller group bid and can reduce the revenue of sellers as well as the probability of winning the buyer group. Therefore, we propose the following rule to increase both spectrum reuse and revenue of sellers in the auction mechanism:

- In dense graphs, we select the grouping algorithm who finds the largest buyer groups to increase the spectrum reuse.
- In sparse graphs, we select the grouping algorithm who finds the largish buyer groups with the high bid.
- In medium-dense graphs, since the size of groups could be medium, these algorithms have approximately similar performance so we can select each one of the aforementioned methods.

After forming the conflict graph $G(t_i|h_0^i)$ and selecting the suitable buyer grouping algorithm for every spectrum type t_i, we find the family of independent sets G_i^b for the interference graph $G(t_i|h_0^i)$ according to the selected algorithm. Therefore, each set in the G_i^b is a buyer group which can purchase a channel with type t_i. Also, since in our system model, we assume that each seller can supply multiple channels with different types, so we put the sellers which supply channels with common type in the same group. The sellers in each group G_i^s compete to each other to sell their channels with the common type t_i which this group is created based on it. The summary of the before-mentioned procedures is shown in *Algorithm 1*.

Algorithm 1: *Graph Creation, Analysis & Grouping (A, C, T, W, S, D^b, D^s)*

$\boldsymbol{G}^b = \emptyset$;
for all $t_i \in T$ ***do***
    Create the candidate buyer set for t_i: $Q_i = \{w_k | w_k \in W \wedge a_{ki} = 1 \wedge d_{ki}^b = 1\}$;
    Find h_0^i*: the smallest channel with type* t_i *belong to the buyers in* Q_i*;*
    Construct conflict graph $G(t_i|h_0^i)$ *on buyer set* Q_i *based on matrix* C_i*;*
    Calculate the density of $G(t_i|h_0)$ *according to (10);*
    Select a suitable grouping method based on the graph density $d\left(G(t_i|h_0^i)\right)$*;*
    Find independent sets $\boldsymbol{IS}_i$ *for* $G(t_i|h_0)$ *according to the selected method;*
    $\boldsymbol{G}_i^b = \boldsymbol{IS}_i$;
    $\boldsymbol{G}^b = \boldsymbol{G}^b \cup \boldsymbol{G}_i^b$;
    Form the seller group G_i^s *for* t_i: $G_i^s = \{s_k \in S | d_{ki}^s = 1\}$;
end for
return $\boldsymbol{G}^b, \boldsymbol{G}^s$;

C. Buyer grouping enhancement (EBG)

It can be seen in the often existing buyer grouping algorithms that the buyer grouping procedure is independent of the number of sellers. Consider the candidate buyers set Q_i for the channel type t_i, the family of buyer groups G_i^b and also the seller group G_i^s related to the channel type t_i. This is obvious that increasing the size of groups in G_i^b causes increasing the channel utilization in the auction; therefore we find the maximal independent set to increase the channel utilization. On the other hand, as the size of groups in G_i^b increases, the total number of groups in G_i^b decreases, since these groups compete for a known number $|G_i^s|$ of channels with type t_i, therefore, fewer buyer groups of G_i^b and consequently fewer sellers can win in the auction for this channel type specially if the buyer groups be less than the sellers number, so the seller satisfaction ratio may decrease [12, 13]. In contrast to this problem, we propose an enhancement algorithm for the buyer grouping mechanism in a way that improves both channel utilization and other auction metrics such as seller satisfaction. In the proposed algorithm, we first compare the number of buyer groups in G_i^b with the total number of channels with type t_i to be sold. Let N_{ti} be the total number of channels with type t_i to be sold. Define $BTSR_i$ be the number of buyer groups in G_i^b to the total number of channels with type t_i to be sold ratio, $BTSR_i = |G_i^b|/N_{ti}$. For each type t_i, buyer group enhancement algorithm first obtains $BTSR_i$ parameter. Then, while the number of buyer groups related to each channel type is less than N_{ti}, decompose the biggest buyer group in G_i^b (if it is bigger than $BTSR_i$) into two equal or semi-equal groups. If the size of the biggest buyer group in G_i^b be odd, so we have to decompose it into two groups with one buyer difference, we call them semi-equal groups. The new created groups are replaced with the prime group. The buyer grouping enhancement procedure is shown in *Algorithm 2*.

Algorithm 2: *Buyer-Group-Enhancement* ($\boldsymbol{N}_t$, $\boldsymbol{G}^b$)

```
G^be = ∅;
for all G_i^b ∈ G^b do
    BTSR_i = |G_i^b| / N_{t_i};
    gsize = |G_i^b|;
    while gsize < N_{t_i} do
        Find the group G_ik^b ∈ G_i^b, s.t ∀G_ij^b ∈ G_i^b, |G_ik^b| ≥ |G_ij^b|;
        if |G_ik^b| ≥ BTSR_i then
            Decompose G_ik^b and replace the result instead of G_ik^b in G_i^b;
            Increment gsize;
        elseif break;
    end while
end for
G^be = G^b;
return G^be;
```

D. Adaptive enhanced buyer grouping (AEBG)

In this method, first we use adaptive buyer grouping to assign adaptively the suitable algorithms for buyer grouping by analyzing the density of the related conflict graphs. Then we enhance the performance of the selected buyer grouping algorithm by using the buyer grouping enhancement method.

V. NUMERICAL RESULTS

In the previous section, we proposed the novel algorithms for spectrum buyer grouping that can solve the various challenges faced by the buyer grouping in a practical spectrum auction mechanism. In addition, we proposed three algorithms, including ABG, EBG, and AEDG, to overally improve the auction performance metrics in an adaptive fashion. It is important to note that the computation complexity which these proposed algorithms add to the common algorithms, who only find the maximum independent sets for buyer grouping, is too small and so negligible. In this section, we use simulation to evaluate the performance of these proposed algorithms on various auction metrics.

A. Channel dependent conflict graph formation

We use the Erdős–Rényi model [14] to generate random graphs with various densities for each spectrum type. In this model, a graph $G(n,p)$, where n is the number of nodes, is constructed by connecting nodes in such a way that each edge is included in the graph with probability p independent from every other edge. Therefore, the constructed graph $G(n,p)$ has on average $\binom{n}{2}p$ edges and the distribution of the degree of any particular vertex is binomial [15]. So we can result that the expected graph density D is equal to p:

$$E(D) = \binom{n}{2}p/\binom{n}{2} = p \tag{11}$$

Fig. 3 shows the graph density D versus the probability of connection p for 20-100 buyers and

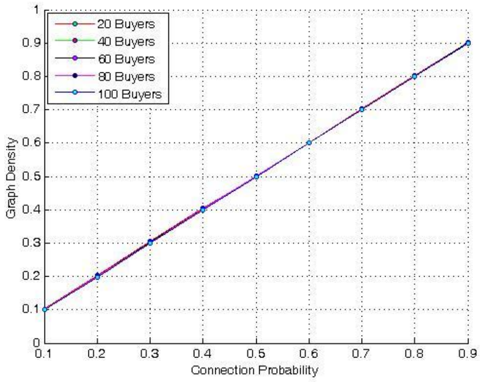

Fig. 3. Graph density vs. probability of connection for different buyer numbers

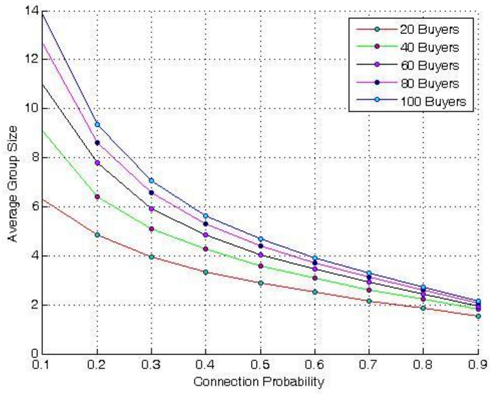

Fig. 4. Average group size vs. probability of connection

averaged over 100 runs according to formula (9). Fig. 3 confirms that formula (11) is established independently of the number of buyers.

Fig. 4 shows the relation between probability p and the average group size. Also, Fig. 5 shows the relation between probability p and the number of groups. As shown in Fig. 4 and Fig. 5, increasing in the probability p cause the graph to be denser and so the average group size to reduce while the number of groups to rise. Hence, connection probability p is a suitable parameter to generate random graphs with various densities and evaluate the performance of the proposed algorithms on various auction metrics in the following sub-sections.

B. Adaptive buyer grouping

In this step, we use the following algorithms for buyer grouping: No Grouping (NG), MIS, Greedy, Greedy-U and Adaptive. The number of sellers and the number of spectrum buyers are set to 10 and 100, respectively, and the probability of connection varies from 0.1 to 0.9. Also, the results are averaged over 100 runs. We compare the average group size as a function of number of buyers when using these algorithms in Fig. 6

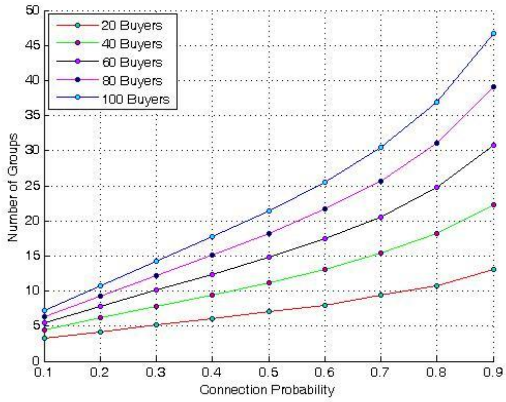

Fig. 5. Number of groups vs. probability of connection

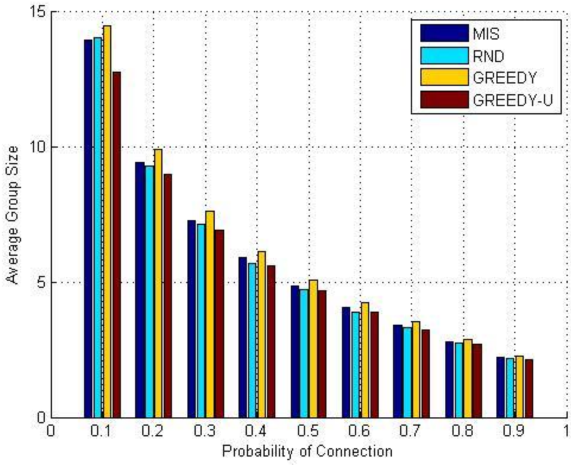

Fig. 6. The comparison of average group size in grouping algorithms

Fig. 6 illustrates that Greedy and Greedy-U have the largest and the smallest average group size, respectively. However, as stated before and shown in Figs. 7-9, although larger group size can increase spectrum reuse but it can reduce the seller satisfaction ration as well as the number of traded channels

Furthermore, Figs. 7-9 illustrate that by changing the type of graph via changing the connection probability *p*, the performance of different algorithms changes. In this regards, in a sparse graph, Greedy-U has the highest performance on the number of traded channels and the seller satisfaction ratio metrics, while, in contrast, Greedy-U shows the lowest performance in a dense graph on these metrics. Also, we see that Greedy works just opposite of the Greedy-U on these performance metrics.

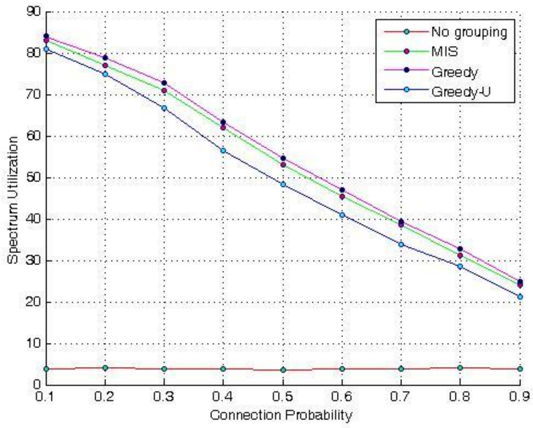

Fig. 7. The comparison of the spectrum utilization in grouping algorithms

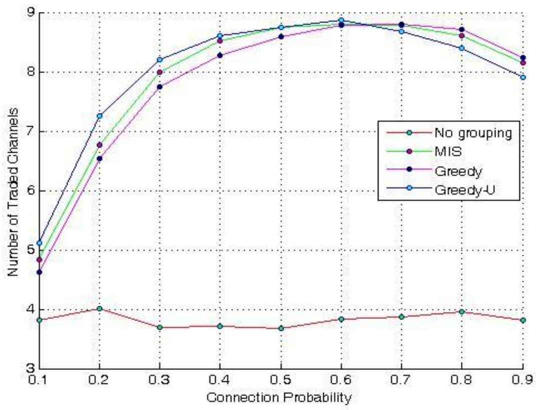

Fig. 8. The comparison of the number of traded channels in grouping algorithms

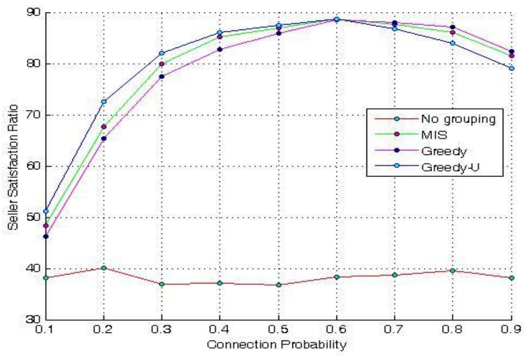

Fig. 9. The comparison of the seller satisfaction ratio in grouping algorithms

As a result, as mentioned before, in adaptive buyer grouping algorithm (ABG), we choose an appropriate grouping algorithm adaptively according to the type of graph information. Of course, there are different definitions for distinguishing between different types of graphs. For instance, authors in [16] distinguish between sparse, normal (or medium-dense) and dense graphs as follow: (sparse: $|E| < |V|$, normal: $|V| \leq |E| < 3|V|$ and dense: $|E| \geq 3|V|$). For adaptive grouping, we use

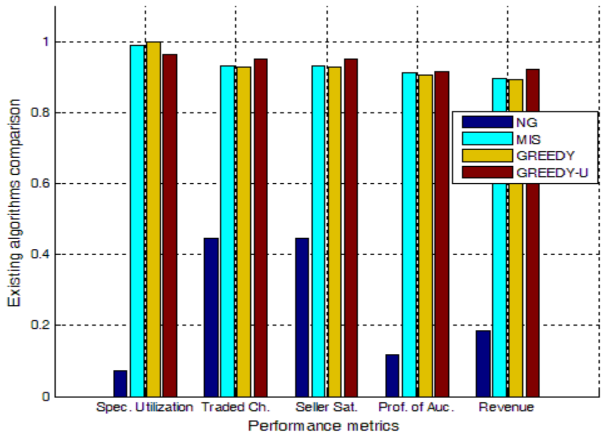

Fig. 10. The comparison of the existing algorithms

following method: in a dense graph ($p \geq 0.6$), we apply a grouping algorithm which finds the largest buyer groups to increase the spectrum reuse, i.e. Greedy. In reverse, in a sparse graph ($p < 0.6$), we apply a grouping algorithm which finds the largish buyer groups with the high bid, i.e. Greedy-U or even MIS depends on the importance of the revenue or the number of traded channels. We consider the auction in which the buyers' bids and the sellers' reserve prices are randomly distributed over [0, 1] and [0, 2], respectively. Since sellers may expect its channel to be sold to more than one buyer due to spectrum reusability, we set the sellers' reserve prices higher than the buyers' bids. Also, the number of spectrum buyers and the number of sellers are set to 100 and 10, respectively. All the results on performance metrics are averaged over connection probability p in duration $(0, 1)$ and 100 runs. Fig. 10 shows the performance metrics results achieved by applying the existing grouping algorithms, including Greedy, Greedy-U and MIS, to the auction. As before-mentioned, these grouping algorithms attempt to find the maximal independent sets of buyers in the corresponding interference graph. So the main focus of them is to increase the spectrum utilization rather than NG. However, the existing algorithms also somewhat improve the other performance metrics. It is important to note that in the existing algorithms, with every increase in the spectrum utilization there will be a decrease in the other performance metrics, as shown in Fig. 10. Another important point is that all the existing algorithms lead to a significant improvement in spectrum utilization compared to the NG. So we are going to improve the other performance metrics.

In order to improve the other performance metrics, ABG adaptively applies Greedy and Greedy-U algorithms for dense graphs and sparse graphs, respectively. Fig. 11 compares the performance of the proposed algorithms, including ABG, EBG and AEBG, with MIS algorithm.

From Fig. 11, we see that ABG produces the higher number of traded channels, seller satisfaction ratio, profit of auctioneer and auction revenue in comparison with MIS.

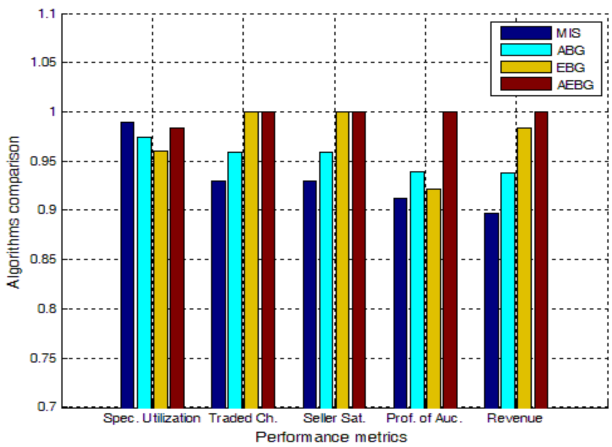

Fig. 11. The comparison of the proposed algorithms with MIS

C. *Buyer grouping enhancement*

Another proposed algorithm to increase the auction performance metrics was EBG which modified the created buyer groups in respect to $BTSR$. In this section, we compare the performance of EBG with MIS. The simulation settings are similar to the previous part. The results also can be seen in Fig. 11. The simulation results illustrate that using EBG in the auction also improves the number of traded channels, seller satisfaction ratio, profit of auctioneer and auction revenue metrics in comparison with MIS.

D. *Adaptive enhanced buyer grouping*

As illustrated in the previous subsections, each of the ABG and EBG methods improved the number of traded channels, seller satisfaction ratio, profit of auctioneer and auction revenue metrics in comparison with MIS from two different aspects. As mentioned before, in AEBG, we jointly apply ABG and EBG methods to the auction. On the other words, we apply the buyer grouping enhancement method to the groups which achieved by the adaptive buyer grouping algorithm. So we expect that AEBG to obtain the two previous-mentioned improvements related to ABG and EBG. Fig. 11 also shows the results achieved using AEBG. As illustrated in Fig. 11, AEBG not only achieves almost similar spectrum utilization in comparison with MIS, but also significantly improves the other important performance metrics.

VI. RELATED WORKS

Auctions have been widely studied in recent years as one of the best-known market-based solutions to improve the efficiency of spectrum use [7][20][22-25]. Truthfulness is the most critical property of auction scheme. An auction could be vulnerable to market manipulation and produce very poor outcomes if this property is not guaranteed [21]. Although the first efforts in the scope of spectrum auctions had not considered truthfulness, such as [17-19], authors in [20] propose the first truthful spectrum auction design. On the other hand, compared with traditional auctions, a significant difference of spectrum auctions is the property of spectrum that can be spatially reused concurrently [3]. The first truthful double auction design with spectrum reuse, called TRUST, is proposed in [7].

To handle spectrum reusability, a buyer grouping procedure has been applied in many existing spectrum auction schemes, in which the buyer's interference conditions are modeled as conflict graph [22-25]. Buyer grouping problem can be transformed into the problem of finding chromatic number or maximum independent set of a graph, which is NP-hard [26]. Several approximate algorithms have been proposed to tackle the spectrum reuse problem [4, 5]. It is important to note that almost none of the proposed algorithms for spectrum buyer grouping have been specifically designed for spectrum allocation. However, buyer grouping in a practical spectrum auction has its own challenges. In this paper, first we illustrated the challenges of buyer grouping in a practical spectrum auction mechanism and then, proposed the novel algorithms for spectrum buyer grouping to solve these challenges.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we illustrated the challenges of buyer grouping in a practical spectrum auction. We proposed the novel algorithms for spectrum buyer grouping that solve the related challenges. We used channel dependent interference graphs to tackle with the problem of frequency-variant channel characteristics caused by spectrum frequency heterogeneity. We designed our algorithms so that maintain truthfulness which is essential to resist market manipulation and ensure auction fairness and efficiency. By extensive simulations, we have shown that our proposed algorithms can not only solve the challenges caused by radio spectrum properties but also provide good performance on various auction metrics.

As for future work, we are going to design a double auction mechanism in heterogeneous environment and investigate the effectiveness of the proposed algorithms in the auction. Another possible direction is to extend the proposed algorithms to be resistant to collusion.

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