Adaptive strategy-proof double auction mechanism for heterogeneous spectrum allocation

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Abstract: Spectrum auctions are one of the best-known solutions to improve the efficiency of spectrum use. However, there can be many challenges in the design of a practical spectrum auction. Heterogeneity is one of the most major challenges. Unfortunately, most of the existing auction designs either do not take into account the various aspects of heterogeneity or assume only the scenario where each seller supplies one distinct channel and each buyer wishes to buy merely one channel. The authors propose a spectrum auction mechanism which considers the various aspects of heterogeneity as well as multi-channel purchasing. They prove that the auction design preserves three important economic aspects including truthfulness, budget balance and individual-rationality. Moreover, most of the existing works only provide the bidders a simple demand format. Their auction mechanism enables bidders to use diverse demand formats. Furthermore, they propose some novel adaptive grouping algorithms to improve the auction’s performance. The simulation results demonstrate good performance of the proposed algorithms on various auction metrics.

1 Introduction

The radio-frequency spectrum is one of the most expensive scarce resources in the wireless communication domain. However, spectrum occupancy measurements have indicated that a significant amount of the licensed spectrum remains unused in many places much of the time [1], so the traditional exclusive licencing spectrum policy leads to the low efficiency of spectrum utilisation. Spectrum auctions are one of the best market-based solutions to improve the efficiency of spectrum use. Heterogeneity is one of the most major challenges in spectrum auctions. Unfortunately, most of the existing designs either do not take into account the various aspects of heterogeneity [2–4] or assume only the scenario where each seller supplies one distinct channel and each buyer wishes to buy merely one channel [5]. Moreover, most of the existing works only provide the bidders a simple demand format while in a well-designed auction, buyers should be able to express different preferences for different spectrums by flexibly requesting channels they would like to obtain [4, 6]. We consider various aspects of heterogeneity as well as multi-channel purchasing in our proposed auction mechanism and enables bidders to use diverse demand formats. In summary, the main contributions of this paper are as follows:

- Our proposed auction mechanism supports spatial reusability as well as various heterogeneity aspects in where each seller (or buyer) can sell (or buy) multiple channels.
- Since buyer grouping affects many auction performance metrics, we propose some adaptive algorithms to improve the existing algorithms.
- To the best of our knowledge, our proposed double auction mechanism is the first one for heterogeneous spectrum transactions which enables bidders to use flexible demand formats.
- We design our spectrum auction mechanism so that it maintains the critical auctions properties: truthfulness, individual-rationality and budget balance.

The rest of this paper is organised as follows. Problem model and related works are reviewed in Section 2. Detailed description of our auction mechanism is presented in Section 3. We prove the economic-robustness of the proposed auction scheme by theoretical analysis in Section 4. We give simulation results in Section 5 and the conclusions in Section 6.

2 Problem model and related works

2.1 Problem model

We assume the scenario that N secondary service providers (called buyers) want to buy spectrum resources of K different types from M spectrum owners (called sellers). To provide this purpose, a single-round double auction with one auctioneer, M sellers and N buyers is held. Let $S = \{s_1, s_2, \ldots, s_M\}$ denotes the set of sellers, $W = \{w_1, w_2, \ldots, w_N\}$ denotes the set of buyers and $T = \{t_1, t_2, \ldots, t_K\}$ denotes the set of spectrum types. Moreover, let the channel set $H$ represents the set of all channels contributed by the sellers in the auction. We assume that each seller can contribute multiple channels of the same spectrum type and each buyer can obtain multiple channels of the same spectrum type. We allow buyers to express their preferences over each spectrum type separately. In addition, we consider that the auction supports flexible bidding requests for buyers described as follows:

- Range request (RR): If buyer $w_n$ requests $d_{nk}$ channels of spectrum type $t_k$, this buyer will obtain $x$ channels, such that $x \in [0, d_{nk}]$.
- Strict request (SR): If buyer $w_n$ requests $d_{nk}$ channels of spectrum type $t_k$, this buyer will obtain either all $d_{nk}$ channels or nothing.

Each buyer $w_n$ submits a $1 \times 4$ bid vector $B_n^k = (t_k, d_{nk}, b_{nk}^s, BT_{nk}^s)$. It means that buyer $w_n$ requests for $d_{nk}$ channels of type $t_k$ with per-channel price $b_{nk}^s$ and bidding type $BT_{nk}^s$, where $BT_{nk}^s$ can be selected from the above-mentioned bidding type set $BT = \{RR, SR\}$. Moreover, each seller $s_m$ submits a $1 \times K$ bid vector $B_m^k = (b_{m1}^k, b_{m2}^k, \ldots, b_{mk}^k)$ for its available spectrum types, where $b_{mk}^k$ means the quoted price offered by the seller $s_m$ for the type $k$. We denote $N \times K$ matrix $B^s$ and $M \times K$ matrix $B^t$ as the bid matrices of all buyers and all sellers, respectively. We represent the true valuation of seller $s_m$ and buyer $w_n$ for the type $t_k$ by $v_{mk}^w$.
and $v_{ik}$, respectively. In the auction, the auctioneer determines the per-channel payment $p_{ik}$ for seller $s_{ik}$ if it wins a channel of type $t_i$ and the per-channel price $p_{ik}$ that buyer $w_i$ should pay if it wins a channel of this type.

2.2 Related works

Truthfulness is the most important property of an auction mechanism. Zhou et al. [6] proposed the first truthful single-sided spectrum auction mechanism. The first truthful double auction design with spectrum reuse, called TRUST, has been proposed in [7]. However, the early works in the field of spectrum auction did not consider the multi-channel scenario. Zhang et al. [2]; Wu and Vaidya [3] proposed some strategy-proof single-sided multi-channel auction mechanisms. On the other hand, heterogeneity is one of the most important challenges in the spectrum auctions. Unfortunately, most of the existing designs, such as the above-mentioned works, did not take into account heterogeneity in their auction mechanisms. Some aspects of heterogeneity in the scope of spectrum auctions were studied in [5, 8, 9]. However, these works only considered some aspects of heterogeneity or assumed only the scenario where each seller supplies one distinct channel and each buyer wishes to buy merely one channel [5]. Furthermore, most of the existing works only provide the bidders a simple demand format. Wang et al. [4] and Feng et al. [6] allowed flexible spectrum bidding, but they did not consider spectrum heterogeneity. To the best of our knowledge, our proposed design is the first truthful multi-channel double auction mechanism that addresses the various aspects of heterogeneous, supports spatial reuseability and enables buyers/sellers to use flexible demand formats and buy/sell multiple channels.

3 Auction design

In this section, we propose our auction mechanism for the allocation of heterogeneous spectrum. Our proposed auction scheme includes the following steps.

3.1 Channel-dependent conflict graph formation

To handle spectrum reusability, a buyer grouping step has been applied in many existing spectrum auction schemes, in which the buyer’s interference conditions are modelled as conflict graph. Since channel characteristics are dependent on the frequency used, we can expect that the shape of the interference regions will be channel dependent [10], so we use channel-dependent interference graphs. Of course, since the channels of each spectrum type show similar propagation and quality characteristics, we assume that these channels are homogeneous and have a same interference graph. Moreover, we present and prove the following theorem for obtaining the conflict graph related to the channels of each spectrum type.

Theorem 1: To group the buyers of each spectrum type $t_i$ in $T$, only the buyer grouping using the interference graph related to the smallest channel of this type in the channel set $H$ can guarantee interference-free transmission.

Proof: Let $H_i$ be the set of all channels of type $t_i$ in the channel set $H$ and $h_0$ be the channel having the smallest frequency channel belonging to $H_i$. We name it the smallest channel of type $t_i$. Moreover, we define $G(t_i|h_0)$ as the interference graph of the buyers based on the frequency channel $h_0$. Now we create the interference graph $G(t_i|h_0)$ and consider each two arbitrary nodes (buyers) $w_i$ and $w_j$ with no-edge between them. We denote $TR(w_i|h_0)$ as the transmission range of $w_i$ using the channel $h_0$, which is defined as the maximum distance from node $w_i$ on the channel $h_0$ where connectivity with another node exists and a transmitted signal from node $w_i$ on the channel $h_0$ can be successfully received. It is clear that as the frequency increases, the path loss also increases [11], and therefore the transmission range decreases.

It means that

$$TR(w_i|h_0) \leq TR(w_i|h_t), \quad \forall w_i \in W, \quad \forall h_t, h_t \in H_i, \quad s.t. \quad h_t \geq h_0$$

(1)

Hence, for every frequency channel $h_i$ belonging to $H_i$, since $h_0$ is the smallest frequency channel in $H_i$, we have $h_0 \leq h_i$ and so

$$TR(w_i|h_i) \leq TR(w_i|h_0), \quad TR(w_i|h_j) \leq TR(w_i|h_0), \quad \forall h_j \in H_i$$

(2)

It means that if $w_i$ and $w_j$ do not interfere with each other in $h_0$, they do not interfere in any other channel belonging to $H_i$. Therefore if we group the buyers using $G(t_i|h_0)$, interference-free transmission will be guaranteed.

Moreover, we show that the buyer grouping using any other interference graph such as $G(t_i|h_j)$ so that $h_j > h_0$ can cause interference between buyers. We create the interference graphs $G(t_i|h_j)$ and consider two nodes $w_i$ and $w_k$ with no-edge between them, so we should have

$$TR(w_i|h_i) + TR(w_k|h_j) < d_{jk}$$

(3)

where $d_{jk}$ denotes the distance between $w_i$ and $w_k$.

From (1) and (3), the following condition can be established for a channel $h_i < h_0$,

$$TR(w_i|h_i) + TR(w_j|h_k) < d_{jk} < TR(w_i|h_j) + TR(w_k|h_j)$$

(4)

It means that $w_i$ and $w_k$ do interfere with each other in $h_i$. Therefore our claim holds.

Let $A = (a_{ij})$ be an $N \times K(0, 1)$-matrix which represents the buyers’ channel type availability such that $a_{ij} = 1$ indicates that the type $t_i$ is available for the buyer $w_j$. Moreover, let $C = (c_{ij})$ be a $K \times N(0, 1)$-matrix which represents the conflict relationships between buyers for each channel type such that $c_{ij} = 0$ means that buyers $w_i$ and $w_j$ do not interfere with each other in type $t_i$. Finally, let $N \times K$ matrix $D^a$ and $M \times K$ matrix $D^b$ represent the $(0, 1)$-demand matrices of buyers and sellers for each type that belongs to $T$, respectively. For instance, $d_{ij}^a = 1$ means that the buyer $w_i$ wants to buy a channel of type $t_j$.

In conflict graph formation, at first, we obtain the set of candidate buyers $Q_i$ for each $t_i \in T$. The set $Q_i$ includes the buyers which have a demand for the channel type $t_i$, and also a channel of type $t_i$ is available for them

$$Q_i = \{ w_i | w_i \in W \wedge a_{ik} = 1 \wedge d_{ik}^a = 1 \}$$

(5)

Then, we create the interference graph $G(t_i|h_0)$ related to the type $t_i$ on the set $Q_i$ according to the adjacency matrix $C$ such that $h_0$ is the smallest channel of type $t_i$ in the channel set $H$.

3.2 Adaptive grouping

Since buyer grouping affects many auction performance metrics, in the following we propose some novel adaptive algorithms to improve the existing algorithms. In adaptive buyer grouping (ABG), selecting the suitable buyer grouping between the existing is done according to the graph density analysis. The enhanced buyer grouping (EBG) algorithm increases the auction performance metrics through modifying the created buyer groups with respect to the buyer-to-seller ratio (BTSR). Meanwhile, the adaptive EBG (AEBG) algorithm jointly applies ABG and EBG methods to the auction.

In this step, we propose three novel adaptive algorithms for spectrum buyer grouping.

3.2.1 ABG algorithm: Several grouping algorithms have been proposed to tackle the spectrum reuse problem. However, as it will
be studied in Section 5, these algorithms have different performances on different types of graphs, including sparse, medium density and dense graphs. Therefore, in the proposed ABG algorithm, we first analyse the density of a conflict graph of buyers and according to the results, select a suitable algorithm for buyer grouping relating to this graph. To find out the type of a graph, we use the graph density parameter which is the ratio of the number of edges in the graph against the number of edges in a complete graph with the same number of vertices. Since a simple undirected graph can have at most \( \frac{|V|^2}{2} \) edges, so its density is calculated as

\[
\text{Graph density} = \frac{|E|}{\binom{|V|}{2}} = \frac{2|E|}{|V|(|V| - 1)} \quad (6)
\]

In (6), \(|E|\) is the number of edges in graph \( G(t_i|h_0) \) related to the type \( t_i \), which is equal to the half of the number of non-zero cells in above-mentioned matrix \( C = (c_{ijk}) \). Moreover, \(|V|\) is the number of vertices in \( G(t_i|h_0) \) related to the type \( t_i \), which is equal to the size of set \( Q_i \), that is, \(|Q_i|\). Therefore we can calculate the density of the conflict graph related to the type \( t_i \) as

\[
d(G(t_i|h_0)) = \frac{|E|}{\binom{|V|}{2}} = \frac{\sum_{i,j,k} |c_{ijk}|}{|Q_i|(|Q_i| - 1)} \quad (7)
\]

Algorithm 1: Graph Creation & Adaptive Grouping

\[
G^b = \emptyset; \\
\text{for all } t_i \in T \text{ do} \\
\quad \text{Create the candidate buyer set } Q_i \text{ for } t_i; \\
\quad \text{Find } h_0^i; \\
\quad \text{Construct conflict graph } G(t_i|h_0^i); \\
\quad \text{Calculate } d(G(t_i|h_0^i)); \\
\quad \text{Select a suitable grouping method based on } d(G(t_i|h_0^i)); \\
\quad \text{Find independent sets } IS_i \text{ for } G(t_i|h_0^i); \\
\quad G_i^b = IS_i; \\
\quad G^b = G^b \cup IS_i; \\
\quad \text{Form the seller group } G_i^s \text{ for } t_i: G_i^s = \{s_k \in S | d_{ks}^i = 1\}; \\
\text{end for} \\
\text{return } G^b, G^s;
\]

Fig. 1 Graph creation and adaptive grouping

Summary of the before-mentioned steps is shown in Algorithm 1 (see Fig. 1).

3.2.2 EBG algorithm: In the majority of the existing algorithms, buyer grouping is independent of the number of sellers. In our proposed EBG algorithm, we compare the number of buyer groups in \( G_i^b \) with the total number of the channels of type \( t_i \) to be sold. Let \( N_t \) be the total number of the channels of type \( t_i \) to be sold. Define BTSR, be the number of the buyer groups in \( G_i^b \) to the total number of the channels of type \( t_i \) to be sold ratio, \( \text{BTSR}_i = \frac{|G_i^b|}{N_t} \). For each type \( t_i \), EBG first obtains BTSR parameter. Then, while the number of buyer groups related to each channel type is less than \( N_t \), decompose the biggest buyer group in \( G_i^b \) (if it is bigger than BTSR) into two equal or semi-equal groups. Algorithm 2 (see Fig. 2) shows the EBG procedure.

3.2.3 AEBG algorithm: In this method, first we apply ABG to assign adaptively the suitable algorithms for buyer grouping. Then, we enhance the performance of the selected algorithm using EBG.

3.3 Winner selection

After grouping, we can consider all buyer groups in \( G_i^b \) as the supper buyers which want to purchase the channels of type \( t_i \). Moreover, we

Algorithm 2: Buyer-Group-Enhancement

\[
G^{be} = \emptyset; \\
\text{for all } G_i^b \in G^b \text{ do} \\
\quad \text{BTSR}_i = \frac{|G_i^b|}{N_t}; \\
\quad gsize = |G_i^b|; \\
\quad \text{while } gsize < N_t \text{ do} \\
\quad \quad \text{Find the group } G_{ik}^b \in G_i^b, \text{ s.t. } V(G_{ik}^b) \subseteq G_i^b, |G_{ik}^b| \geq |G_{ij}^b|; \\
\quad \quad \text{if } |G_{ik}^b| \geq \text{BTSR}_i \text{ then} \\
\quad \quad \quad \text{Decompose } G_{ik}^b \text{ and replace the result instead of } G_{ik}^b \text{ in } G_i^b; \\
\quad \quad \quad \text{Increment } gsize; \\
\quad \quad \text{elseif } \text{break; } \\
\quad \text{end while} \\
\text{return } G^{be} = G^b; 
\]

Fig. 2 Buyer-group-enhancement
consider all sellers in $G_i^t$ as the sellers which want to sell the channels of this type. Let $G_j^t$ represent the $j$th buyer group in $G_i^t$ which can purchase a channel of type $t_i$. To determine the bid of each buyer group $G_j^t$ in $G_i^t$, we use uniform pricing rather than discriminatory pricing to achieve individual-rationality and truthfulness in the auction. We present the following theorem for obtaining the buyer group bids.

**Theorem 2:** To achieve individual-rationality and truthfulness in the auction, the group bid $\pi_i^k$ for the channel type $t_i$, under per-group uniform pricing, should not be exceeded than the product of the smallest per-channel buyer bid and the number of buyers, in that group.

**Proof:** From individual-rationality, each buyer $w_n$ must not be charged more than its bid for the type $t_i$, $b_{in}^t$. Moreover, under uniform pricing, the auctioneer charges equally the buyers in the same group. Using these conditions and considering the McAfee’s design [13], the theorem can be easily proved. Since a similar proof was presented in [7] and because of space limitations, we omit the proof. □

In winner determination phase, the auctioneer first calculates the per-channel buyers group bid $\pi_i^k$ relating to the buyer group $G_j^t$ for the channel type $t_i$, according to Theorem 2 as

$$\pi_i^k = \min\{b_{in}^t | w_i \in G_j^t \}$$

Moreover, the demand interval of the group $G_j^t$ for the type $t_i$ is calculated as

$$gd_i^k(\min) = \max\{d_{in}^t | w_i \in G_j^t \land BT_{in}^t = SR\}$$

$$gd_i^k(\max) = \max\{d_{in}^t | w_i \in G_j^t \}$$

According to (9), minimum demand of a buyer group is the maximum demand with type of SR in that group. Then, for each $t_i$, the auctioneer sorts the related buyer groups’ bid list $\pi_i$ in non-increasing order. Of course, auctioneer tries to assign the maximum possible number of channels for group $G_j^t$ in the demanded interval of $[gd_i^k(\min), gd_i^k(\max)]$. We represent the sorted bids and their related buyer groups by $\pi_i^{\text{bs}}$ and $G_i^{\text{bs}}$, respectively. Moreover, the auctioneer sorts the seller bid list $\pi_s^{\text{bs}}$ corresponding to the type $t_i$ in non-decreasing order. We represent the sorted bids and their related sellers by $\pi_s^{\text{bs}}$ and $G_i^{\text{bs}}$, respectively. We define $k_i$ as the last profitable trade type for $t_i$ as

$$k_i = \arg \max_{1 \leq \text{length}(\pi_i^{\text{bs}})} \pi_i^{\text{bs}} \geq \pi_s^{\text{bs}}$$

Thus, the preliminary auction winners for the channels of type $t_i$ are the buyers belonging to the first $k_i - 1$ buyer groups in $G_i^{\text{bs}}$ and the first $k_i - 1$ sellers in $G_i^{\text{bs}}$. We calculate the number of winning channels of type $t_i$ as

$$\text{wc}_{t_i} = \sum_{m=1}^{k_i-1} d_{mi}$$

To determine the final winners of the auction, the auctioneer tries to assign these wc, channels from the set of preliminary winning sellers to the set of preliminary winning buyer groups according to their demand intervals and with respect to their orders in the lists. If the number of available channels $ac_n$, for a buyer group $G_j^t$ is less than its minimum demand $gd_i^k(\min)$, auctioneer removes the buyer group $G_j^t$ from the winning buyer list, else auctioneer adds this buyer group to the final winning buyer group list and allocates whatever is possible, $gd_i^k = \min\{ac_n, gd_i^k(\max)\}$. At last, the winning buyers for the channels of type $t_i$ are the members of the final winning buyer groups. Moreover, the winning sellers for this channel type are the preliminary winning sellers who have assigned a channel of type $t_i$ to a final winning buyer group. After determining the winning buyer groups $G_j^t$ and allocating the possible channels $\pi_i$ to them, auctioneer assigns, for every winning buyer group $G_j^t$, $\pi_i^k$ channels to the members of $G_j^t$ in such a way that each buyer $w_n$ with bidding type of SR is assigned equal to his demand $d_{in}^t$, but each buyer $w_n$ with bidding type of RR is assigned $d_{in}^t = \min(d_{in}^t, \pi_i)$ channels.

### 3.4 Pricing

After the winner selection, the auctioneer calculates the price of the winning buyers and the payment of the winning sellers for each type $t_i$. To maintain truthfulness, the auctioneer pays, per channel, each winning seller belonging to the winning seller set $G_j^t$ by the next seller’s bid after the last winning seller in $\pi_i^k$, $\pi_i^k$, which can be the $k_i$th seller’s bid in $\pi_i^k$. Of course, if $(k_i - 1)$th seller has been removed from the winning seller list, $\pi_i^k$ is the biggest bid of the sellers in which have been removed from the preliminary winning seller list. Moreover, the auctioneer charges, per channel, each winning buyer group belonging to the family of winning buyer set $G_j^t$ by the next buyer group’s bid after the last winning buyer group in $\pi_i^k$, $\pi_i^k$, which can be the $k_i$th buyer group’s bid in $\pi_i^k$. Similarly, if $(k_i - 1)$th buyer group has been removed from the winning buyer group list, $\pi_i^k$ is the smallest bid of the buyer groups in which have been removed from the preliminary winning buyer group list. Finally, this per-channel buyer’s group price $\pi_i$ is shared by all members in each winning buyer group, so per-channel price $P_i$ and total charge $P_i$ to a buyer $w_n$ belonging to the winning buyer group $G_j^t$ for the channel type $t_i$ are

$$P_i = P_i / |G_j^t|, \quad P_i = d_{in}^t P_i / |G_j^t|$$

Moreover, no charges and no payments are made to loosing buyers and loosing sellers, respectively. The detailed procedure for winner selection and pricing is shown in Algorithm 3 (see Fig. 3).

### 4 Proofs of economic properties

A double auction is truthful if and only if no seller $m$ or buyer $n$ can improve its own utility by bidding untruthfully ($B_{mk} \neq v_{mk}$ or $B_{nk} \neq v_{nk}$). $U_{mk}(v_{mk}) \geq U_{mk}(B_{mk})$, $U_{nk}(v_{nk}) \geq U_{nk}(B_{nk})$, $\forall v_{mk} \in S$, $\forall v_{nk} \in W$, $\forall m \in T$. (14)

A double auction is individual rational if no winning seller is paid less than its bid and no winning buyer pays more than its bid

$$P_{mk} \geq B_{mk}, \quad P_{nk} \leq B_{nk}, \quad \forall v_{mk} \in S, \quad \forall v_{nk} \in W, \quad \forall m \in T$$

A double auction is budget balanced if the auctioneer’s profit $\Phi$ is non-negative

$$\Phi = \sum_{1 \leq m \leq n} P_{mk} - \sum_{1 \leq n \leq m} P_{mk} \geq 0$$

In the following, we prove that our auction mechanism has the above-mentioned properties.

**Theorem 3:** The auction mechanism is individually rational.

**Proof:** According to (8), for $\forall t_i \in T$ and $\forall v_{nk} \in G_i^{\text{bs}}$, we have

$$\pi_i = \min\{b_{in}^t | w_i \in G_j^t \}, |G_j^t| \leq b_{in}^t, |G_j^t|$$

If $G_j^t \in G_i^{\text{bs}}$, combining (13) and (17) and according to the winner
Algorithm 3: Winner Selection and Pricing

for all $G^b_i \in \mathcal{G}^b$ do
  for all $G^s_i \in \mathcal{G}^s$ do
    $\pi^b_{ij} = \min\{b^b_{k|}, |G^b_{ij}|\};$
    $\pi^s_{ij} = \min\{b^s_{k|}, |G^s_{ij}|\};$
  end for
  $\pi^b_{n} = \text{sorted } \pi^b_{i} \text{ in non-increasing order};$
  $\pi^s_{n} = \text{sorted } \pi^s_{i} \text{ according to } \pi^b_{n};$
end for

for all $G^s_i \in \mathcal{G}^s$ do
  $\pi^s_{i} = \{b^s_{k|}, S^s_k \in \mathcal{G}^s_i\};$
  $\pi^s_{n} = \text{sorted } \pi^s_{i} \text{ in non-decreasing order};$
  $\mathcal{G}^s_i = \text{sorted } \pi^s_{i} \text{ according to } \pi^s_{n};$
end for

for all $t_i \in \mathcal{T}$ do
  $k_i = \text{argmax}_{l \leq \min\{\text{length}(\pi^b_{j}), \text{length}(\pi^s_{j})\}} \pi^b_{n};$

  Select the first $k_i - 1$ buyer groups in $\mathcal{G}^b_i$ as the preliminary winning buyers $G^b_{i,wi};$

  Select the first $k_i - 1$ sellers in $\mathcal{G}^s_i$ as the preliminary winning sellers $G^s_{i,si};$

  Find final winners from the preliminary winning lists & allocate channels to them:
    if $acn_i \geq gd^b_{ij}(\min)$ then $G^b_{i} = G^b_{i,wi} \cup G^b_{i,ij}; gd^b_{ij} = \min(acn_i, gd^b_{ij}(\max));$
  for all $w_k \in \mathcal{G}^b_{i,wi} \cap \mathcal{G}^b_{i,ij}$ do
    if $\mathcal{B}^b_{ki} = \mathcal{R}$ then assign $a^b_{ki} = \min(d^b_{ki}, g^s_{ki}) \text{ channels};$
    if $\mathcal{B}^b_{ki} = \mathcal{S}$ then assign $a^b_{ki} = d^b_{ki} \text{ channels};$
  end for

  if $G^s_{i} \text{ has assigned a channel to } G^b_{i,wi} \text{ then } G^s_{i} = G^s_{i,si} \cup G^s_{i,ij};$

  $P^b_i = \frac{\pi^b_{n}}{|G^b_{i}|}; P^s_i = \frac{\pi^s_{n}}{|G^s_{i}|};$

  for all $s_k \in \mathcal{G}^s_{i,si}$ do $P^p_{ki} = s^s_{ki};$ end for

  for all $G^s_i \in \mathcal{G}^s_{i,si}$ do
    for all $w_k \in \mathcal{G}^b_{i,wi} \cap \mathcal{G}^b_{i,ij}$ do
      $P^p_{ki} = \frac{s^s_{ki}}{|G^s_{i}|} \text{ end for}$
  end for

end for

Fig. 3 Winner selection and pricing

determination and pricing algorithm, we obtain

$$\pi^b_{i} \leq \frac{b^b_{k|}}{|G^b_{i}|} \text{ and } \frac{b^s_{k|}}{|G^s_{i}|} = b^s_{m} (18)$$

Therefore the per-channel clearing price $P^b_{mi}$ for the winning buyer $w_m$ and the channel type $t_i$ is no more than its per-channel bid for this channel type, $b^b_{m}.$

Moreover, according to Fig. 3 and the sorting method, for $\forall s_m \in \mathcal{G}^s_{i,si}$ and $\forall t_i \in \mathcal{T},$ we have

$$b^s_{m} \leq \pi^s_{i} = P^p_{mi} = P^s_{mi} (19)$$

It means that no winning seller will receive less than its request and so the theorem holds.

Theorem 4: The auction mechanism is budget-balanced.

Proof: According to (11), for each winning group related to the type $t_i$, we have

$$\pi^b_{ni} \geq \pi^s_{ni}, \ |G^b_{ni}| = |G^s_{ni}| (20)$$

From (20), we obtain

$$|G^b_{ni}| \cdot \pi^b_{ni} - |G^s_{ni}| \cdot \pi^s_{ni} \geq 0 (21)$$

Therefore, the auctioneer’s profit $\Phi$ is always no <0

$$\Phi = \sum_{i \leq s} (|G^b_{ni}| \cdot \pi^b_{ni} - |G^s_{ni}| \cdot \pi^s_{ni}) \geq 0 (22)$$

Therefore, the theorem holds.

To prove truthfulness, first we need to demonstrate that the winner determination mechanism is monotonic for both buyers and
There are only two possible cases:

Case 1: \( b_{ni}^{b} > \pi_{ij}^{w} \). As \( b_{ni}^{b} > b_{ni}^{h} \), so the group bid will remain unchanged by bidding \( b_{ni}^{b} \), \( \pi_{ij}^{w} = \pi_{ij}^{w} \). Therefore the auction result will remain unchanged and \( w_{n} \) will win the channel type \( t_{i} \) in the auction.

Case 2: \( b_{ni}^{h} = \pi_{ij}^{w} = \min \{ b_{k}^{h} \mid w_{k} \in G_{ij} \} \). As \( b_{ni}^{h} > b_{ni}^{h} \), so the group bid will be greater by bidding \( b_{ni}^{b} \), \( \pi_{ij}^{w} > \pi_{ij}^{w} \). Moreover, \( w_{n} \) is a winner by bidding \( b_{ni}^{b} \), so according to the winner determination algorithm, \( \pi_{ij}^{w} \geq \pi_{ij}^{w} \). Then \( \pi_{ij}^{w} \geq \pi_{ij}^{w} \), and therefore \( w_{n} \) will also win in the auction for the channel type \( t_{i} \) by bidding \( b_{ni}^{b} \).

Proof: Since, according to the winner determination algorithm, the buyer bids of relating to the other channel types other than \( t_{i} \) do not effect on the winner determination for the channel type \( t_{i} \), we consider the bids of buyers other than \( w_{n} \) only for the channel type \( t_{i}, B_{(t_{i})} \). Without loss of generality, we suppose that \( \pi_{n} \in G_{ij} \). There are only two possible cases:

Case 1: \( b_{ni}^{h} > \pi_{ij}^{w} \). As \( b_{ni}^{h} > b_{ni}^{h} \), so the group bid will remain unchanged by bidding \( b_{ni}^{h} \), \( \pi_{ij}^{w} = \pi_{ij}^{w} \). Therefore the auction result will remain unchanged and \( w_{n} \) will win the channel type \( t_{i} \) in the auction.

Case 2: \( b_{ni}^{h} = \pi_{ij}^{w} = \min \{ b_{k}^{h} \mid w_{k} \in G_{ij} \} \). As \( b_{ni}^{h} > b_{ni}^{h} \), so the group bid will be greater by bidding \( b_{ni}^{b} \), \( \pi_{ij}^{w} > \pi_{ij}^{w} \). Moreover, \( w_{n} \) is a winner by bidding \( b_{ni}^{b} \), so according to the winner determination algorithm, \( \pi_{ij}^{w} \geq \pi_{ij}^{w} \). Then \( \pi_{ij}^{w} \geq \pi_{ij}^{w} \), and therefore \( w_{n} \) will also win in the auction for the channel type \( t_{i} \) by bidding \( b_{ni}^{b} \).

Lemma 2: Given \( B_{(t_{i})}^{b} = B_{(t_{i})}^{b} \backslash \{ b_{ni}^{b} \} \) and \( B_{t_{i}}^{b} \), if seller \( s_{n} \) wins in the auction for the channel type \( t_{i} \), it also wins by bidding \( b_{ni}^{h} < b_{ni}^{h} \) for this channel type.

Proof: Similarly, we consider only \( B_{(t_{i})}^{b} \). Since \( s_{n} \) is a winner seller by bidding \( b_{ni}^{h} \), so according to the winner determination algorithm, \( b_{ni}^{h} \leq \pi_{ij}^{w} \). Then \( b_{ni}^{h} \leq \pi_{ij}^{w} \), and therefore \( s_{n} \) will also win in the auction for the type \( t_{i} \) by bidding \( b_{ni}^{h} \).

Lemma 3: Given \( B_{(t_{i})}^{b} = B_{(t_{i})}^{b} \backslash \{ b_{ni}^{b} \} \) and \( B_{t_{i}}^{h} \), if buyer \( w_{n} \) wins in the auction for the channel type \( t_{i} \), bidding \( b_{ni}^{h} \) and \( b_{ni}^{h} \), the prices charged to \( w_{n} \) are the same.

Proof: Similarly, we only consider \( B_{(t_{i})}^{b} \) and \( B_{t_{i}}^{b} \). Without the loss of generality, let \( b_{ni}^{h} > b_{ni}^{h} \). According to Lemma 1, by increasing a winning buyer’s bid, the auction results will remain unchanged, as well as the position of \( k_{i} \) in the sorted list of the group bids related to the type \( t_{i} \). Since without changing the demands, the price is only dependent on position \( k_{i} \), the prices charged for buyer \( w_{n} \) by bidding \( b_{ni}^{h} \) and \( b_{ni}^{h} \) are the same.

Lemma 4: Given \( B_{(t_{i})}^{b} = B_{(t_{i})}^{b} \backslash \{ b_{ni}^{b} \} \) and \( B_{t_{i}}^{b} \), if seller \( s_{n} \) wins in the auction for the channel type \( t_{i} \) by bidding \( b_{ni}^{h} \) and \( b_{ni}^{h} \), the payment paid to \( s_{n} \) is the same for both.

Proof: Similarly, we only consider \( B_{(t_{i})}^{b} \) and \( B_{t_{i}}^{b} \). Since \( s_{n} \) wins the auction by bidding \( b_{ni}^{h} \) and \( b_{ni}^{h} \), the payment is decided by a seller ranked after \( m \), which remains unchanged in both cases. Therefore the claim holds.

Theorem 5: The auction mechanism is truthful for buyers.

Proof: It is required to show that any buyer \( w_{n} \) cannot increase its utility for one channel of type \( t_{i} \) by bidding other than its valuation for this channel type, \( b_{ni}^{h} \neq v_{ni} \). There are four possible cases for bidding of one buyer. In the following, we examine our claim for these cases:

Case 1: If \( w_{n} \) bids either truthfully or untruthfully, he looses in the auction.

In this case, since buyer \( w_{n} \) looses in the auction for both bids \( b_{ni}^{h} \) and \( v_{ni} \), this buyer charged with zero for both bids, leading to the same utility of zero.

Case 2: Buyer \( w_{n} \) wins in the auction only if he bids truthfully.

According to Lemma 1, this case happens only if \( b_{ni}^{h} < v_{ni} \). According to Theorem 3, the clearing price for the winning buyer \( w_{n} \) is no more than its bid for this channel, so the utility of winning buyer \( w_{n} \) by bidding \( v_{ni}^{w} \) is non-negative. On the other hand, the utility of losing buyer \( w_{n} \) by bidding \( b_{ni}^{h} \) is zero. Therefore our claim holds.

Case 3: Buyer \( w_{n} \) wins in the auction only if he bids untruthfully.

According to Lemma 1, this case happens only if \( b_{ni}^{h} > v_{ni} \). Let buyer \( w_{n} \) for bidding the channel type \( t_{i} \) placed in group \( G_{ij} \). Since buyer \( w_{n} \) bids by bidding greater than \( v_{ni}^{w} \), \( w_{n} \) should have offered the smallest bid in its group when bidding \( v_{ni}^{w} \), denoted by \( \pi_{ij}^{w} \). Therefore we have

\[
\pi_{ij}^{w} = \min \{ b_{k}^{h} \mid w_{k} \in G_{ij} \} = v_{ni}^{w} \quad | G_{ij} | \quad (23)
\]

Moreover, since \( w_{n} \) wins by bidding \( b_{ni}^{h} \) and losses by bidding \( v_{ni}^{w} \), its group bid should meet inequality \( \pi_{ij}^{w} \geq P_{ij}^{b} \geq \pi_{ij}^{w} \), where \( \pi_{ij}^{w} \) represents the group bid of \( G_{ij} \) when \( w_{n} \) bids \( b_{ni}^{h} \) and \( P_{ij}^{b} \) represents the price charged to this group when \( w_{n} \) wins in the auction by bidding \( b_{ni}^{h} \). Therefore, the utility of buyer \( w_{n} \) by bidding \( b_{ni}^{h} \) is

\[
\left\{ v_{ni}^{w} - P_{ij}^{b} \right\} \quad | G_{ij} | \quad 0 \quad (24)
\]

Therefore the utility of winning buyer \( w_{n} \) by bidding \( v_{ni}^{w} \) is \( 0 \). On the other hand, since buyer \( w_{n} \) looses in the auction by bidding \( v_{ni}^{w} \), its utility is zero for this bid. Therefore, our claim holds.

Case 4: If \( w_{n} \) bids either truthfully or untruthfully, he wins in the auction.

From Lemma 3, buyer \( w_{n} \) is charged by equal price for both bids \( b_{ni}^{h} \) and \( v_{ni}^{w} \). Therefore its utility is the same for both bids and the claim holds. From the aforementioned cases, we result that no buyer can increase its utility by bidding untruthfully. It means our auction mechanism is truthful for buyers.

Theorem 6: The auction mechanism is truthful for sellers.

Proof: It is required to show that any seller \( s_{n} \) cannot obtain higher utility for one channel of type \( t_{i} \) by bidding other than its valuation for this channel, \( b_{ni}^{h} \neq v_{ni} \). Similar to the previous theorem, there are also four possible cases for bidding of one seller, so we should examine our claim for these cases. It can be done in the way of the previous theorem. We regret that because of space limitations.

5 Numerical results

In this section, we use simulation to examine the performance of the proposed mechanisms on various auction metrics.

5.1 Impact of flexible bidding

As we mentioned before, our proposed auction mechanism enables spectrum buyers to use flexible demand formats, which is very desirable to the buyers. Here we show that this feature can also increase the number of traded channels in the auction. For this
purpose, we consider the auction in which the buyers’ bids as well as the sellers’ reserve prices are randomly distributed over (0, 1]. The number of sellers is set to 20 and the number of buyers varies from 10 to 100. Moreover, the results are averaged over 100 runs. We assume that each seller contributes one channel. We consider two bidding methods: flexible bidding and simple bidding. In the flexible bidding, the demand of each buyer is randomly set as 1, 2 or 3. In the simple bidding, each buyer requests for only one channel. As shown in Figs. 4 and 5, using flexible bidding in the auction increases the number of traded channels as well as the seller satisfaction ratio in comparison with simple bidding.

5.2 Evaluating the performance of the proposed algorithms

We use the Erdős–Rényi model [14] to generate random graphs with various densities. In this model, a graph $G(n, p)$, where $n$ denotes the number of nodes, is constructed by connecting nodes such that each edge is included in the graph with probability $p$ independent from other edges. Therefore the constructed graph $G(n, p)$ has on average $\binom{n}{2}p$ edges and the distribution of the degree of any particular vertex is binomial [15]. Therefore we can conclude that the expected graph density $D$ is equal to $p$

$$E(D) = \binom{n}{2}p \binom{n}{2} = p$$

(25)

In this section, we compare the performance of the proposed grouping algorithms, including ABG, EBG and AEBG, with some existing grouping algorithms, including no grouping, maximal independent set (MIS), Greedy, Greedy-U [16, 17]:

- MIS: It assigns channels by finding the MISs of the conflict graph.
- Greedy-U: To form a group, it recursively chooses a node with the minimal degree in the current conflict graph, eliminates the chosen node and its neighbours, and updates the degree values of the remaining nodes.
- Greedy: It is the same as the Greedy-U, except that it chooses the nodes based on its original degree value.

The number of sellers and the number of buyers are set to 10 and 100, respectively, and the probability of connection $p$ varies from 0.1
Moreover, the results are averaged over 100 runs. Fig. 6 illustrates that Greedy-U and Greedy have the largest and the smallest average group size, respectively. However, as stated before and shown in Figs. 7–9, although larger group size can increase spectrum reuse, but it can reduce the seller satisfaction ratio as well as the number of traded channels. Furthermore, Figs. 7–9 illustrate that by changing the type of graph via changing $p$, the performance of different algorithms changes. In this regards, in a sparse graph, Greedy has the highest performance on the number of traded channels and the seller satisfaction ratio metrics, whereas, in contrast, Greedy shows the lowest performance in a dense graph on these metrics. Moreover, we see that Greedy-U works just opposite of the Greedy on these performance metrics. Therefore, in ABG algorithm, we use Greedy-U and Greedy in dense graph ($p \geq 0.6$) and sparse graph ($p < 0.6$), respectively. Another proposed algorithm to increase the auction performance metrics was EBG that would modify the created buyer groups in respect to BTSR. Moreover, AEBG algorithm, jointly apply ABG and EBG methods to the auction. On the other words, we apply the EBG method to the groups which achieved by the ABG algorithm. Therefore we expect that AEBG to obtain both improvements of ABG and EBG, as shown in Fig. 10. As illustrated in Fig. 10, AEBG not only achieves a good spectrum utilisation in comparison with the existing algorithms, but also significantly improves the other important performance metrics.

6 Conclusions

We have proposed a multi-channel double auction mechanism which supports heterogeneous spectrums as well as homogeneous spectrums. In this auction, secondary service providers are able to express their preferences over each spectrum type separately. We have proved that our auction design can not only tackle the challenges induced by various aspects of heterogeneity in the spectrum auction, but also preserve three important economic aspects including truthfulness, budget balance and individual-rationality. Furthermore, our auction mechanism enables bidders to use flexible demand formats. We have shown that using the flexible bidding improves the number of traded channels as well as the seller satisfaction ratio in the auction. On the other hand, since buyer grouping affects many auction metrics, we have proposed some adaptive algorithms to improve the existing buyer grouping algorithms. The simulation results illustrate that using the proposed algorithms in the auction not only achieves almost similar spectrum utilisation in comparison with other existing algorithms, but also improves the other important performance metrics.

7 References


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