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Adaptive strategy-proof double auction mechanism for heterogeneous spectrum allocation

Mojtaba Mazoochi[1] ✉, *Mohammad Ali Pourmina*[1], *Hamidreza Bakhshi*[2]

[1]*Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran*
[2]*Department of Electrical Engineering, Shahed University, Tehran, Iran*
✉ *E-mail: m.mazoochi@srbiau.ac.ir*

Abstract: Spectrum auctions are one of the best-known solutions to improve the efficiency of spectrum use. However, there can be many challenges in the design of a practical spectrum auction. Heterogeneity is one of the most major challenges. Unfortunately, most of the existing auction designs either do not take into account the various aspects of heterogeneity or assume only the scenario where each seller supplies one distinct channel and each buyer wishes to buy merely one channel. The authors propose a spectrum auction mechanism which considers the various aspects of heterogeneity as well as multi-channel purchasing. They prove that the auction design preserves three important economic aspects including truthfulness, budget balance and individual-rationality. Moreover, most of the existing works only provide the bidders a simple demand format. Their auction mechanism enables bidders to use diverse demand formats. Furthermore, they propose some novel adaptive grouping algorithms to improve the auction's performance. The simulation results demonstrate good performance of the proposed algorithms on various auction metrics.

1 Introduction

The radio-frequency spectrum is one of the most expensive scarce resources in the wireless communication domain. However, spectrum occupancy measurements have indicated that a significant amount of the licenced spectrum remains unused in many places much of the time [1], so the traditional exclusive licencing spectrum policy leads to the low efficiency of spectrum utilisation. Spectrum auctions are one of the best market-based solutions to improve the efficiency of spectrum use. Heterogeneity is one of the most major challenges in spectrum auctions. Unfortunately, most of the existing designs either do not take into account the various aspects of heterogeneity [2–4] or assume only the scenario where each seller supplies one distinct channel and each buyer wishes to buy merely one channel [5]. Moreover, most of the existing works only provide the bidders a simple demand format while in a well-designed auction, buyers should be able to express different preferences for different spectrums by flexibly requesting channels they would like to obtain [4, 6]. We consider various aspects of heterogeneity as well as multi-channel purchasing in our proposed auction mechanism and enables bidders to use diverse demand formats. In summary, the main contributions of this paper are as follows:

- Our proposed auction mechanism supports spatial reusability as well as various heterogeneity aspects in where each seller (or buyer) can sell (or buy) multiple channels.
- Since buyer grouping affects many auction performance metrics, we propose some adaptive algorithms to improve the existing algorithms.
- To the best of our knowledge, our proposed double auction mechanism is the first one for heterogeneous spectrum transactions which enables bidders to use flexible demand formats.
- We design our spectrum auction mechanism so that it maintains the critical auctions properties: truthfulness, individual-rationality and budget balance.

The rest of this paper is organised as follows. Problem model and related works are reviewed in Section 2. Detailed description of our auction mechanism is presented in Section 3. We prove the economic-robustness of the proposed auction scheme by theoretical analysis in Section 4. We give simulation results in Section 5 and the conclusions in Section 6.

2 Problem model and related works

2.1 Problem model

We assume the scenario that N secondary service providers (called buyers) want to buy spectrum resources of K different types from M spectrum owners (called sellers). To provide this purpose, a single-round double auction with one auctioneer, M sellers and N buyers is held. Let $S = \{s_1, s_2, \dots, s_M\}$ denotes the set of sellers, $W = \{w_1, w_2, \dots, w_N\}$ denotes the set of buyers and $T = \{t_1, t_2, \dots, t_K\}$ denotes the set of spectrum types. Moreover, let the channel set H represents the set of all channels contributed by the sellers in the auction. We assume that each seller can contribute multiple channels of the same spectrum type and each buyer can obtain multiple channels of the same spectrum type. We allow buyers to express their preferences over each spectrum type separately. In addition, we consider that the auction supports flexible bidding requests for buyers described as follows:

- *Range request (RR):* If buyer w_n requests d^b_{nk} channels of spectrum type t_k, this buyer will obtain x channels, such that $x \in [0, d^b_{nk}]$.
- *Strict request (SR):* If buyer w_n requests d^b_{nk} channels of spectrum type t_k, this buyer will obtain either all d^b_{nk} channels or nothing.

Each buyer w_n submits a 1×4 bid vector $\boldsymbol{B}^b_n = (t_k, d^b_{nk}, b^b_{nk}, \mathrm{BT}^b_{nk})$. It means that buyer w_n requests for d^b_{nk} channels of type t_k with per-channel price b^b_{nk} and bidding type BT^b_{nk}, where BT^b_{nk} can be selected from the above-mentioned bidding type set $\mathrm{BT} = \{\mathrm{RR}, \mathrm{SR}\}$. Moreover, each seller s_m submits a $1 \times K$ bid vector $\boldsymbol{B}^s_m = (b^s_{m1}, b^s_{m2}, \dots, b^s_{mK})$ for its available spectrum types, where b^s_{mk} means the quoted price offered by the seller s_m for the type k. We denote $N \times 4$ matrix $\boldsymbol{B}^b$ and $M \times K$ matrix $\boldsymbol{B}^s$ as the bid matrices of all buyers and all sellers, respectively. We represent the true valuation of seller s_m and buyer w_n for the type t_k by v^s_{mk}

and v^{b}_{nk}, respectively. In the auction, the auctioneer determines the per-channel payment P^{s}_{mk} for seller s_m if it wins a channel of type t_k and the per-channel price P^{s}_{nk} that buyer w_n should pay if it wins a channel of this type.

2.2 Related works

Truthfulness is the most important property of an auction mechanism. Zhou *et al.* [6] proposed the first truthful single-sided spectrum auction mechanism. The first truthful double auction design with spectrum reuse, called TRUST, has been proposed in [7]. However, the early works in the field of spectrum auction did not consider the multi-channel scenario. Zhang *et al.* [2]; Wu and Vaidya [3] proposed some strategy-proof single-sided multi-channel auction mechanisms. On the other hand, heterogeneity is one of the most important challenges in the spectrum auctions. Unfortunately, most of the existing designs, such as the above-mentioned works, did not take into account heterogeneity in their auction mechanisms. Some aspects of heterogeneity in the scope of spectrum auctions were studied in [5, 8, 9]. However, these works only considered some aspects of heterogeneity or assumed only the scenario where each seller supplies one distinct channel and each buyer wishes to buy merely one channel [5]. Furthermore, most of the existing works only provide the bidders a simple demand format. Wang *et al.* [4] and Feng *et al.* [6] allowed flexible spectrum bidding, but they did not consider spectrum heterogeneity. To the best of our knowledge, our proposed design is the first truthful multi-channel double auction mechanism that addresses the various aspects of heterogeneous, supports spatial reusability and enables buyers/sellers to use flexible demand formats and buy/sell multiple channels.

3 Auction design

In this section, we propose our auction mechanism for the allocation of heterogeneous spectrum. Our proposed auction scheme includes the following steps.

3.1 Channel-dependent conflict graph formation

To handle spectrum reusability, a buyer grouping step has been applied in many existing spectrum auction schemes, in which the buyer's interference conditions are modelled as conflict graph. Since channel characteristics are dependent on the frequency used, we can expect that the shape of the interference regions will be channel dependent [10], so we use channel-dependent interference graphs. Of course, since the channels of each spectrum type show similar propagation and quality characteristics, we assume that these channels are homogeneous and have a same interference graph. Moreover, we present and proof the following theorem for obtaining the conflict graph related to the channels of each spectrum type.

Theorem 1: To group the buyers of each spectrum type $t_i \in T$, only the buyer grouping using the interference graph related to the smallest channel of this type in the channel set H can guarantee interference-free transmission.

Proof: Let H_i be the set of all channels of type t_i in the channel set H and h_0 be the channel having the smallest frequency channel belonging to H_i, we named it the smallest channel of type t_i. Moreover, we define $G(t_i/h_k)$ as the interference graph of the buyers based on the frequency channel h_k. Now we create the interference graph $G(t_i/h_0)$ and consider each two arbitrary nodes (buyers) w_j and w_k with no-edge between them. We denote $\mathrm{TR}(w_j/h_k)$ as the transmission range of w_j using the channel h_k, which is defined as the maximum distance from node w_j on the channel h_k where connectivity with another node exists and a transmitted signal from node w_j on the channel h_k can be successfully received. It is clear that as the frequency increases, the path loss also increases [11], and therefore the transmission range decreases. It means that

$$\mathrm{TR}(w_j|h_l) \le \mathrm{TR}(w_j|h_s), \quad \forall w_j \in W, \quad \forall h_l,\ h_s \in H_i, \quad \text{s.t.} \quad h_l \ge h_s \tag{1}$$

Hence, for every frequency channel h_l belonging to H_i, since h_0 is the smallest frequency channel in H_i, we have $h_0 \le h_l$ and so

$$\mathrm{TR}(w_j|h_l) \le \mathrm{TR}(w_j|h_0),\ \ \mathrm{TR}(w_k|h_l) \le \mathrm{TR}(w_k|h_0), \quad \forall h_l \in H_i \tag{2}$$

It means that if w_j and w_k do not interfere with each other in h_0, they do not interfere in any other channel belonging to H_i. Therefore if we group the buyers using $G(t_i/h_0)$, interference-free transmission will be guaranteed.
Moreover, we show that the buyer grouping using any other interference graph such as $G(t_i/h_l)$ so that $h_l > h_0$ can cause interference between buyers. We create the interference graphs $G(t_i/h_l)$ and consider two nodes w_j and w_k with no-edge between them, so we should have

$$\mathrm{TR}(w_j|h_l) + \mathrm{TR}(w_k|h_l) < d_{jk} \tag{3}$$

where d_{jk} denotes the distance between w_j and w_k.

From (1) and (3), the following condition can be established for a channel $h_s < h_l$

$$\mathrm{TR}(w_j|h_l) + \mathrm{TR}(w_k|h_l) < d_{jk} < \mathrm{TR}(w_j|h_s) + \mathrm{TR}(w_k|h_s) \tag{4}$$

It means that w_j and w_k do interfere with each other in h_s. Therefore our claim holds. □

Let $\boldsymbol{A} = (a_{ij})$ be an $N \times K(0, 1)$-matrix which represents the buyers' channel type availability such that $a_{ij} = 1$ indicates that the type t_j is available for the buyer w_i. Moreover, let $\boldsymbol{C} = (c_{ijk})$ be a $K \times N \times N(0, 1)$-matrix which represents the conflict relationships between buyers for each channel type such that $c_{ijk} = 0$ means that buyers w_j and w_k do not interfere with each other in type t_i. Finally, let $N \times K$ matrix $\boldsymbol{D}^{\mathrm{b}}$ and $M \times K$ matrix $\boldsymbol{D}^{\mathrm{s}}$ represent the (0, 1)-demand matrices of buyers and sellers for each type that belongs to T, respectively. For instance, $d^{\mathrm{b}}_{ij} = 1$ means that the buyer w_i wants to buy a channel of type t_j.

In conflict graph formation, at first, we obtain the set of candidate buyers Q_i for each $t_i \in T$. The set Q_i includes the buyers which have a demand for the channel type t_i, and also a channel of type t_i is available for them

$$Q_i = \{w_k | w_k \in W \wedge a_{ki} = 1 \wedge d^{\mathrm{b}}_{ki} = 1\} \tag{5}$$

Then, we create the interference graph $G(t_i|h^i_0)$ related to the type t_i on the set Q_i according to the adjacency matrix $\boldsymbol{C}$ such that h^i_0 is the smallest channel of type t_i in the channel set H.

3.2 Adaptive grouping

Since buyer grouping affects many auction performance metrics, in the following we propose some novel adaptive algorithms to improve the existing algorithms. In adaptive buyer grouping (ABG), selecting the suitable buyer grouping between the existing is done according to the graph density analysis. The enhanced buyer grouping (EBG) algorithm increases the auction performance metrics through modifying the created buyer groups with respect to the buyer-to-seller ratio (BTSR). Meanwhile, the adaptive EBG (AEBG) algorithm jointly applies ABG and EBG methods to the auction.

In this step, we propose three novel adaptive algorithms for spectrum buyer grouping.

3.2.1 ABG algorithm: Several grouping algorithms have been proposed to tackle the spectrum reuse problem. However, as it will

Algorithm 1: *Graph Creation & Adaptive Grouping*

$\boldsymbol{G}^b = \emptyset$;
for all $t_i \in T$ ***do***
Create the candidate buyer set Q_i *for* t_i;
Find h_0^i;
Construct conflict graph $G(t_i|h_0^i)$;
Calculate $d\left(G(t_i|h_0^i)\right)$;
Select a suitable grouping method based on $d\left(G(t_i|h_0^i)\right)$;
Find independent sets $\boldsymbol{IS}_i$ *for* $G(t_i|h_0)$;
$\boldsymbol{G}_i^b = \boldsymbol{IS}_i$;
$\boldsymbol{G}^b = \boldsymbol{G}^b \cup \boldsymbol{G}_i^b$;
Form the seller group G_i^s *for* t_i: $G_i^s = \{s_k \in S | d_{ki}^s = 1\}$;
end for
return $\boldsymbol{G}^b, \boldsymbol{G}^s$;

Fig. 1 *Graph creation and adaptive grouping*

be studied in Section 5, these algorithms have different performances on different types of graphs, including sparse, medium density and dense graphs. Therefore, in the proposed ABG algorithm, we first analyse the density of a conflict graph of buyers and according to the results, select a suitable algorithm for buyer grouping relating to this graph. To find out the type of a graph, we use the graph density parameter which is the ratio of the number of edges in the graph against the number of edges in a complete graph with the same number of vertices [12]. Since a simple undirected graph can have at most $\binom{|V|}{2}$ edges, so its density is calculated as

$$\text{Graph density} = |E| \Big/ \binom{|V|}{2} = 2|E|/|V|(|V|-1) \qquad (6)$$

In (6), $|E|$ is the number of edges in graph $G(t_i|h_0^i)$ related to the type t_i, which is equal to the half of the number of non-zero cells in above-mentioned matrix $\boldsymbol{C} = (c_{ijk})$. Moreover, $|V|$ is the number of vertices in $G(t_i|h_0^i)$ related to the type t_i, which is equal to the size of set Q_i, that is, $|Q_i|$. Therefore we can calculate the density of the conflict graph related to the type t_i as

$$d\left(G(t_i|h_0^i)\right) = |E| \Big/ \binom{|V|}{2} = \sum_{i,j,k} |c_{ijk}|/|Q_i|(|Q_i|-1) \qquad (7)$$

Summary of the before-mentioned steps is shown in Algorithm 1 (see Fig. 1).

3.2.2 EBG algorithm: In the majority of the existing algorithms, buyer grouping is independent of the number of sellers. In our proposed EBG algorithm, we compare the number of buyer groups in $\boldsymbol{G}_i^{\mathrm{b}}$ with the total number of the channels of type t_i to be sold. Let N_{t_i} be the total number of the channels of type t_i to be sold. Define BTSR_i be the number of the buyer groups in $\boldsymbol{G}_i^{\mathrm{b}}$ to the total number of the channels of type t_i to be sold ratio, $\mathrm{BTSR}_i = |\boldsymbol{G}_i^{\mathrm{b}}|/N_{t_i}$. For each type t_i, EBG first obtains BTSR_i parameter. Then, while the number of buyer groups related to each channel type is less than N_{t_i}, decompose the biggest buyer group in $\boldsymbol{G}_i^{\mathrm{b}}$ (if it is bigger than BTSR_i) into two equal or semi-equal groups. Algorithm 2 (see Fig. 2) shows the EBG procedure.

3.2.3 AEBG algorithm: In this method, first we apply ABG to assign adaptively the suitable algorithms for buyer grouping. Then, we enhance the performance of the selected algorithm using EBG.

3.3 Winner selection

After grouping, we can consider all buyer groups in $\boldsymbol{G}_i^{\mathrm{b}}$ as the supper buyers which want to purchase the channels of type t_i. Moreover, we

Algorithm 2: *Buyer-Group-Enhancement*

$\boldsymbol{G}^{be} = \emptyset$;
for all $G_i^b \in \boldsymbol{G}^b$ ***do***
$BTSR_i = |\boldsymbol{G}_i^b|/N_{t_i}$;
$gsize = |\boldsymbol{G}_i^b|$;
while $gsize < N_{t_i}$ ***do***
Find the group $G_{ik}^b \in \boldsymbol{G}_i^b$, *s.t* $\forall G_{ij}^b \in \boldsymbol{G}_i^b$, $|G_{ik}^b| \geq |G_{ij}^b|$;
if $|G_{ik}^b| \geq BTSR_i$ ***then***
Decompose G_{ik}^b *and replace the result instead of* G_{ik}^b *in* $\boldsymbol{G}_i^b$;
Increment gsize;
elseif *break;*
end while
end for
$\boldsymbol{G}^{be} = \boldsymbol{G}^b$;
return $\boldsymbol{G}^{be}$;

Fig. 2 *Buyer-group-enhancement*

consider all sellers in $\boldsymbol{G}_i^{\mathrm{s}}$ as the sellers which want to sell the channels of this type. Let G_{ij}^{b} represent the jth buyer group in $\boldsymbol{G}_i^{\mathrm{b}}$ which can purchase a channel of type t_i. To determine the bid of each buyer group G_{ij}^{b} in $\boldsymbol{G}_i^{\mathrm{b}}$, we use uniform pricing rather than discriminatory pricing to achieve individual-rationality and truthfulness in the auction. We present the following theorem for obtaining the buyer group bids.

Theorem 2: To achieve individual-rationality and truthfulness in the auction, the group bid $\boldsymbol{\pi}_{ij}^{\mathrm{b}}$ for the channel type t_i, under per-group uniform pricing, should not be exceeded than the product of the smallest per-channel buyer bid and the number of buyers, in that group.

Proof: From individual-rationality, each buyer w_n must not be charged more than its bid for the type t_i, b_{ni}^{b}. Moreover, under uniform pricing, the auctioneer charges equally the buyers in the same group. Using these conditions and considering the McAfee's design [13], the theorem can be easily proved. Since a similar proof was presented in [7] and because of space limitations, we omit the proof. □

In winner determination phase, the auctioneer first calculates the per-channel buyers group bid π_{ij}^{b} relating to the buyer group G_{ij}^{b} for the channel type t_i according to Theorem 2 as

$$\pi_{ij}^{\mathrm{b}} = \mathrm{Min}\{b_{ki}^{\mathrm{b}} | w_k \in G_{ij}^{\mathrm{b}}\}.|G_{ij}^{\mathrm{b}}| \tag{8}$$

Moreover, the demand interval of the group G_{ij}^{b} for the type t_i is calculated as

$$gd_{ij}^{\mathrm{b}}(\min) = \mathrm{Max}\{d_{ki}^{\mathrm{b}} | w_k \in G_{ij}^{\mathrm{b}} \wedge \mathrm{BT}_{ki}^{\mathrm{b}} = \mathrm{SR}\} \tag{9}$$

$$gd_{ij}^{\mathrm{b}}(\max) = \mathrm{Max}\{d_{ki}^{\mathrm{b}} | w_k \in G_{ij}^{\mathrm{b}}\} \tag{10}$$

According to (9), minimum demand of a buyer group is the maximum demand with type of SR in that group. Then, for each t_i, the auctioneer sorts the related buyer groups' bid list $\boldsymbol{\pi}_i^{\mathrm{b}}$ in non-increasing order. Of course, auctioneer tries to assign the maximum possible number of channels for group G_{ij}^{b} in the demanded interval of $[gd_{ij}^{\mathrm{b}}(\min), gd_{ij}^{\mathrm{b}}(\max)]$. We represent the sorted bids and their related buyer groups by $\boldsymbol{\pi}_i^{\mathrm{bs}}$ and $\boldsymbol{G}_i^{\mathrm{bs}}$, respectively. Moreover, the auctioneer sorts the seller bid list $\boldsymbol{\pi}_i^{\mathrm{s}}$ corresponding to the type t_i in non-decreasing order. We represent the sorted bids and their related sellers by $\boldsymbol{\pi}_i^{\mathrm{ss}}$ and $\boldsymbol{G}_i^{\mathrm{ss}}$, respectively. We define k_i as the last profitable trade for type t_i

$$k_i = \underset{l \le \min\{\mathrm{length}(\boldsymbol{\pi}_i^{\mathrm{ss}}), \mathrm{length}(\boldsymbol{\pi}_i^{\mathrm{bs}})\}}{\mathrm{argmax}} \quad \boldsymbol{\pi}_{il}^{\mathrm{bs}} \ge \boldsymbol{\pi}_{il}^{\mathrm{ss}} \tag{11}$$

Thus, the preliminary auction winners for the channels of type t_i are the buyers belonging to the first $k_i - 1$ buyer groups in $\boldsymbol{G}_i^{\mathrm{bs}}$ and the first $k_i - 1$ sellers in $\boldsymbol{G}_i^{\mathrm{ss}}$. We calculate the number of winning channels of type t_i as

$$\mathrm{wcn}_i = \sum\nolimits_{m=1}^{k_i-1} d_{mi}^{\mathrm{s}} \tag{12}$$

To determine the final winners of the auction, the auctioneer tries to assign these wcn_i channels from the set of preliminary winning sellers to the set of preliminary winning buyer groups according to their demand intervals and with respect to their orders in the lists. If the number of available channels acn_i for a buyer group G_{ij}^{b} is less than its minimum demand $gd_{ij}^{\mathrm{b}}(\min)$, auctioneer removes the buyer group G_{ij}^{b} from the winning buyer list, else auctioneer adds this buyer group to the final winning buyer group list and allocates whatever is possible, $ga_{ij}^{\mathrm{b}} = \min(\mathrm{acn}_i, gd_{ij}^{\mathrm{b}}(\max))$. At last, the winning buyers for the channels of type t_i are the members of the final winning buyer groups. Moreover, the winning sellers for this channel type are the preliminary winning sellers who have assigned a channel of type t_i to a final winning buyer group. After determining the winning buyer groups $\boldsymbol{G}_i^{\mathrm{bw}}$ and allocating the possible channels $\boldsymbol{ga}_i^{\mathrm{b}}$ to them, auctioneer assigns, for every winning buyer group G_{ij}^{bw}, ga_{ij}^{b} channels to the members of G_{ij}^{b} in such a way that each buyer w_k with bidding type of SR is assigned equal to his demand d_{ki}^{b}, but each buyer w_k with bidding type of RR is assigned $a_{ki}^{\mathrm{b}} = \min(d_{ki}^{\mathrm{b}}, ga_{ij}^{\mathrm{b}})$ channels.

3.4 Pricing

After the winner selection, the auctioneer calculates the price of the winning buyers and the payment of the winning sellers for each type t_i. To maintain truthfulness, the auctioneer pays, per channel, each winning seller belonging to the winning seller set $\boldsymbol{G}_i^{\mathrm{sw}}$ by the next seller's bid after the last winning seller in $\boldsymbol{\pi}_i^{\mathrm{ss}}$, π_i^{sn}, which can be the k_ith seller's bid in $\boldsymbol{\pi}_i^{\mathrm{ss}}$. Of course, if $(k_i - 1)$th seller has been removed from the winning seller list, π_i^{sn} is the biggest bid of the sellers in which have been removed from the preliminary winning seller list. Moreover, the auctioneer charges, per channel, each winning buyer group belonging to the family of winning buyer set $\boldsymbol{G}_i^{\mathrm{bw}}$ by the next buyer group's bid after the last winning buyer group in $\boldsymbol{\pi}_i^{\mathrm{bs}}$, $\boldsymbol{\pi}_i^{\mathrm{b}n}$, which can be the k_ith buyer group's bid in $\boldsymbol{\pi}_i^{\mathrm{bs}}$. Similarly, if $(k_i - 1)$th buyer group has been removed from the winning buyer group list, $\pi_i^{\mathrm{b}n}$ is the smallest bid of the buyer groups in which have been removed from the preliminary winning buyer group list. Finally, this per-channel buyer's group price P_i^{b} is shared by all members in each winning buyer group, so per-channel price P_{ki}^{b} and total charge P_{ki}^{bt} to a buyer w_k belonging to the winning buyer group G_{ij}^{b} for the channel type t_i are

$$P_{ki}^{\mathrm{b}} = P_i^{\mathrm{b}}/|G_{ij}^{\mathrm{b}}|, \quad P_{ki}^{\mathrm{bt}} = a_{ki}^{\mathrm{b}}.P_i^{\mathrm{b}}/|G_{ij}^{\mathrm{b}}| \tag{13}$$

Moreover, no charges and no payments are made to loosing buyers and loosing sellers, respectively. The detailed procedure for winner selection and pricing is shown in Algorithm 3 (see Fig. 3).

4 Proofs of economic properties

A double auction is truthful if and only if no seller m or buyer n can improve its own utility by bidding untruthfully ($B_{mk}^{\mathrm{s}} \ne v_{mk}^{\mathrm{s}}$ or $B_{nk}^{\mathrm{b}} \ne v_{nk}^{\mathrm{b}}$)

$$\begin{aligned} &U_{mk}^{\mathrm{s}}(v_{mk}^{\mathrm{s}}) \ge U_{mk}^{\mathrm{s}}(B_{mk}^{\mathrm{s}}), \quad U_{nk}^{\mathrm{b}}(v_{nk}^{\mathrm{b}}) \ge U_{nk}^{\mathrm{b}}(B_{nk}^{\mathrm{b}}), \quad \forall s_m \in S, \\ &\forall w_n \in W, \quad \forall t_k \in T \end{aligned} \tag{14}$$

A double auction is individual rational if no winning seller is paid less than its bid and no winning buyer pays more than its bid

$$P_{mk}^{\mathrm{s}} \ge B_{mk}^{\mathrm{s}}, \ P_{nk}^{\mathrm{b}} \le B_{nk}^{\mathrm{b}}, \quad \forall s_m \in S, \quad \forall w_n \in W, \quad \forall t_k \in T \tag{15}$$

A double auction is budget balanced if the auctioneer's profit Φ is non-negative

$$\Phi = \sum_{\substack{1 \le n \le N \\ 1 \le k \le K}} P_{nk}^{\mathrm{b}} - \sum_{\substack{1 \le m \le M \\ 1 \le k \le K}} P_{mk}^{\mathrm{s}} \ge 0 \tag{16}$$

In the following, we prove that our auction mechanism has the above-mentioned properties.

Theorem 3: The auction mechanism is individually rational.

Proof: According to (8), for $\forall t_i \in T$ and $\forall w_n \in G_{ij}^{\mathrm{b}}$, we have

$$\pi_{ij}^{\mathrm{b}} = \mathrm{Min}\{b_{si}^{\mathrm{b}} | w_s \in G_{ij}^{\mathrm{b}}\}.|G_{ij}^{\mathrm{b}}| \le b_{ni}^{\mathrm{b}}.|G_{ij}^{\mathrm{b}}| \tag{17}$$

If $G_{ij}^{\mathrm{b}} \in \boldsymbol{G}_i^{\mathrm{bw}}$, combining (13) and (17) and according to the winner

Algorithm 3: *Winner Selection and Pricing*

for all $\boldsymbol{G}_i^b \in \boldsymbol{G}^b$ **do**
  for all $G_{ij}^b \in \boldsymbol{G}_i^b$ **do**
    $\pi_{ij}^b = min\{b_{ki}^b | w_k \in G_{ij}^b\}.|G_{ij}^b|$;
    $gd_{ij}^b(min), gd_{ij}^b(max)$;
  end for
  $\boldsymbol{\pi}_i^{bs}$ = sorted $\boldsymbol{\pi}_i^b$ in non-increasing order;
  $\boldsymbol{G}_i^{bs}$ = sorted $\boldsymbol{G}_i^b$ according to $\boldsymbol{\pi}_i^{bs}$;
end for
for all $\boldsymbol{G}_i^s \in \boldsymbol{G}^s$ **do**
  $\boldsymbol{\pi}_i^s = \{b_{ki}^s | s_k \in \boldsymbol{G}_i^s\}$;
  $\boldsymbol{\pi}_i^{ss}$ = sorted $\boldsymbol{\pi}_i^s$ in non-decreasing order;
  $\boldsymbol{G}_i^{ss}$ = sorted $\boldsymbol{G}_i^s$ according to $\boldsymbol{\pi}_i^{ss}$;
end for
for all $t_i \in T$ **do**
  $k_i = argmax_{l \le min\{length(\boldsymbol{\pi}_i^{ss}), length(\boldsymbol{\pi}_i^{bs})\}} \pi_{il}^{bs} \ge \pi_{il}^{ss}$;
  Select the first $k_i - 1$ buyer groups in $\boldsymbol{G}_i^{bs}$ as the preliminary winning buyers $\boldsymbol{G}_i^{bpw}$;
  Select the first $k_i - 1$ sellers in $\boldsymbol{G}_i^{ss}$ as the preliminary winning sellers $\boldsymbol{G}_i^{spw}$;
  Find final winners from the preliminary winning lists &allocate channels to them:
    if $acn_i \ge gd_{ij}^b(min)$ **then** $\boldsymbol{G}_i^{bw} = \boldsymbol{G}_i^{bw} \cup G_{ij}^{bpw}$; $ga_{ij}^b = min(acn_i, gd_{ij}^b(max))$;
  for all $w_k \in G_{ij}^{bpw} \cap \boldsymbol{G}_i^{bw}$ **do**
    if $BT_{ki}^b = RR$ **then** assign $a_{ki}^b = min(d_{ki}^b, ga_{ij}^b)$ channels;
    if $BT_{ki}^b = SR$ **then** assign $a_{ki}^b = d_{ki}^b$ channels;
  end for
  if G_{ij}^{spw} has assigned a channel to $\boldsymbol{G}_i^{bw}$ **then** $\boldsymbol{G}_i^{sw} = \boldsymbol{G}_i^{sw} \cup G_{ij}^{spw}$;
  $P_i^s = \pi_i^{sn}$, $P_i^b = \pi_i^{bn}$;
  for all $s_k \in \boldsymbol{G}_i^{sw}$ **do** $P_{ki}^{st} = a_{ki}^s . P_i^s$ **end for**
  for all $G_{ij}^b \in \boldsymbol{G}_i^{bw}$ **do**
    for all $w_k \in G_{ij}^b$ **do** $P_{ki}^{bt} = a_{ki}^b . P_i^b / |G_{ij}^b|$ **end for**
  end for
end for

Fig. 3 *Winner selection and pricing*

determination and pricing algorithm, we obtain

$$P_{ni}^{\mathrm{b}} = \frac{P_i^{\mathrm{b}}}{|G_{ij}^{\mathrm{b}}|} \le \frac{\pi_{ij}^{\mathrm{b}}}{|G_{ij}^{\mathrm{b}}|} \le \frac{b_{ni}^{\mathrm{b}} \cdot |G_{ij}^{\mathrm{b}}|}{|G_{ij}^{\mathrm{b}}|} = b_{ni}^{\mathrm{b}} \tag{18}$$

Therefore the per-channel clearing price P_{ni}^{b} for the winning buyer w_n and the channel type t_i is no more than its per-channel bid for this channel type, b_{ni}^{b}.
Moreover, according to Fig. 3 and the sorting method, for $\forall s_m \in \boldsymbol{G}_i^{\mathrm{sw}}$ and $\forall t_i \in T$, we have

$$b_{mi}^{\mathrm{s}} \le \pi_i^{sn} = P_i^{\mathrm{s}} = P_{mi}^{\mathrm{s}} \tag{19}$$

It means that no winning seller will receive less than its request and so the theorem holds. □

Theorem 4: The auction mechanism is budget-balanced.

Proof: According to (11), for each winning group related to the type t_i, we have

$$\pi_i^{\mathrm{bs}} \ge \pi_i^{\mathrm{ss}}, \quad |\boldsymbol{G}_i^{\mathrm{bw}}| = |\boldsymbol{G}_i^{\mathrm{sw}}| \tag{20}$$

From (20), we obtain

$$|\boldsymbol{G}_i^{\mathrm{bw}}| \cdot \pi_i^{\mathrm{bs}} - |\boldsymbol{G}_i^{\mathrm{sw}}| \cdot \pi_i^{\mathrm{ss}} \ge 0 \tag{21}$$

Therefore, the auctioneer's profit Φ is always no <0

$$\Phi = \sum_{1 \le i \le K} (|\boldsymbol{G}_i^{\mathrm{bw}}| \cdot \pi_i^{\mathrm{bs}} - |\boldsymbol{G}_i^{\mathrm{sw}}| \cdot \pi_i^{\mathrm{ss}}) \ge 0 \tag{22}$$

Therefore, the theorem holds. □

To prove truthfulness, first we need to demonstrate that the winner determination mechanism is monotonic for both buyers and sellers, and also the pricing procedure is bid-independent. The following lemmas prove these claims.

Lemma 1: Given $\boldsymbol{B}^{\mathrm{b}}_{-(ni)} = \boldsymbol{B}^{\mathrm{b}} \backslash \{b^{\mathrm{b}}_{ni}\}$ and $\boldsymbol{B}^{\mathrm{s}}$, if buyer w_n wins in the auction for a channel of type t_i, it also wins by bidding $b'^{\mathrm{b}}_{ni} > b^{\mathrm{b}}_{ni}$ for this channel type.

Proof: Since, according to the winner determination algorithm, the buyer bids of relating to the other channel types other than t_i do not effect on the winner determination for the channel type t_i, we consider the bids of buyers other than w_n only for the channel type t_i, $\boldsymbol{B}^{\mathrm{b}}_{(-ni)}$. Without loss of generality, we suppose that $w_n \in G^{\mathrm{b}}_{ij}$. There are only two possible cases:

Case 1: $b^{\mathrm{b}}_{ni} > \pi^{\mathrm{b}}_{ij}$ As $b'^{\mathrm{b}}_{ni} > b^{\mathrm{b}}_{ni}$, so the group bid will remain unchanged by bidding b'^{b}_{ni}, $\pi'^{\mathrm{b}}_{ij} = \pi^{\mathrm{b}}_{ij}$. Therefore the auction result will remain unchanged and w_n will win the channel type t_i in the auction.
Case 2: $b^{\mathrm{b}}_{ni} = \pi^{\mathrm{b}}_{ij} = \min\{b^{\mathrm{b}}_{ki} | w_k \in G_{ij}\}$: As $b'^{\mathrm{b}}_{ni} > b^{\mathrm{b}}_{ni}$, so the group bid will be greater by bidding b'^{b}_{ni}, $\pi'^{b}_{ij} > \pi^{\mathrm{b}}_{ij}$. Moreover, w_n is a winning buyer by bidding b^{b}_{ni}, so according to the winner determination algorithm, $\pi^{\mathrm{b}}_{ij} \geq \pi^{\mathrm{b}n}_{i}$. Then $\pi'^{\mathrm{b}}_{ij} \geq \pi^{\mathrm{b}n}_{i}$, and therefore w_n will also win in the auction for the channel type t_i by bidding b'^{b}_{ni}. □

Lemma 2: Given $\boldsymbol{B}^{\mathrm{s}}_{-(mi)} = \boldsymbol{B}^{\mathrm{s}} \backslash \{b^{\mathrm{s}}_{mi}\}$ and $\boldsymbol{B}^{\mathrm{b}}$, if seller s_m wins in the auction for the channel type t_i, it also wins by bidding $b'^{\mathrm{s}}_{mi} < b^{\mathrm{s}}_{mi}$ for this channel type.

Proof: Similarly, we consider only $\boldsymbol{B}^{\mathrm{s}}_{(-m)i}$. Since s_m is a winning seller by bidding b^{s}_{mi}, so according to the winner determination algorithm, $b^{\mathrm{s}}_{mi} \leq \pi^{sn}_{i}$. Then $b'^{\mathrm{s}}_{mi} \leq \pi^{sn}_{i}$, and therefore s_m will also win in the auction for the type t_i by bidding b'^{s}_{mi}. □

Lemma 3: Given $\boldsymbol{B}^{\mathrm{b}}_{-(ni)} = \boldsymbol{B}^{\mathrm{b}} \backslash \{b^{\mathrm{b}}_{ni}\}$ and $\boldsymbol{B}^{\mathrm{s}}$, if buyer w_n wins in the auction for the channel type t_i by bidding b^{b}_{ni} and b'^{b}_{ni}, the prices charged to w_n are the same.

Proof: Similarly, we only consider $\boldsymbol{B}^{\mathrm{b}}_{i} \backslash \{b^{\mathrm{b}}_{ni}\}$ and $\boldsymbol{B}^{\mathrm{s}}_{i}$. Without the loss of generality, let $b'^{\mathrm{b}}_{ni} > b^{\mathrm{b}}_{ni}$. According to *Lemma 1*, by increasing a winning buyer's bid, the auction results will remain unchanged, as well as the position of k_i in the sorted list of the group bids related to the type t_i. Since without changing the demands, the price is only dependent on position k_i, the prices charged for buyer w_n by bidding b^{b}_{ni} and b'^{b}_{ni} are the same. □

Lemma 4: Given $\boldsymbol{B}^{\mathrm{s}}_{-(mi)} = \boldsymbol{B}^{\mathrm{s}} \backslash \{b^{\mathrm{s}}_{mi}\}$ and $\boldsymbol{B}^{\mathrm{b}}$, if seller s_m wins in the auction for the channel type t_i by bidding b^{s}_{mi} and b'^{s}_{mi}, the payment paid to s_m is the same for both.

Proof: Similarly, we only consider $\boldsymbol{B}^{\mathrm{s}}_{i} \backslash \{b^{\mathrm{s}}_{mi}\}$ and $\boldsymbol{B}^{\mathrm{b}}_{i}$. Since s_m wins the auction by bidding b^{s}_{mi} and b'^{s}_{mi}, the payment is decided by a seller ranked after m, which remains unchanged in both cases. Therefore the claim holds. □

Theorem 5: The auction mechanism is truthful for buyers.

Proof: It is required to show that any buyer w_n cannot increase its utility for one channel of type t_i by bidding other than its valuation for this channel type, $b'^{\mathrm{b}}_{ni} \neq v^{\mathrm{b}}_{ni}$. There are four possible cases for bidding of one buyer. In the following, we examine our claim for these cases:

Case 1: If w_n bids either truthfully or untruthfully, he looses in the auction.

In this case, since buyer w_n looses in the auction for both bids b'^{b}_{ni} and v^{b}_{ni}, this buyer charged with zero for both bids, leading to the same utility of zero.

Case 2: Buyer w_n wins in the auction only if he bids truthfully.

According to Lemma 1, this case happens only if $b'^{\mathrm{b}}_{ni} < v^{\mathrm{b}}_{ni}$. According to Theorem 3, the clearing price for the winning buyer w_n is no more than its bid for this channel, so the utility of winning buyer w_n by bidding v^{b}_{ni} is non-negative. On the other hand, the utility of loosing buyer w_n by bidding b'^{b}_{ni} is zero. Therefore our claim holds.

Case 3: Buyer w_n wins in the auction only if he bids untruthfully.

According to Lemma 1, this case happens only if $b'^{\mathrm{b}}_{ni} > v^{\mathrm{b}}_{ni}$. Let buyer w_k for bidding the channel type t_i placed in group G^{b}_{ij}. Since buyer w_n wins by bidding greater than v^{b}_{ni}, w_n should have offered the smallest bid in its group when bidding v^{b}_{ni}, denoted by π^{b}_{ij}. Therefore we have

$$\pi^{\mathrm{b}}_{ij} = \mathrm{Min}\{b^{\mathrm{b}}_{ki} | w_k \in G^{\mathrm{b}}_{ij}\}.|G^{\mathrm{b}}_{ij}| = v^{\mathrm{b}}_{ni}.|G^{\mathrm{b}}_{ij}| \tag{23}$$

Moreover, since w_n wins by bidding b'^{b}_{ni} and looses by bidding v^{b}_{ni}, its group bid should meet inequality $\pi'^{\mathrm{b}}_{ij} \geq P'^{\mathrm{b}}_{i} \geq \pi^{\mathrm{b}}_{ij}$, where π'^{b}_{ij} represents the group bid of G^{b}_{ij} when w_n bids b'^{b}_{ni} and P'^{b}_{i} represents the price charged to this group when w_n wins in the auction by bidding b'^{b}_{ni}. Therefore, the utility of buyer w_n by bidding b'^{b}_{ni} is

$$v^{\mathrm{b}}_{ni} - \frac{P'^{\mathrm{b}}_{i}}{|G^{\mathrm{b}}_{ij}|} \leq v^{\mathrm{b}}_{ni} - \frac{\pi^{\mathrm{b}}_{ij}}{|G^{\mathrm{b}}_{ij}|} = 0 \tag{24}$$

Therefore the utility of winning buyer w_k by bidding v^{b}_{ni} is ≤0. On the other hand, since buyer w_n looses in the auction by bidding v^{b}_{ni}, its utility is zero for this bid. Therefore, our claim holds.

Case 4: If w_n bids either truthfully or untruthfully, he wins in the auction.

From Lemma 3, buyer w_n is charged by equal price for both bids b'^{b}_{ni} and v^{b}_{ni}. Therefore its utility is the same for both bids and the claim holds.
From the aforementioned cases, we result that no buyer can increase its utility by bidding untruthfully. It means our auction mechanism is truthful for buyers. □

Theorem 6: The auction mechanism is truthful for sellers.

Proof: It is required to show that any seller s_m cannot obtain higher utility for one channel of type t_i by bidding other than its valuation for this channel, $b'^{\mathrm{s}}_{mi} \neq v^{\mathrm{s}}_{mi}$. Similar to the previous theorem, there are also four possible cases for bidding of one seller, so we should examine our claim for these cases. It can be done in the way of the previous theorem. We regret that because of space limitations. □

5 Numerical results

In this section, we use simulation to examine the performance of the proposed mechanisms on various auction metrics.

5.1 Impact of flexible bidding

As we mentioned before, our proposed auction mechanism enables spectrum buyers to use flexible demand formats, which is very desirable to the buyers. Here we show that this feature can also increase the number of traded channels in the auction. For this

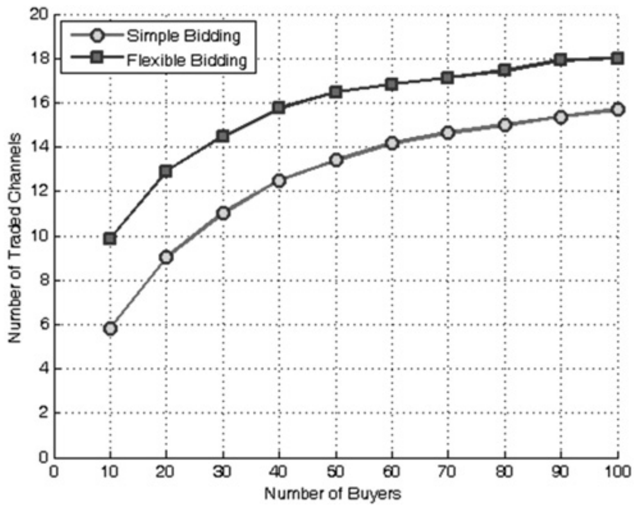

Fig. 4 *Number of traded channels achieved by flexible bidding and simple bidding*

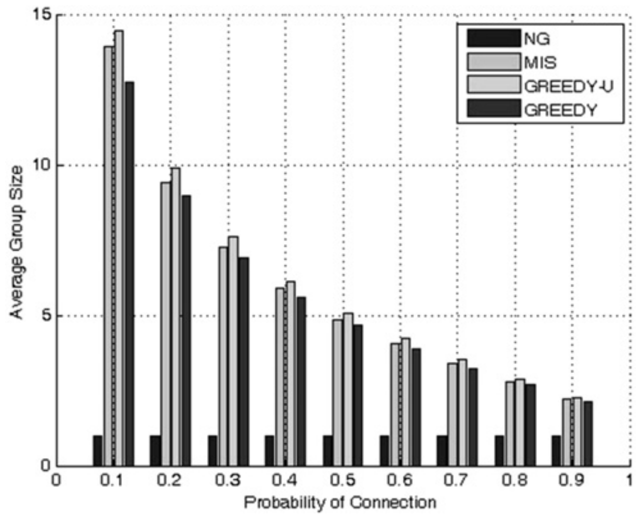

Fig. 6 *Comparison of average group size in the existing grouping algorithms*

purpose, we consider the auction in which the buyers' bids as well as the sellers' reserve prices are randomly distributed over (0, 1]. The number of sellers is set to 20 and the number of buyers varies from 10 to 100. Moreover, the results are averaged over 100 runs. We assume that each seller contributes one channel. We consider two bidding methods: flexible bidding and simple bidding. In the flexible bidding, the demand of each buyer is randomly set as 1, 2 or 3. In the simple bidding, each buyer requests for only one channel. As shown in Figs. 4 and 5, using flexible bidding in the auction increases the number of traded channels as well as the seller satisfaction ratio in comparison with simple bidding.

5.2 Evaluating the performance of the proposed algorithms

We use the Erdős–Rényi model [14] to generate random graphs with various densities. In this model, a graph $G(n, p)$, where n denotes the number of nodes, is constructed by connecting nodes such that each edge is included in the graph with probability p independent from other edges. Therefore the constructed graph $G(n, p)$ has on average $\binom{n}{2}p$ edges and the distribution of the degree of any particular vertex is binomial [15]. Therefore we can conclude that the expected graph density D is equal to p

$$E(D) = \binom{n}{2}p \Big/ \binom{n}{2} = p \tag{25}$$

In this section, we compare the performance of the proposed grouping algorithms, including ABG, EBG and AEBG, with some existing grouping algorithms, including no grouping, maximal independent set (MIS), Greedy, Greedy-U [16, 17]:

- *MIS:* It assigns channels by finding the MISs of the conflict graph.
- *Greedy-U:* To form a group, it recursively chooses a node with the minimal degree in the current conflict graph, eliminates the chosen node and its neighbours, and updates the degree values of the remaining nodes.
- *Greedy:* It is the same as the Greedy-U, except that it chooses the nodes based on its original degree value.

The number of sellers and the number of buyers are set to 10 and 100, respectively, and the probability of connection p varies from 0.1

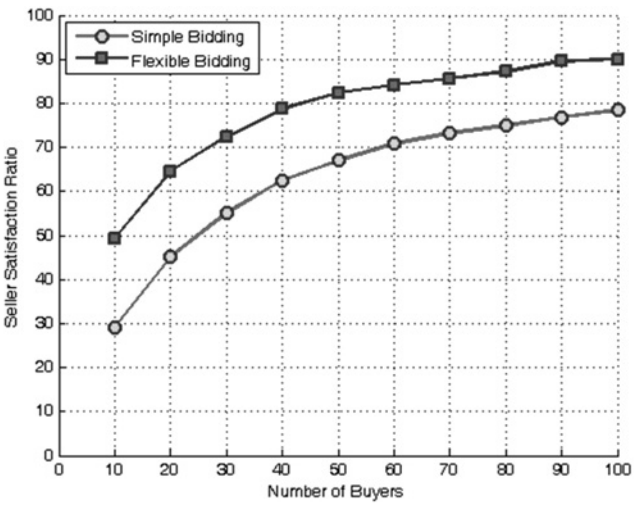

Fig. 5 *Seller satisfaction ratios achieved by flexible bidding and simple bidding*

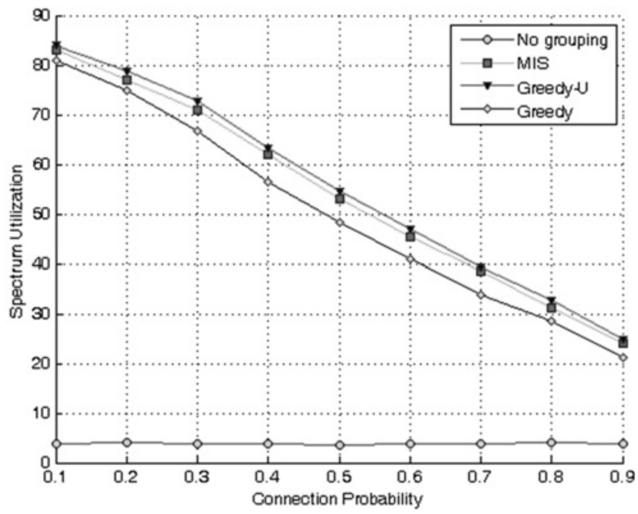

Fig. 7 *Comparison of the spectrum utilisation in the existing grouping algorithms*

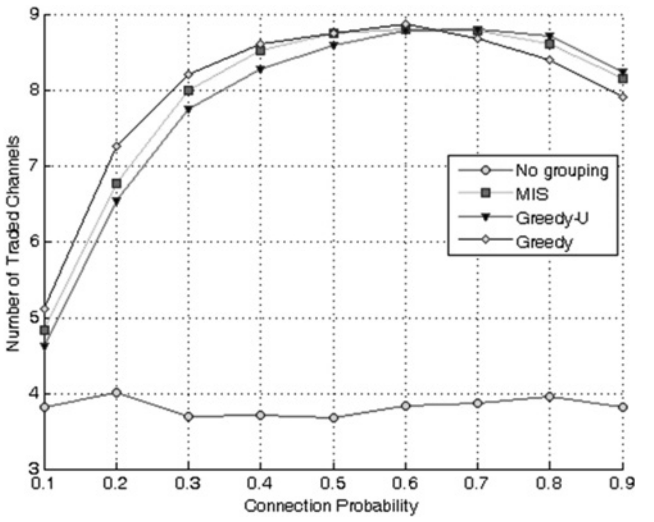

Fig. 8 *Comparison of the number of traded channels in the existing grouping algorithms*

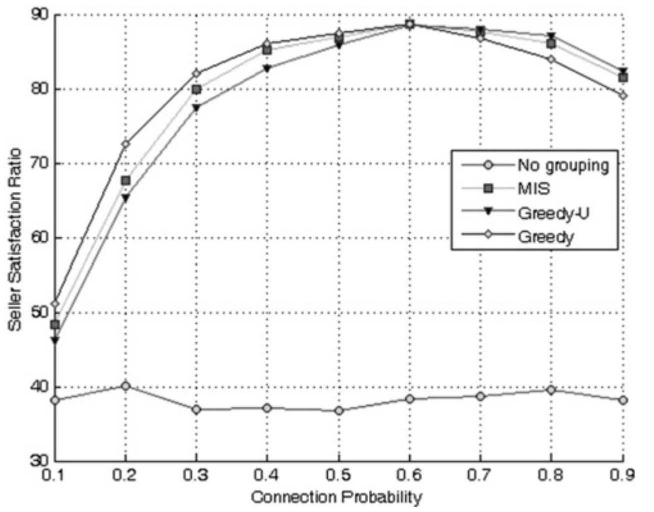

Fig. 9 *Comparison of the seller satisfaction ratio in the existing grouping algorithms*

to 0.9. Moreover, the results are averaged over 100 runs. Fig. 6 illustrates that Greedy-U and Greedy have the largest and the smallest average group size, respectively. However, as stated before and shown in Figs. 7–9, although larger group size can increase spectrum reuse, but it can reduce the seller satisfaction ratio as well as the number of traded channels.

Furthermore, Figs. 7–9 illustrate that by changing the type of graph via changing p, the performance of different algorithms changes. In this regards, in a sparse graph, Greedy has the highest performance on the number of traded channels and the seller satisfaction ratio metrics, whereas, in contrast, Greedy shows the lowest performance in a dense graph on these metrics. Moreover, we see that Greedy-U works just opposite of the Greedy on these performance metrics. Therefore, in ABG algorithm, we use Greedy-U and Greedy in dense graph ($p \geq 0.6$) and sparse graph ($p < 0.6$), respectively. Another proposed algorithm to increase the auction performance metrics was EBG that would modify the created buyer groups in respect to BTSR. Moreover, AEBG algorithm, jointly apply ABG and EBG methods to the auction. We consider the auction in which the buyers' bids and the sellers' reserve prices are randomly distributed over [0, 1] and [0, 2], respectively. Moreover, the number of spectrum buyers and the

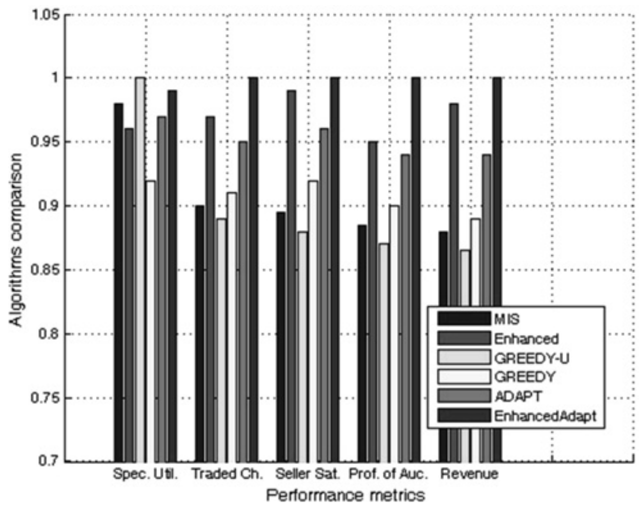

Fig. 10 *Comparison of the proposed algorithms with the existing algorithms*

number of sellers are set to 100 and 10, respectively. All the results on performance metrics are averaged over connection probability p in duration (0, 1) and 100 runs. Fig. 10 compares the performance of the proposed algorithms, including ABG, EBG and AEBG, with the existing algorithms. As illustrated in Fig. 10, each of the ABG and EBG methods improves the number of traded channels, seller satisfaction ratio, profit of auctioneer and auction revenue metrics in comparison with the existing algorithms from two different aspects. As mentioned before, in AEBG, we jointly apply ABG and EBG methods to the auction. On the other words, we apply the EBG method to the groups which achieved by the ABG algorithm. Therefore we expect that AEBG to obtain both improvements of ABG and EBG, as shown in Fig. 10. As illustrated in Fig. 10, AEBG not only achieves a good spectrum utilisation in comparison with the existing algorithms, but also significantly improves the other important performance metrics.

6 Conclusions

We have proposed a multi-channel double auction mechanism which supports heterogeneous spectrums as well as homogeneous spectrums. In this auction, secondary service providers are able to express their preferences over each spectrum type separately. We have proved that our auction design can not only tackle the challenges induced by various aspects of heterogeneity in the spectrum auction, but also preserve three important economic aspects including truthfulness, budget balance and individual-rationality. Furthermore, our auction mechanism enables bidders to use flexible demand formats. We have shown that using the flexible bidding improves the number of traded channels as well as the seller satisfaction ratio in the auction. On the other hand, since buyer grouping affects many auction metrics, we have proposed some adaptive algorithms to improve the existing buyer grouping algorithms. The simulation results illustrate that using the proposed algorithms in the auction not only achieves almost similar spectrum utilisation in comparison with other existing algorithms, but also improves the other important performance metrics.

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