An improved method with interval-valued intuitionistic fuzzy setting to failure mode and effects analysis based on complex proportional assessment

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ABSTRACT

Failure mode and effects analysis (FMEA) is a bottom-up analytical method that helps industrial companies to identify and prioritize their potential failure modes. Identifying serious failure modes and performing corrective actions on the right spot should be done to prevent the customer experience the failure. Despite of the extended use of FMEA in various industries; the conventional FMEA is debatable due to some deficiencies reported in some studies in the field such as possessing equal weights for all three risk factors which may result in biased results in the final prioritization. Furthermore, considering input judgments as crisp data makes it inadequate, in term of considering human uncertainties and ambiguities in FMEA calculation. Therefore, this paper proposes a new method based on group decision-making concept that extends complex proportional assessment (COPRAS) method to improve calculations in FMEA. In addition, this method regards human judgment uncertainty as fuzzy inputs under interval-valued intuitionistic fuzzy environment, as well as Einstein geometric operators for computational tasks. A practical manufacturing example is used to examine applicability and effectiveness of this method. As it will be illustrated, this method has a capability to overcome FMEA's shortage and improves FMEA and decision results.

Keywords: Failure mode and effects analysis (FMEA), interval-valued intuitionistic fuzzy sets, complex proportional assessment

1 INTRODUCTION

Extensive pervasive effort on enhancing competitiveness advantages is the major challenge for manufacturing industries. Therefore, industries have to continually improve their performance in terms of quality and costs, in order to secure the customer satisfaction. Failure mode and effects analysis (FMEA) is a well-known approach, developed to prevent customers from being subject to undesirable and unacceptable faults. FMEA enables organizations to prioritize the failures risks, plan and perform corrective actions, in the right time, to reduce the probability of failures and their

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subsequent drawbacks, i.e., customer dissatisfaction and reduced competitiveness advantages [1-5].

Traditional FMEA is performed through calculating and prioritizing the risk priority number (RPN) of failure modes. RPN index is obtained by multiplying three risk factors including severity of failures (S), likelihood of occurrence (O) and probability of missing the detection of the failures before reaching to customer (D). These factors are evaluated by a cross-functional FMEA teams including experts from different areas. Considering recent studies in this field reveals that despite extensive use of conventional FMEA in various industries, there are some major debates in RPN computation method, including: 1. it is assumed that O, S and D have an equal importance; 2. despite of different risks hidden in different failures, their combinations of O, S and D may produce the same RPN value, and therefore, it may result in false decisions [6].

Mentioned FMEA's deficiencies have been a motivation for several studies, to improve and modify conventional FMEA, by proposing new techniques. There are several studies employing decision making methods in developing new FMEA approaches include technique for order of preference by similarity to ideal solution (TOPSIS) [7], analytical hierarchy process (AHP) [8], Hu et al. [9], VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian))[10], decision making trial and evaluation laboratory technique (DEMATEL) [5], and extended MULTIMOORA by Liu et al. [11] In addition to methods mentioned above, the complex proportional assessment (COPRAS) method proposed by Zavadskas et al. [12], is one of MADM's methods that has been widely used for solving different problems [13, 14]. However, the COPRAS method has not been applied in FMEA.

The issue of ambiguity hidden in experts' evaluation, and difficulty of precise evaluation of failure modes has been investigated by many studies. To this end, fuzzy sets theory is employed to consider uncertainty and vagueness in FMEA [3, 15,16]. The concept of interval-valued intuitionistic fuzzy sets (IVIFSs) introduced by Atanassov and Gargov [17] is a generalized form of intuitionistic fuzzy sets (IFSs) theory, which has an important role in improving decision modeling. It considers membership and non-membership functions as interval values; and hence, has more potential to consider vague situations rather than IFSs [18].

Based on explanations provided above, a new IVIF-method for FMEA is reported in this article which is based on COPRAS method that has been widely used in different decision-making problems; however, it must be applied to FMEA problems. Moreover, the analysis is carried out under an IVIF-environment to manage vagueness and uncertainty hidden in the decisions, and retains more information about experts' evaluation through considering membership and non-membership functions as interval values. Finally, the controversial mathematical form of RPN formula is replaced by Einstein geometric operators reported by Wang and Liu [19], that makes the interpretation of FMEA results reasonable and practical.

The rest of this paper is organized as follows. In section 2, some IVIFs and relevant preliminaries needed to this work are presented. Section 3 presents the proposed IVIF-decision method and Section 4 examine the applicability and performance of new method by illustrating a practical example and follows by discussion. Finally, the conclusion is expressed in sections 5.

2 PRELIMINARIES

In this section, the key concepts of IVIFSs theory are explained.

Definition 1: Let X be an ordinary finite, nonempty set. An IVIFS in X is defined as [19]:

$$\tilde{A} = \{\langle x, \bar{\mu}_{\tilde{A}}(x), \bar{\vartheta}_{\tilde{A}}(x)\rangle\}$$

Where $\bar{\mu}_{\tilde{A}}(x) \subset [0,1]$ and $\bar{\vartheta}_{\tilde{A}}(x) \subset [0,1]$ are interval-valued membership and non-membership functions, respectively. In fact, $\bar{\mu}_{\tilde{A}}$ and $\bar{\vartheta}_{\tilde{A}}$ are closed intervals instead of real numbers and $sup\bar{\mu}_{\tilde{A}}(x) + sup\bar{\vartheta}_{\tilde{A}}(x) \leq 1$, $\forall x \in X$.

In this paper, an IVIF $\tilde{A}$ is denoted by $\langle [a_{\tilde{A}}, b_{\tilde{A}}], [c_{\tilde{A}}, d_{\tilde{A}}]\rangle$, where $[a_{\tilde{A}}, b_{\tilde{A}}] \subset [0,1]$, $[c_{\tilde{A}}, d_{\tilde{A}}] \subset [0,1]$. For each element x, $\tilde{\pi} = [1 - b_{\tilde{A}}(x) - d_{\tilde{A}}(x), 1 - a_{\tilde{A}}(x) - c_{\tilde{A}}(x)]$, is called hesitancy degree of an IVIF of $x \in X$ in $\tilde{A}$. It can be seen that $\tilde{\pi} \subset [0,1]$.

$$\tilde{\alpha}_1 + \tilde{\alpha}_2 = \langle \left[\frac{a_1 + a_2}{1 + a_1 a_2}, \frac{b_1 + b_2}{1 + b_1 b_2}\right], \left[\frac{c_1 c_2}{1 + (1 - c_1)(1 - c_2)}, \frac{d_1 d_2}{1 + (1 - d_1)(1 - d_2)}\right]\rangle \tag{1}$$

$$\tilde{\alpha}_1 \times \tilde{\alpha}_2 = \langle \left[\frac{a_1 a_2}{1 + (1 - a_1)(1 - a_2)}, \frac{b_1 b_2}{1 + (1 - b_1)(1 - b_2)}\right], \left[\frac{c_1 + c_2}{1 + c_1 c_2}, \frac{d_1 + d_2}{1 + d_1 d_2}\right]\rangle \tag{2}$$

$$\lambda\tilde{\alpha} = \langle \left[\frac{(1 + a)^\lambda - (1 - a)^\lambda}{(1 + a)^\lambda + (1 - a)^\lambda}, \frac{(1 + b)^\lambda - (1 - b)^\lambda}{(1 + b)^\lambda + (1 - b)^\lambda}\right], \left[\frac{2c^\lambda}{(2 - c)^\lambda + c^\lambda}, \frac{2d^\lambda}{(2 - d)^\lambda + d^\lambda}\right]\rangle, \lambda > 0 \tag{3}$$

$$\tilde{\alpha}^\lambda = \langle \left[\frac{2a^\lambda}{(2 - a)^\lambda + a^\lambda}, \frac{2b^\lambda}{(2 - b)^\lambda + b^\lambda}\right], \left[\frac{(1 + c)^\lambda - (1 - c)^\lambda}{(1 + c)^\lambda + (1 - c)^\lambda}, \frac{(1 + d)^\lambda - (1 - d)^\lambda}{(1 + d)^\lambda + (1 - d)^\lambda}\right]\rangle, \lambda > 0 \tag{4}$$

Definition 2: Let $\tilde{A}_1$ and $\tilde{A}_2$ be two IVIFNs, then score function $S(\tilde{A})$ and accuracy function $H(\tilde{A})$ will be obtained by the following form [19]:

$$S(\tilde{A}) = \frac{1}{4}[2 + a_{\tilde{A}} - c_{\tilde{A}} + b_{\tilde{A}} - d_{\tilde{A}}] \tag{5}$$

$$H(\tilde{A}) = a_{\tilde{A}} + b_{\tilde{A}} - 1 + \frac{c_{\tilde{A}} + d_{\tilde{A}}}{2} \tag{6}$$

where $S(\tilde{A}) \in [0,1]$ and $H(\tilde{A}) \in [-1,1]$. The larger value of $S(\tilde{A})$ the higher the IVIF is. On this basis, the largest and the smallest IVIFN are $\langle [1,1], [0,0]\rangle$ and $\langle [0,0], [1,1]\rangle$ respectively.

Definition 3: Let $\tilde{\alpha}_j = \langle [a_j, b_j], [c, d_j]\rangle$ $(j = 1,2, \dots, n)$ be a collection of IVIFNs, then an interval-intuitionistic fuzzy Einstein weighted geometric ($IVIFWG_\omega^\varepsilon$) operator is defined as follows [19]:

$$IVIFWG_\omega^\varepsilon(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \tilde{\alpha}_1^{\omega_1} \times \tilde{\alpha}_2^{\omega_2} \times \dots \times \tilde{\alpha}_n^{\omega_n}$$
$$= \langle \left[\frac{2\prod_{j=1}^n a_j^{\omega_j}}{\prod_{j=1}^n (2 - a_j)^{\omega_j} + \prod_{j=1}^n a_j^{\omega_j}}, \frac{2\prod_{j=1}^n b_j^{\omega_j}}{\prod_{j=1}^n (2 - b_j)^{\omega_j} + \prod_{j=1}^n b_j^{\omega_j}}\right], \left[\frac{\prod_{j=1}^n (1 + c_j)^{\omega_j} - \prod_{j=1}^n (1 - c_j)^{\omega_j}}{\prod_{j=1}^n (1 + c_j)^{\omega_j} + \prod_{j=1}^n (1 - c_j)^{\omega_j}}, \frac{\prod_{j=1}^n (1 + d_j)^{\omega_j} - \prod_{j=1}^n (1 - d_j)^{\omega_j}}{\prod_{j=1}^n (1 + d_j)^{\omega_j} + \prod_{j=1}^n (1 - d_j)^{\omega_j}}\right]\rangle \tag{7}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $\tilde{\alpha}_j (j = 1,2, \dots, n)$ such that $\omega_j \in [0,1], j = 1,2, \dots, n$ and $\sum_{j=1}^n \omega_j = 1$.

Explained by Wang and Liu [19], ($IVIFWG_{\omega}^{\varepsilon}$) is an appropriate alternative to the algebraic counterpart, which gives the same smooth approximations as the algebraic operator.

3 PROPOSED IVIF-DECISION METHOD

Step 1. Establishment of cross-functional FMEA team

FMEA is a group analysis that requires cross-functional teams from different areas of study. Due to sensitivity of failures detection issue, establishing this team is first and one of the most important steps in FMEA.

Step 2. Evaluation of failure modes with respect to risk factors

Supposing there is a cross-functional FMEA team including l members, $TM_k(k = 1,2,\dots,l)$; and m potential failure modes, $FM_i(i = 1,2,\dots,m)$ assessed by experts with respect to n risk factors, $RF_j(j = 1,2,\dots,n)$, using IVIF linguistic terms. Then, the performance matrix is established by representing these terms as IVIF numbers, $\tilde{x}_{ij}^{k} = \langle[a_{\tilde{A}}(x), b_{\tilde{A}}(x)], [c_{\tilde{A}}(x), d_{\tilde{A}}(x)]\rangle$. Moreover, relative weights for each risk factor, $\tilde{w}_j^k = \langle[a_{\tilde{w}}^k, b_{\tilde{w}}^k], [c_{\tilde{w}}^k, d_{\tilde{w}}^k]\rangle$ and relative importance of each member in FMEA team $\tilde{\lambda}_k$ satisfying $\sum_{k=1}^{l} \lambda_k = 1$ should be considered. Following matrix shows the k_{th} expert's assessment of m failure modes with respect to n risk factors:

$$X^{(k)} = (\tilde{x}_{ij}^{(k)})_{m\times n} = \begin{bmatrix} \langle[a_{11}^{(k)}, b_{11}^{(k)}], [c_{11}^{(k)}, d_{11}^{(k)}]\rangle & \dots & \langle[a_{1m}^{(k)}, b_{1m}^{(k)}], [c_{1m}^{(k)}, d_{1m}^{(k)}]\rangle \\ \vdots & \ddots & \vdots \\ \langle[a_{n1}^{(k)}, b_{n1}^{(k)}], [c_{n1}^{(k)}, d_{n1}^{(k)}]\rangle & \cdots & \langle[a_{mn}^{(k)}, b_{mn}^{(k)}], [c_{mn}^{(k)}, d_{mn}^{(k)}]\rangle \end{bmatrix}$$

Step 3. Aggregation of experts' assessments and criteria subjective weights using $IVIFWG_{\omega}^{\varepsilon}$ operator mentioned by Eq. (7).

Step 4. Establishment of the weighted performance matrix obtained by following Eq. (8):

$$\hat{\tilde{x}}_{ij} = \tilde{x}_{ij}.\tilde{w}_j$$
$$= \langle\left[\frac{a_{\tilde{x}_{ij}} a_{\tilde{w}_j}}{1 + \left(1 - a_{\tilde{x}_{ij}}\right)\left(1 - a_{\tilde{w}_j}\right)}, \frac{b_{\tilde{x}_{ij}} b_{\tilde{w}_j}}{1 + \left(1 - b_{\tilde{x}_{ij}}\right)\left(1 - b_{\tilde{w}_j}\right)}\right], \left[\frac{c_{\tilde{x}_{ij}} + c_{\tilde{w}_j}}{1 + c_{\tilde{x}_{ij}} c_{\tilde{w}_j}}, \frac{d_{\tilde{x}_{ij}} + d_{\tilde{w}_j}}{1 + d_{\tilde{x}_{ij}} d_{\tilde{w}_j}}\right]\rangle \quad (8)$$

where $\tilde{x}_{ij}$ and $\tilde{w}_j$ are denoted by $\langle\left[a_{\tilde{x}_{ij}}, b_{\tilde{x}_{ij}}\right], \left[c_{\tilde{x}_{ij}}, d_{\tilde{x}_{ij}}\right]\rangle$ and $\langle\left[a_{\tilde{w}_j}, b_{\tilde{w}_j}\right], \left[c_{\tilde{w}_j}, d_{\tilde{w}_j}\right]\rangle$, respectively.

Step 5. Summing the values of criteria for benefit and cost separately. As all three risk factors are benefit criteria, there is only need to calculate the index of benefit criteria' summation:

$$\tilde{P}_i = \sum_{j=1}^{n} \hat{\tilde{x}}_{ij} \quad (9)$$

Step 6. Calculating the relative weight of each alternative (Q_i). Because cost criteria summation is equal to zero, Q_i will be calculated as following:

$$Q_i = S(\tilde{P}_i) + \frac{\sum_{i=1}^{m} S(\tilde{R}_i)}{S(\tilde{R}_i) \sum_{i=1}^{m} \frac{1}{S(\tilde{R}_i)}} \qquad \textbf{(10)}$$

where $\tilde{P}_i = \langle [a_{\tilde{P}_i}, b_{\tilde{P}_i}], [c_{\tilde{P}_i}, d_{\tilde{P}_i}] \rangle$ is benefit criteria' summation of i_{th} failure.

Step 7. Prioritizing the alternatives. The more the Q_i index, the more the risk of failure modes and higher priority of the alternatives.

4 APPLICABLE MANUFACTURING EXAMPLE AND DISCUSSION

In this section, the applicability of the proposed new IVIF-COPRAS method in FMEA is shown by a practical example adapted from Liu et al. [20]. This example involves developing new horizontal directional drilling (HDD) machine that is a complex product with several multidisciplinary sub-systems (e.g., hydraulic system, electric system and engine system). Hence, conducting FMEA can enhance reliability and reduce the likelihood of faults occurrence. Failures assessment is performed by experts as illustrated in Table 1; and translated to IVIF-performance matrix using linguistic terms. Tables 2 and 3 show linguistic variables used for rating failures evaluation [21], and relative importance weights [22] and their relevant IVIF numbers.

Table 1. Assessed information on nine failure modes by five FMEA team members

Risk Factors		*O*						*S*				*D*				
Team members		TM1	TM2	TM3	TM4	TM5	TM1	TM2	TM3	TM4	TM5	TM1	TM2	TM3	TM4	TM5
Failure Modes	FM1	M	ML	M	MH	M	ML	MH	ML	M	M	M	L	ML	ML	ML
	FM2	M	MH	H	MH	MH	MH	MH	H	M	H	MH	MH	ML	MH	M
	FM3	ML	ML	L	M	M	MH	M	H	MH	M	H	H	H	MH	MH
	FM4	L	M	L	M	M	M	MH	ML	M	M	VL	ML	VL	L	VL
	FM5	M	ML	M	M	M	M	H	MH	MH	M	L	ML	VL	L	VL
	FM6	H	MH	M	MH	M	H	H	MH	MH	H	L	M	L	EL	VL
	FM7	VH	MH	VH	MH	H	MH	MH	H	MH	MH	MH	M	MH	M	M
	FM8	ML	M	L	ML	EH	EH	H	EH	H	VH	ML	ML	L	L	L
	FM9	M	M	ML	M	ML	ML	M	ML	ML	M	M	M	M	ML	ML
Weights		H	M	H	M	M	H	VH	H	VH	H	VL	L	L	M	L

Since the FMEA team members have different expertise, it is better to assign them different weights. In this example, these weights are assumed to be 0.15, 0.20, 0.25, 0.10 and 0.30. After

establishing IVIF-matrix of FMEA team members assessments of these opinions are aggregated into the unique assessment by using Eq.(6). The aggregated assessments are provided in the following:

$$X = (\tilde{x}_{ij})_{9\times 3} = \begin{bmatrix} \langle[0.488,0.538],[0.361,0.411]\rangle & \langle[0.476,0.527],[0.372,0.418]\rangle & \langle[0.378,0.430],[0.457,0.507]\rangle \\ \langle[0.608,0.658],[0.241,0.291]\rangle & \langle[0.643,0.693],[0.206,0.251]\rangle & \langle[0.516,0.566],[0.333,0.383]\rangle \\ \vdots & \vdots & \vdots \\ \langle[0.443,0.493],[0.406,0.456]\rangle & \langle[0.448,0.498],[0.401,0.446]\rangle & \langle[0.458,0.508],[0.391,0.441]\rangle \end{bmatrix}$$

Table 2. Linguistic terms and their corresponding IVIF-values

Linguistic terms	IVIF numbers
Extremely Bad/Low (EL)	$\langle[0.02,0.05],[0.90,0.95]\rangle$
Very Bad/low (VL)	$\langle[0.10,0.15],[0.70,0.75]\rangle$
Bad/low (L)	$\langle[0.25,0.30],[0.55,0.60]\rangle$
Medium Bad/Low (ML)	$\langle[0.40,0.45],[0.45,0.50]\rangle$
Fair/Medium (M)	$\langle[0.50,0.55],[0.35,0.40]\rangle$
Medium Good/High (MH)	$\langle[0.60,0.65],[0.25,0.30]\rangle$
Good/High (H)	$\langle[0.70,0.75],[0.15,0.20]\rangle$
Very Good/High (VH)	$\langle[0.80,0.85],[0.05,0.10]\rangle$
Extremely Good/High (EH)	$\langle[0.90,0.95],[0.02,0.05]\rangle$

Table 3. Linguistic terms for the relative importance of criteria

Linguistic terms	IVIF numbers
Very Good/High (VH)	$\langle[0.80,0.90],[0.05,0.10]\rangle$
Good/High (H)	$\langle[0.55,0.70],[0.10,0.20]\rangle$
Medium Good/high (MH)	$\langle[0.45,0.60],[0.15,0.30]\rangle$
Fair/Medium (M)	$\langle[0.30,0.50],[0.20,0.40]\rangle$
Medium Bad/low (ML)	$\langle[0.25,0.40],[0.35,0.50]\rangle$
Bad/low (L)	$\langle[0.10,0.30],[0.45,0.60]\rangle$
Very Bad/low (VL)	$\langle[0.00,0.10],[0.70,0.90]\rangle$

The weighted performance matrix is obtained by Eq. (8).Then, the values for benefit criteria summation ($\tilde{P}_i$) and relative weight of each alternative (Q_i) are calculated as illustrated in Table 4.

Table 4. The values of $\tilde{P}_i$, $\tilde{R}_i$ and Q_i

Failure Modes	$\tilde{P}_i$	$\tilde{R}_i$	Q_i	Rank
FM1	$\langle[0.376,0.618],[0.052,0.129]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.407	7
FM2	$\langle[0.507,0.760],[0.013,0.049]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.602	3
FM3	$\langle[0.402,0.682],[0.035,0.096]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.476	5
FM4	$\langle[0.354,0.557],[0.056,0.135]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.360	9
FM5	$\langle[0.427,0.642],[0.030,0.086]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.476	6
FM6	$\langle[0.503,0.720],[0.013,0.048]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.582	4
FM7	$\langle[0.529,0.785],[0.011,0.045]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.629	2
FM8	$\langle[0.574,0.792],[0.004,0.022]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.670	1
FM9	$\langle[0.347,0.598],[0.066,0.153]\rangle$	$\langle[0.0,0.0],[0.0,0.0]\rangle$	0.363	8

According to Tables 4, failure modes 8 and 7 have higher priorities than the others. As shown in Table 1, they have been assessed as important and very important). Therefore, corrective actions should be performed for these failure modes. Failures 2, 6, 3, 5 and 1 are in the next levels of priorities; this can be seen in experts judgments that evaluated them important, (but less important than failures 7 and 8); and finally failures 9 and 4 have the least values of index Q_i.

It can be concluded that this method has an acceptable performance, and possesses the capacity for further development in this field. In comparison with other FMEA methods, this paper proposed a method based on proportional assessment technique. This method has been widely developed in different decision-making situations; however, there is still no focus on this method to improve the superiority of FMEA. Moreover, interval-valued intuitionistic fuzzy environment is considered for managing vagueness and uncertainty hidden in experts' evaluations. Finally, the questionable mathematical form of RPN formula is replaced with Einstein geometric operators. As a result, the interpretation of FMEA is more reasonable.

5 CONCLUSION

Competitive nature of global marketing compels industries to continually improve their performance in terms of the quality and costs, in order to guarantee the customer satisfaction. Failure mode and effects analysis (FMEA) is an analytical approach developed to help organizations plan and conduct corrective action, to reduce the probability of faults and improve their performance. Despite of its extensive use in different industries, some shortages are identified that debates conventional FMEA calculations. Considering equal weights for all three risk factors may result in wrong RPN priority; and thus, incorrect decisions. Conducting the analysis in crisp terms is another issue that results in elimination of ambiguity and uncertainty hidden in experts' judgments. Since FMEA is a group analysis that requires a team for final evaluation, it can be seen as a group decision-making problem and therefore, can be solved by decision-making methods. Complex proportional assessment (COPRAS) method is one of MADM's methods that has been widely used to solve various decision-making problems. However, there is still lack of effort in developing COPRAS method in FMEA field.

This paper presents a new method which extends COPRAS under interval-valued intuitionistic fuzzy environment, using Einstein geometric operators to conduct FMEA. As a result, weak points of traditional FMEA are reduced and different relative weights for risk factors as well as uncertainty of experts' judgments are considered. Furthermore, the questionable mathematical form of RPN formula is replaced by the Einstein geometric operators that make the interpretation of FMEA result reasonable and practical. Further research will concentrate on improving other possibility theories for representing uncertainty and vagueness hidden in human judgments like D-S evidence theory.

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