DEVELOPMENT OF A MATHEMATICAL MODEL FOR THE PULSE-FREE PART OF THE NEAR FAULT EARTHQUAKES FOR THEIR TIME DOMAIN SIMULATION

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Abstract

The complicated nature of near-field earthquakes and its effects on the structures, especially MDOF systems, have questioned the credibility of design spectra usage for these kinds of seismological events. In comparison to the far field ground motions, a velocity pulse can be detected at the beginning of near-field earthquakes. This pulse is main factor for many disastrous effects on flexible structures as they can impart a considerable amount of energy to the low frequency structural systems. Therefore, it is desired to provide a mathematical representation of the near-field records for problems which lack recorded motion with those specific characteristics. Many researchers have tried to propose a mathematical model for the velocity pulse present in near field earthquakes. However, neglecting the residual (pulse-free) part of those records could lead to certain inadequacies of the resulting simulated near field records. In this study, using Papageorgiou's pulse model, a simplified model for the Fourier transform of the residual part of near-fault records is proposed for their simulation. The model can be calibrated using a number of parameters to properly match the target records. Subsequently, regression analyses are performed in order to relate the model's parameters to the seismological characteristics of the event. Finally, an algorithm is presented to combine the velocity pulse obtained from Papageorgiou's model and the residual record generated using the proposed mathematical model. Various parameters such as, moment magnitude, epicentral distance, etc., are involved in producing the residual record. MATLAB program is used for numerical analyses.

Keywords: Velocity Pulse, Pulse Shape, Background Earthquake, Near-Fault Earthquakes.

1. INTRODUCTION

It is believed that unlike the ordinary earthquake records, consideration of near-fault earthquakes require much more attention than what is simply being used now as response spectrum method. These types of earthquake excitations possess certain characteristics that make the simulation of near-fault earthquakes impossible by simplified methods. Velocity time histories of near-fault earthquakes contain pulses that form their unique behavior. In other words, a "near-fault earthquake" is an earthquake featured with a large energy pulse at the beginning of its record. These pulses are exclusive for the forward direction of wave propagation from source that occurs in the fault-normal direction of ground motion [1].

Different attempts made for defining a proper pulse form to represent the near-fault records have succeeded from some point of views, raising hopes for their simulation. This is of great importance since it can improve the process of analysis and design of structures located in the vicinity of active faults. Various studies have implied that the response of a structure subjected to these pulses can be correlated with the "form of pulse", "pulse intensity' and "ratio of system's fundamental period to the dominant period of pulse". In the last couple of decades, many efforts have been made to determine an efficient pulse form with not much success. Using Hall's 3 pulses model, Alavi and Krawinkler investigated their suitability in reproduction of the recorded near-fault earthquakes' response spectra [1]. Due to their simple form, these pulses were unable to capture the complicated characteristics of the near-fault earthquakes.

Somerville utilized splines to model the ground motion's largest cycle with smooth curves[2]. Having compared with Alavi and Krawinkler's model, Somerville realized that more half cycles should be included in the model for more complete description of these ground motions. Menun and Fu proposed a 5-parameter model whose parameters can be obtained through a nonlinear regression analysis [3]. However, other researchers like Zhu & Xin-Le believed that having a constant period and limited number of half cycles(4 cycles at most) for simulation may lead to unrealistic results for certain types of ground motions [4].

Maveroeidis and Papageorgiou have proposed the following model for ground velocity with 5 parameters [5]:

$$v(t)=\begin{cases}\frac{A}{2}\{1+\cos(\frac{2\pi f_p}{\gamma}(t-t_0))\}\cos(2\pi f_p(t-t_0)+\theta) & t_0-\frac{\gamma}{2f_p}<t<t_0+\frac{\gamma}{2f_p}\\ 0 & otherwise\end{cases} \quad (1)$$

Where A, f_p, θ, γ *and* t_0 represent the pulse's amplitude, dominant frequency, phase lag, zero crossing and envelope function's peak time respectively. Figure 1 shows a schematic representation of Papageorgiou's model. Similar to Menun and Fu's model, this model also has difficulty in predicting spectral values for short-period region of response spectra. Therefore, they used the "Specific Barrier Method" to produce the high frequency part of these earthquakes:

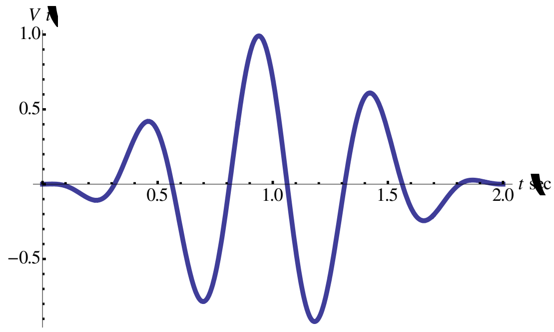

Figure 1: A Schematic Representation of Papageorgiou's Model[3]

Knowing that a pulse cannot reflect all features of a near-fault earthquake, the aim of this work is to reproduce the remaining part of the record (residual or background), once the main pulse is extracted from original record. Having assessed the previous models, the Papageorgiou's model is adopted here to simulate the pulse-like signal of the near field earthquakes. Then, some regressive equations will be presented to simulate the near-fault earthquakes according to the seismological characteristics of a specific site.
In this study, all 91 near-fault records identified by Baker out of over 3500 NGR records of Berkeley University are considered [6]. All these records have PGV's greater than 30 cm/sec^2 and the moment magnitudes greater than 5.5. Response spectra method is used for determining the response of SDOF systems to earthquake excitation.

2. SIMULATION OF THE RESIDUAL EARTHQUAKES

As it was mentioned earlier, there are two major components in any near-fault earthquakes, e.g. the pulse-like and the residual ground motions. Taking advantage of the comprehensive study done by Baker in reproducing the main pulse of near-fault earthquakes [6], the resulting residual records presented in that work is used here for regression analyses and simulation of the residual part of near-fault earthquakes. Different mathematical functions with different number of parameters are considered in the regression analyses, using the Fourier amplitudes of 91 Baker's residual records with different frequency bandwidths as the optimality parameters. Figure 2 shows the Fourier amplitude of a residual record that is obtained by pulse extraction from the Coyote Lake 1979 earthquake recorded at Gilroy Array #6 station [6].
A variety of functions can be assumed for variation of Fourier amplitude over a preferably limited frequency domain of the residual records. Sensitivity analyses performed indicate that eliminating frequencies of the Fourier spectrum of the residual records with insignificant Fourier amplitudes, does not lead to any appreciable change in the spectral shape of the overall record, i,e., the combined residual record and the main pulse. For example, considering a constant Fourier amplitude distribution for the residual records as shown in Figure 3, one could obtain the time domain acceleration record by taking its inverse Fourier transform:

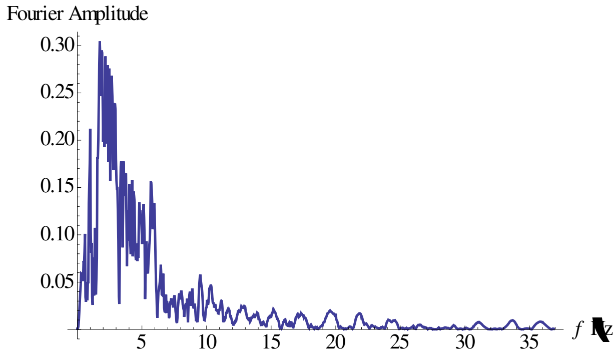

Figure 2: Fourier Amplitude Spectra for Residual Record of Coyote Lake 1979 Earthquake Recorded at Gilroy Array #6 Station

$$a(t) = 2\frac{A}{\pi t}\sin(\frac{\omega_2 - \omega_1}{2}t)\cos(\frac{\omega_2 + \omega_1}{2}t) \qquad (2)$$

In which A, ω_1 , *and* ω_2 are the Fourier amplitude and the lower and upper bounding frequencies respectively. The acceleration and velocity time histories resulted from equation (2) are shown in Figure 4. However, the resulting displacement time history would asymptotically converge to a non-zero final value that needs to be corrected.

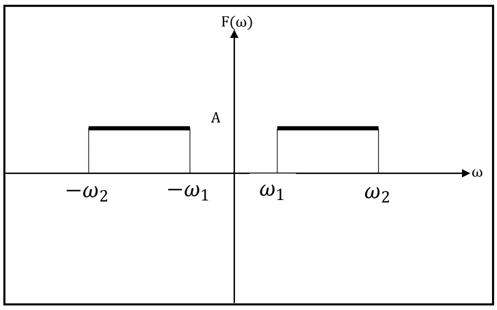

Figure 3: A Simple Fourier Amplitude Distribution Function Considered for the Residual Earthquakes

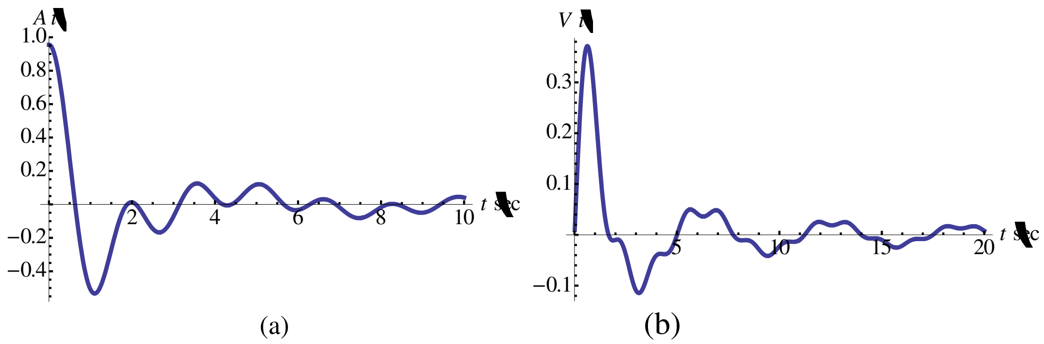

Figure 4: Schematic (a)Acceleration, and (b) Velocity Time Histories of the Considered Fourier Amplitude Distribution Function.

Also, in order to satisfy the zero initial acceleration of the considered signal in a general sense, one can write:

$$a(0) = \ddot{x}(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega(0)}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}(R(\omega) + i\,Z(\omega))\,d\omega \xrightarrow{\text{neglecting imaginary part}}$$
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} R(\omega)\,d\omega \xrightarrow{\text{If } R(\omega) \text{ is even}} \frac{2}{2\pi}\int_{0}^{\infty} R(\omega)\,d\omega = 0 \rightarrow \int_{0}^{\infty} R(\omega)\,d\omega = 0 \qquad (2)$$

Therefore, in the selection of any Fourier amplitude distribution function for the residual record, the above zero initial acceleration condition should be satisfied. In this study, a constant distribution function, as shown in Fig. 5, is considered for the real part of the Fourier transform of the residual record. The imaginary part of its Fourier transform could be assumed any function such that the related integral in Eq. (3) becomes zero. For simplicity, the imaginary part of the Fourier transform is assumed to be zero in this study.

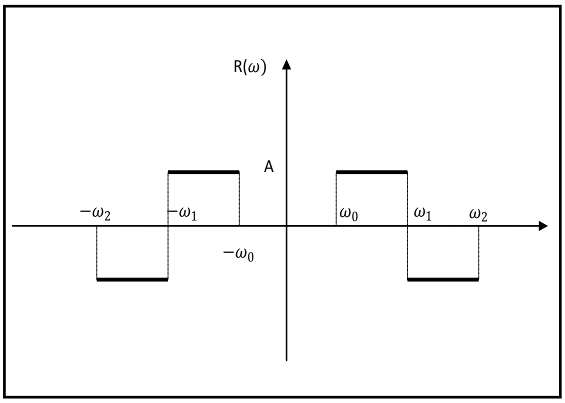

Figure 5: The Real Part of Fourier Transform Function Which Satisfies the Zero Initial Acceleration Condition.

Taking inverse Fourier transform of the function shown in Fig. 5, leads to the following relation for the residual acceleration $a(t)$:

$$a(t) = \frac{4A\sin^2(\frac{\Delta\omega}{2}t)\,\sin(\omega_1 t)}{\pi t} \quad (4)$$

Where ω_1 is the median frequency and $\Delta\omega$ is equal to the absolute difference between either ω_1 and ω_0 or ω_1 and ω_2. The time histories of the resulting acceleration and velocity of the residual record are shown in Fig. 6. As one can see, the zero initial acceleration condition is satisfied by the assumed Fourier amplitude distribution function.

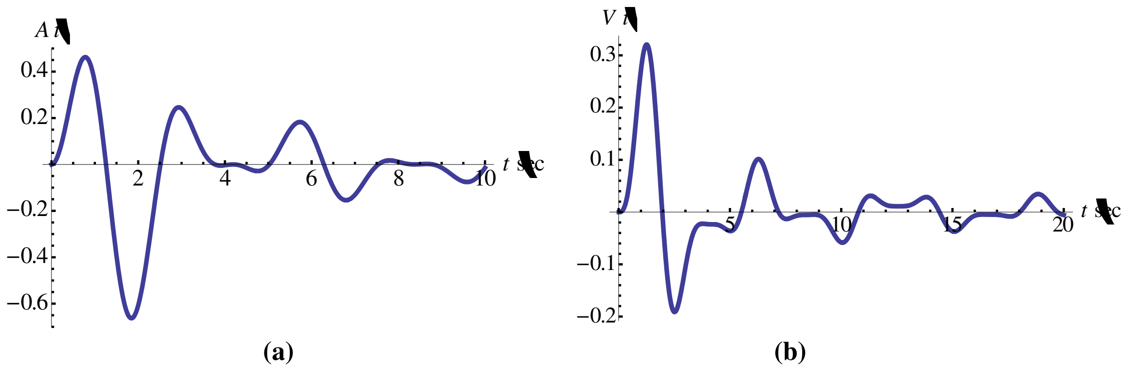

Figure 6: (a) Acceleration, and (b)Velocity Time Histories of the Corrected Fourier Amplitude Distribution Function.

To investigate the efficiency of this model in simulating the residual records, a MATLAB based program was prepared to calculate the Fourier transform of the existing residual records and to determine a number of parameters such as the maximum Fourier amplitude and the corresponding dominant frequency for each residual record. Then, a sensitivity analysis is performed to determine the domain of frequencies(ω_0-ω_2) and the resulting average Fourier amplitude in that domain so that the considered simplified constant banded Fourier amplitude can represent the residual record with least error. As an example, Figure 7 shows, the frequencies ω_0 and ω_2 that are determined such that the average Fourier amplitude of the record is around 0.5 $F(\omega)_{max}$ in which the $F(\omega)_{max}$ is the maximum Fourier amplitude of the earthquake record. The average of the Fourier amplitude within the prescribed frequency domain ω_0 - ω_2, which is represented by A in Eq. (4), is a key parameter which affects the outcome of this approach to a great extent, and therefore should be optimally determined.

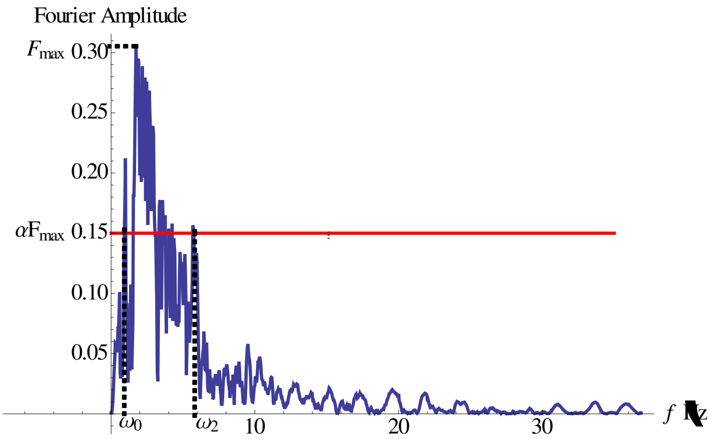

Figure 7: The Procedure for Determining the Model's Parameters

3. COMBINING THE RESIDUAL EARTHQUAKE WITH THE PULSE-LIKE MOTION

Having defined the parameters of the proposed method, the reproduced residual record should be combined with a pulse-like motion in order to obtain a synthetic near-fault record. As it was mentioned before, Papageorgiou's model is used for generating the pulse-like part of near-fault records.

In combining these two parts, the pulse-like ground motion starts from the beginning of, i.e., $t=0$., while the residual record is added to the pulse motion after a time delay and right after the pulse-like motion reaches its maximum amplitude. That is due to the fact that the velocity pulse caused by the forward directivity occurs at the beginning of the near fault records. A sensitivity analysis performed on the magnitude of the time delay between the outset of the pulse motion and the initiation of the residual records on the system's spectral values shows its insignificant effect on the Fourier amplitude of the synthetic records. This procedure is repeated for all the 91 records of Baker's study. The results of synthetic records reveals that the best agreement between the spectral responses of synthetic and real records is achieved for the average Fourier amplitude equal to 0.6 $F(\omega)_{max}$.

4. REGRESSION RELATIONSHIPS FOR THE MODEL PARAMETERS

In generating the near-fault earthquakes for potential sites with insufficient number of near fault earthquake records, the source characteristics and its distance to the site should be correlated to the model's parameters. The results of the synthetic records(combination of the pulse-like motion and the residual ground motions) and real records of Baker's database is used as a basis to develop such relationships.

It is more convenient to categorize the existing earthquake database according to the soil type, in which the Rodriguez's soil classification was adopted. As an option, the three parameters of the model, i.e., the median frequency ω_m, the frequency bandwidth $\Delta\omega$, and the effective Fourier amplitude, A, can be related to the important earthquake characteristic, i.e., earthquake magnitude and the epicentral distance represented by M_w and R respectively. Therefore, the following attenuation relationship is considered for the regression analyses:

$$Ln(Z) = a + bM + cLn(R) \tag{5}$$

Where *Z, M* and R are the model parameters, moment magnitude, and the site to source distance respectively. Using a simple least square analysis, the obtained regression parameters, *a*, *b*, and *c* are listed in Table 1) for the specified soil types. Due to insufficient data, the soil type E has been eliminated from the database and the earthquake records on soil types A and B as well as C and D are categorized as separate soil types.

Table 1: The Calculated Coefficients to Reproduce the Parameters of the Model for Soil Types (C, D) and (A,B)

Soil type C,D	ω_c	$\Delta\omega$	A
a	5.07	3.11	-4.39
b	-0.34	-0.10	0.44
c	-0.12	-0.22	-0.32

Soil type A,B	ω_c	$\Delta\omega$	A
a	8.56	16.23	-5.16
b	-1.07	-2.75	1.23
c	0.51	1.59	-1.79

4. Generation Of Synthetic Near-Fault Records

In order to verify the efficiency of the proposed model in generating the near-fault earthquakes, the reproduced synthetic records are compared with the original records of Baker's database. First, pulse's dominant period is determined using the relationship suggested by Papgeorgiou [3]:

$$Log(T_p) = -2.9 + 0.5M_w \tag{6}$$

Then, Eq. 1 is used to simulate the pulse-like motion of the record assuming, $t_0 = \gamma / 2f_p$ so that the pulse-like motion starts at t = 0. Since the value of parameter γ is not known in advance, a sensitivity analysis is carried out by assigning different values to that parameter in order to have a better match between the simulated and the original ground motions(such as 1.2, 2 and 3). The parameter ν was assumed to be equal to 45°. Due to the existing difference between the Alavi and Krawinkler's database and Baker's earthquake records, a new regressive relation was developed to correlate the pulse's effective intensity and the earthquake parameters as the following (instead of using Alavi and Krawinkler's relation):

$$Ln(v_{eff}) = 3.18 + 0.19M_w - 0.19Ln(R) \tag{7}$$

Given the seismological information of Baker's collection, a synthetic pulse-free residual record can now be generated. Finally, combining the pulse-like motion and the residual record would result in the synthetic ground motion. As an example, Figs. 8 and 9 compare the response spectrum parameters of the combined synthetic and the original records. It seems that compared to the case of Fig. 9, better compliance with the original record is achieved for the case shown in Fig. 8.

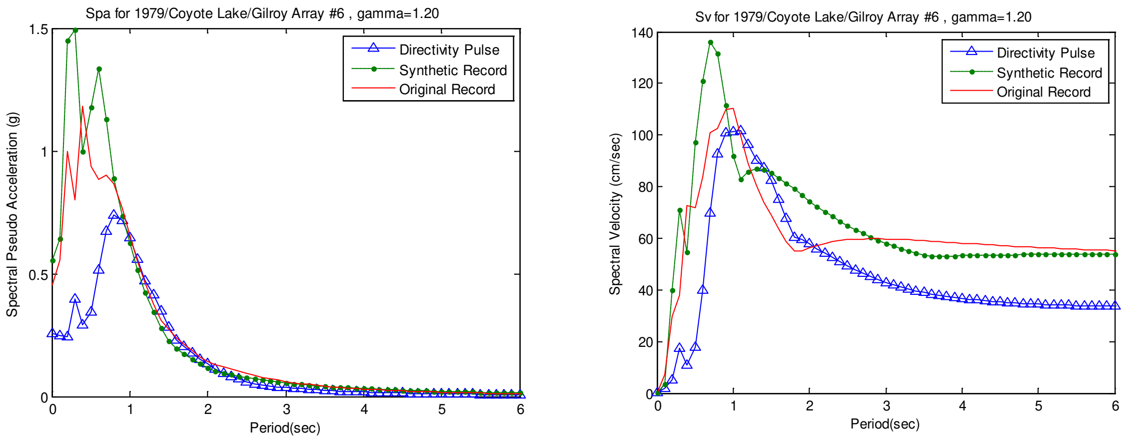

Figure 8: The Response Spectrum Parameters of the Simulated and Original Records for 1979 Coyote Lake / Gilroy Array #6.

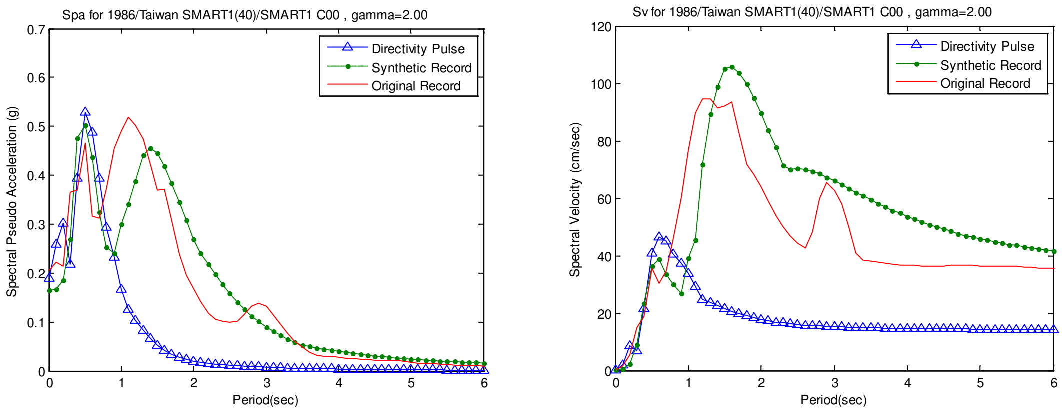

Figure 9: The Response Spectrum Parameters of the Simulated and Original Records for 1986 /Taiwan SMART1(40) /SMART1 C00.

5. CONCLUSION

Various attempts have been made to develop simple mathematical models to represent the near fault earthquakes with forward directivity without much success. Ignoring the effect of background or residual earthquake that mainly affects the low period region of the response spectra is among the main concerns. In this study, an approach is proposed to model the residual records without using a complex random vibration based solution for the problem. Using Papageorgiou's model for the pulse-like part of the ground motion, a simplified three parameters model is proposed for the Fourier transform of the residual part of near-fault records. To calibrate the three parameters of model, i.e., frequency domain, effective Fourier amplitude and the median frequency, a parametric study is carried out using Baker's database. After calculation of the model parameters, regression equations were developed relating the value of those parameters to the earthquake records important characteristics such as the earthquake magnitude and the site to source distance. The suggested algorithm for simulating the near-fault earthquake records, include the simulation of pulse-type ground motion using Papageorgiou's model and the generation of residual earthquake using the proposed approach. Finally, the combination of these two parts leads to the simulated near fault earthquake records. The limited results of the parametric studies indicate that the proposed model has been partially successful in improving the deficiencies of the synthetic near fault ground motions within the low period region of their response spectra.

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