A Self-Starting Control Chart for Simultaneous Monitoring of Mean and Variance of Autocorrelated Simple Linear Profile

Amirhossein Amiri*, Reza Ghashghaei, Peyman Khosravi
Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran
amiri@shahed.ac.ir, r.qashqaei@shahed.ac.ir, pay.khosravi@shahed.ac.ir

Abstract – **Sometimes, quality of a process can be described by a functional relationship between response variables and explanatory variables which called profile. In some situations, there is an autocorrelation structure within a profile. Most of the times in real practice there is no enough data to estimate the process parameters. In this case, we can use a self-starting control chart which does not need preliminary data to start monitoring in start-up stages. In this paper, we consider a simple linear profile in the presence of a first order autoregressive (AR(1)) autocorrelation structure within profile and propose a self-starting control chart to monitor mean and variance of a simple linear profile simultaneously.**

Keywords - **Self-starting control chart, simultaneous monitoring, Recursive residuals, Simple linear profile.**

I. INTRODUCTION

Statistical process monitoring (SPM) is a useful method applied to monitor industrial processes. Control charts are one of the most important tools which are used to monitor the industrial processes. In some cases, the quality of process is characterized by a relationship between a response variable and one or more explanatory variables which is called profile in the literature. Two control charts are developed to monitor simple linear profiles in Phase II [1]. Three EWMA control charts which monitor intercept, slope and standard deviation in simple linear profiles [2]. A cumulative sum (CUSUM) control chart is proposed to monitor simple linear profiles in Phase II [3]. Some monitoring schemes are proposed for simple linear profiles by using variable sample size [4].

In SPM, it is needed to distinguish between monitoring procedures of Phases I and II. In Phase I, we need to analyze the process to estimate process parameters and investigate if the process is in-control (IC). In Phase II monitoring, the process parameters are presumed to be known. However, maybe there is always no enough data to perform Phase I analysis. Hence, it is necessary to apply a kind of control chart which can start monitoring the process without the need of large amount of preliminary observations. Self-starting control chart updates the parameters estimation with each new observation and checks for out-of-control (OC) condition simultaneously. A self-starting CUSUM control chart is developed for location and scale, by using some theoretical properties of residuals independency [5]. A self-starting multivariate exponentially weighted moving average (MEWMA) control chart is proposed in [6]. They transformed the unknown process parameters vector into known process parameters vector with the same dimension in their study. A self-starting control chart is developed to monitor process mean and variance simultaneously based on likelihood ratio test (LRT) method and EWMA procedure [7]. A self-starting control chart for high-dimensional short run processes is proposed which can monitor the process at the start-up stages without sufficient initial data [8]. A self-starting control chart to monitor simple linear profiles is developed in [9]. The proposed control chart enables to detect shifts in the intercept, slope, and/or standard deviation.

In all aforesaid studies, it is assumed that the error terms in the model are i.i.d normal random variables. However, in some cases this assumption can be violated. Within profile autocorrelation in Phase I is addressed by [10] using a linear mixed model (LMM). Authors in [11] investigated simple linear profiles over time with autocorrelation between profiles. [12] proposed a monitoring scheme to monitor polynomial profiles in Phase I over time .

A real case about autocorrelation within simple linear profiles is discussed in [13] and [14]. In this example, apples are sampled from apple trees randomly and the diameters of the selected apples are measured. In this case, the diameter and the time are modeled by a simple linear profile. As mentioned in [13] and [14], there is within profile AR(1) autocorrelation structure. If we want to monitor the process from the initial stages of the process or enough data is not available for Phase I studies and correct estimations of the process, a self-starting chart should be used. Moreover, simultaneous monitoring of location and variability of a quality characteristic is discussed by some researchers in the literature. Hence, in this paper, we develop a self-starting control chart for monitoring the regression parameters and error standard deviation simultaneously in AR(1) autocorrelated simple linear profiles.

The remainder of this paper is organized as follows: in the next section, we present problem formulation. In Section III, we propose a self-starting sum of squares exponentially weighted moving average (SS-EWMA) control chart. The simulation studies and performance evaluation are presented in Section IV. The concluding remarks and future researches are given in the final section.

II. PROBLEM FORMULATION

If (x_i, y_{ij}) is the jth random sample observed over the time, the relationship between y_{ij} and x_i under in-control situation presumed to be as follows:

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij}, \quad ; i = 1,2,...,n \ , \qquad \varepsilon_{ij} = \rho\varepsilon_{(i-1)j} + a_{ij} \ , \tag{1}$$

where ε_{ij} 's are correlated error terms and a_{ij} 's follow $N(0,1)$ [14]. y is response variable and x is the explanatory variable. Since there is no enough data in many processes in real practice, the parameters A_0, A_1 and σ^2 are not known a priori and should be estimated by using (2), (3) and (4), respectively as they are used in [1], [2] and [9].

$$b_{1j} = \frac{S_{xy}(j)}{S_{xx}}, \tag{2}$$

$$b_{0j} = \bar{y}_j - b_{1j}\bar{x}, \tag{3}$$

$$MSE_j = \frac{1}{n-2}\sum_{i=1}^{n}(y_{ij} - b_{1j}x_i - b_{0j})^2, \tag{4}$$

where $\bar{y}_j = (1/n)\sum_{i=1}^{n} y_{ij}$, $\bar{x} = (1/n)\sum_{i=1}^{n} x_i$, $S_{xx} = \sum_{i=1}^{n}(x_{ij} - \bar{x})^2$, and $S_{xy}(j) = \sum_{i=1}^{n}(x_i - \bar{x})y_{ij}$.

Note that the AR(1) structure between error terms in (1), results in autocorrelated observations at different x values in each profile [14], so the observations in each profile can be written as follows:

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij} \quad \text{and}$$
$$y_{(i-1)j} = A_0 + A_1 x_{(i-1)} + \varepsilon_{(i-1)j}$$

then we have

$$y_{ij} - (A_0 + A_1 x_i) = \varepsilon_{ij} = \rho[y_{(i-1)j} - (A_0 + A_1 x_i)] + a_{ij} \tag{5}$$

[14] proposed a transformation method for observations to eliminate the within profile autocorrelation in simple linear regression profile. This transformation is given in (6) as:

$$Y'_{ij} = Y_{ij} - \rho Y_{(i-1)j} \ . \tag{6}$$

If we substitute the simple linear model (1) in Y_{ij} and $Y_{(i-1)j}$ available in (6), a simple linear profile with independent error terms is obtained as follows:

$$Y'_{ij} = A_0(1-\rho) + A_1(X_i - \rho X_{i-1}) + (\varepsilon_{ij} - \rho\varepsilon_{(i-1)j}) \ , \quad i = 1,2,...,n \tag{7}$$

Hence, we have

$$Y'_{ij} = A'_0 + A'_1 X'_i + a_{ij} \ , \tag{8}$$

where $Y'_{ij} = Y_{ij} - \rho Y_{(i-1)j}$, $X'_{ij} = X_{ij} - \rho X_{i-1}$, $A'_0 = A_0(1-\rho)$, and $A'_1 = A_1$, and a_{ij} 's follow $N(0,\sigma^2)$. In this paper, we assumed that ρ is a known parameter.

Since it is assumed that there is no enough initial sample at the start-up stages to estimate the process parameters, we used recursive residuals to design a self-starting control chart which can be applied at the beginning of the process to monitor simple linear profile. Now according to [9] suppose that there are $m-1$ in-control existing data and $m, m+1,...$ future samples with size n . If all the $m, m+1,...$ in-control existing data and $m, m+1,...$ future data pooled together in one sample, i.e. $\{(x_i, y_{ij}), i = 1,2,...,n,$ $j = 1,2,...,m-1, m, m+1...\}$, then we can calculate the standardized recursive residuals for each future sample by using (9) as it is applied in [9].

$$e_{ij} = \left(y_{(j-1)n+i} - \mathbf{z}_i'\boldsymbol{\beta}_{(j-1)n+i-1}\right) \div [S_{(j-1)n+i-1} \times (1 + \mathbf{z}_i'(\mathbf{X}'_{(j-1)n+i-1}\mathbf{X}_{(j-1)n+i-1})^{-1}\mathbf{z}_i)]^{1/2}, \tag{9}$$

$$i = 1,2,...,n \ , \ j = m, m+1,...$$

where

$$\mathbf{z}'_i = (1, x_i), \tag{10}$$

$$\mathbf{y}'_{(j-1)n+i-1} = (y_1, y_2, y_3, ..., y_{(j-1)n+i-1}), \tag{11}$$

$$\mathbf{X}'_{(j-1)n+i-1} = (\overbrace{z_1, z_2, ..., z_n, z_1, z_2, ..., z_n, ...,}^{(j-1)\times n} z_1, z_2, ..., z_{i-1}), \tag{12}$$

$$\boldsymbol{\beta}_t = (\mathbf{X}'_t\mathbf{X}_t)^{-1}\mathbf{X}'_t\mathbf{y}_t, \tag{13}$$

$$S_t = \frac{1}{t-2}(\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta}_t)'(\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta}_t). \tag{14}$$

And for simplicity let $y_{(j-1)n+i} = y_{ij}$, $i = 1,2,...,n$, $j = 1,2,...$ [9]. Note that (10)-(14) are also applied by [9].

The e_{ij} value of each observation depends on estimated regression parameters ($\boldsymbol{\beta}$), standard deviation (S) and the $\mathbf{X}'\mathbf{X}$ value of previous observations, so to avoid the high volume of calculations to reach each observation's e_{ij} , [9] used recursive formulas (15), (16) and (17) as:

$$\left(\mathbf{X}_t'\mathbf{X}_t\right)^{-1} = \left(\mathbf{X}_{t\text{-}1}'\mathbf{X}_{t\text{-}1}\right)^{-1} - \frac{\left(\mathbf{X}_{t\text{-}1}'\mathbf{X}_{t\text{-}1}\right)^{-1}\mathbf{z}_i\mathbf{z}_i'\left(\mathbf{X}_{t\text{-}1}'\mathbf{X}_{t\text{-}1}\right)^{-1}}{1+\mathbf{z}_i'\left(\mathbf{X}_{t\text{-}1}'\mathbf{X}_{t\text{-}1}\right)^{-1}\mathbf{z}_i}, \quad (15)$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \left(\mathbf{X}_{t\text{-}1}'\mathbf{X}_{t\text{-}1}\right)^{-1}\mathbf{z}_i(\mathbf{y}_t - \mathbf{z}_i'\boldsymbol{\beta}_{t-1}), \quad (16)$$

$$S_t^2 = \frac{(t-3)S_{t-1}^2 + (e_{ij})^2}{t-2}, \quad (17)$$

where $t = (j-1)n + i$.

Reference [15] showed that under the in-control linear model e_{ij} follows t distribution with $(j-1)n+i-3$ degrees of freedom [9]. Also, it is proved by [16] that the e_{ij} 's are independent [9]. Hence, by using a transformation, the Q_{ij} statistic which is called *Q*-statistic by [17] is obtained as follows:

$$Q_{ij} = \Phi^{-1}[T_{(j-1)n+i-3}(e_{ij})], \quad (18)$$

where Φ^{-1} denotes the inverse of cumulative distribution function of standard normal random variable, $T_\nu(.)$ is the cumulative distribution function of the *t* distribution with ν degrees of freedom [9]. So, $\{ Q_{ij}, i = 1,2,...,n, \quad j = 1,2,...,m-1,m,m+1...\}$ is a sequence of random variables which are independent and follow $N(0,1)$ [9].

III. Proposed Self-Starting SS-EWMA Control Chart

[18] proposed a chart known as the semicircle chart and [19] has developed this kind of control charts to monitor multivariate profiles in Phase II. This control chart can also identify which parameter (mean or variance) has changed.

This paper proposes the combination of transformation technique in [14], self-starting control chart in [9] and SS-EWMA in [20] to develop a self-staring control chart to monitor the location and variability of AR(1) simple linear profiles. This is the main contribution of the paper.

When a special cause after some subgroups (denoted by τ) happens, the *Q*-statistics' distribution when $j = \tau+1, \tau+2,....$ is different from their distribution when $j = 1,2,...,\tau$. This difference is used to detect assignable cause in process.

In order to monitor residuals we need $\bar{Q}_j = (1/n)\sum_{i=1}^{n} Q_{ij}$ and $S_{Q_j}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Q_{ij} - \bar{Q}_j)^2$ as mean and variance for *j*th sample, respectively. These estimators are unbiased, independent and also follow different distributions. The following equations are used to change the statistics' distribution to $N(0,1)$ for $\bar{Q}_j$ and $S_{Q_j}^2$, respectively.

$$Z_j = \frac{\sqrt{n}\left(\bar{Q}_j\right)}{\sigma_{Q_j}}, \quad (19)$$

$$F_j = \Phi^{-1}\left\{H\left[\frac{(n-1)S_{Q_j}^2}{\sigma_{Q_j}^2}; n-1\right]\right\}, \quad (20)$$

where Z_j follows $N(0,1)$, $H[X;\upsilon] = P(X \le x)$ is chi-square cumulative distribution function of X with υ degrees of freedom and Φ^{-1} is the inverse of the standard normal cumulative distribution function [19]. Now Z_j and F_j have the same distribution. For the *j*th random sample, the EWMA statistic is:

$$U_j = \theta Z_j + (1-\theta)U_{j-1}. \quad j = 1,2,... \quad (21)$$

In (21), U_0 is the starting point which is equal to zero and the smoothing parameter (θ) takes the value in (0,1] interval.

The EWMA statistics derived from F_j for monitoring process variability is given by

$$V_j = \theta F_j + (1-\theta)\, V_{j-1}, \quad j = 1,2,... \quad (22)$$

In (22), $V_0 = 0$. According to (21) and (22), the self-starting-SS-EWMA control chart statistic is formed as (23) [19].

$$EW_j = U_j^2 + V_j^2 . \quad (23)$$

Since EW_j is the sum of squares of U_j and V_j which are EWMA statistics, then the chart designed based on EW_j referred to as a self-starting SS-EWMA control chart [19]. self-starting SS-EWMA statistic represents the equation of a circle with radius equal to $\sqrt{UCL}$. U_j and V_j follow the normal distribution. EW_j follows a chi-square distribution with 2 degrees of freedom when it is divided by $\sigma_{U_j}^2$, Since $\frac{U_j}{\sigma_{U_j}}$ and $\frac{V_j}{\sigma_{V_j}}$ are independent, identical and have standard normal distribution.

$$\frac{EW_j}{\sigma_{U_j}^2} = \frac{U_j^2}{\sigma_{U_j}^2} + \frac{V_j^2}{\sigma_{U_k}^2} \sim \chi^2(2) \quad (24)$$

where

$$\sigma_{U_j}^2 = \sigma_{V_j}^2 = \frac{\theta}{2-\theta}[1-(1-\theta)^{2j}]. \quad (25)$$

Hence,

$$E(EW_j) = 2\sigma_{U_j}^2 , \quad (26)$$

$$\text{Var}(EW_j) = 4\sigma_{U_j}^4 . \quad (27)$$

Since $EW_j > 0$, then the self-starting SS-EWMA chart has merely upper control limit (*UCL)* given in (28):

$$UCL = E(EW_j) + L\sqrt{\text{Var}(EW_j)} = \frac{2\theta}{2-\theta}(1-(1-\theta)^{2j})(1+L) . \quad (28)$$

UCL in (28) is obtained by simulation to achieve a desired in-control ARL. If $EW_j > UCL$ then the control chart signals an out-of-control condition.

IV. Performance evaluation

In this section, we use an illustrative example to evaluate the ARL performance of the proposed chart through 10000 simulation runs. The smoothing parameter θ in all the EWMA statistics is set equal to 0.2. *L* in SS-EWMA and self-starting SS-EWMA is set equal to 3.815 and 3.675, respectively to guarantee the ARL_0 equal to 200 for each control chart. The profile model used in this paper is:

$$y_{ij} = 3 + 2x_i + \varepsilon_{ij},$$
$$\varepsilon_{ij} = \rho\varepsilon_{(i-1)j} + a_{ij},$$

where a_{ij} follows $N(0,1)$ [14]. The explanatory variable *x* values are considered to be 2, 4, 6 and 8.

The results in Table I showed the out-of-control (OC) ARLs of self-starting SS-EWMA control chart under different change point values (τ) and SS-EWMA control chart which is designed in Phase II. In this table, λ is the magnitude of shift in intercept and δ is the magnitude of shift in slope, respectively in unit of sigma. Also, γ is the shift value in standard deviation. The results showed that when the number of historical samples observed before occurring a shift increases, the performance of the self-starting SS-EWMA chart improves under all shifts in the intercept, slope and standard deviation. Similar to the results in [9], when the number of reference samples (τ) increases, the performance of self-starting control chart for monitoring a simple liner profile gets better

.

TABLE I
OC ARLs Of Self-Starting SS-EWMA Chart Under Different Change Points (τ) And SS-EWMA Chart With True Parameters (ARL_0=200)

		τ values of Self-starting SS-EWMA			
	λ	20	50	100	SS-EWMA
$\beta_0 + \lambda\sigma$	0.4	82.54	31.77	19.37	15.05
	0.8	7.60	5.46	5.18	4.97
	1.2	3.49	3.16	3.07	3.04
	1.6	2.50	2.33	2.30	2.27
	2	2.03	1.94	1.91	1.89
	δ				
$\beta_1 + \delta\sigma$	0.05	143.42	91.99	62.84	34.36
	0.1	45.95	16.16	11.44	10.07
	0.15	9.90	6.04	5.70	5.39
	0.2	4.54	3.97	3.76	3.67
	0.25	3.29	2.98	2.88	2.85
	γ				
$\gamma\sigma$	1.4	9.01	6.72	6.12	9.69
	1.8	4.48	3.92	3.71	4.15
	2.2	3.10	2.72	2.62	2.71
	2.6	2.37	2.11	2.07	2.12
	3	1.96	1.78	1.73	1.74

The results of Table II shows the effect of different values of autocorrelation coefficient on the OC ARL performance of the proposed self-starting chart under different shifts in the intercept ($\beta_0 + \lambda\sigma$), slope ($\beta_1 + \delta\sigma$), and error standard deviation ($\gamma\sigma$)[14]. We consider $\rho = 0.1$ and $\rho = 0.9$. The results show that when the value of autocorrelation coefficient increases, the out-of-control ARLs of the self-starting SS-EWMA control chart increase and the performance of the control chart deteriorates.

V. Conclusions and future researches

In this paper, we proposed a self-starting control chart to monitor the location and variability simultaneously in the situation that there is AR(1) autocorrelation structure within a simple linear profile when there are no enough initial samples at the start-up stages for a satisfactory estimation. We also applied a SS-EWMA chart to monitor the parameters of an autocorrelated simple linear profile simultaneously in Phase II. The results showed that if the correlation coefficient gets smaller, performance of the proposed control chart improves. Also, the larger values of reference samples lead to better parameters estimation and as a result the better performance of the proposed control chart. Developing self-starting control chart for monitoring autocorrelated general linear profiles as well as investigating more complicated autocorrelation structures are fruitful areas for future researches.

TABLE II

OUT-OF-CONTROL ARL VALUES FOR $\rho = 0.1$ AND $\rho = 0.9$ UNDER DIFFERENT VALUES OF SHIFTS IN INTERCEPT, SLOPE AND STANDARD DEVIATION WHEN $\tau = 20$ (ARL_0=200)

	λ	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
$\beta_0 + \lambda\sigma$	$\rho = 0.1$	162.73	96.52	37.31	11.18	5.62	4.07	3.28	2.80	2.45	2.23
	$\rho = 0.9$	189.03	188.69	187.85	185.97	182.09	176.94	175.49	167.04	163.81	158.57
	δ	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
$\beta_1 + \delta\sigma$	$\rho = 0.1$	182.56	147.5	99.24	58.31	25.55	12.30	7.15	5.11	4.17	3.58
	$\rho = 0.9$	188.58	179.5	163.6	140.9	128.1	104.2	78.86	59.50	39.08	29.12
	γ	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
$\gamma\sigma$	$\rho = 0.1$	20.22	9.71	6.27	4.60	3.71	3.15	2.70	2.37	2.13	1.97
	$\rho = 0.9$	46.74	15.64	7.78	5.30	4.08	3.29	2.77	2.47	2.21	1.97

REFERENCES

[1] L. Kang, S. Albin, "On-line monitoring when the process yields a linear."*Journal of Quality Technology*, vol. 32, no. 4, pp. 418-426, 2000.

[2] K. Kim, M. A. Mahmoud, W. H. Woodall, "On the monitoring of linear profiles." *Journal of Quality Technology*, vol. 35, no. 3, pp. 317, 2003.

[3] A. Saghaei, M. Mehrjoo, A. Amiri, “Monitoring simple linear profiles using cumulative sum control charts”. *Amirkabir Journal of Science & Research-Mechanical Engineering*, vol. 41, no. 2, pp.73-82, 2010.

[4] R. B. Kazemzadeh, A. Amiri, B. Kouhestani, “Monitoring simple linear profiles using variable sample size schemes”. *Journal of Statistical Computation and Simulation*, vol. 86, no. 15, pp. 2923-2945, 2016.

[5] D. M. Hawkins, “Self-starting CUSUM charts for location and scale”. *The Statistician*, vol. 36, no. 4, pp. 299-316, 1987.

[6] D. M. Hawkins, E. M. Maboudou-Tchao, “Self-starting multivariate exponentially weighted moving average control charting”. *Technometrics*, vol. 49, no. 2, pp. 199-209, 2007.

[7] Z. Li, J. Zhang, Z. Wang, “Self-starting control chart for simultaneously monitoring process mean and variance”. *International Journal of Production Research*, vol. 48, no. 15, pp. 4537-4553, 2010.

[8] Y. Li, Y. Liu, C. Zou, W. Jiang, “A self-starting control chart for high-dimensional short-run processes”. *International Journal of Production Research*, vol. 52, no. 2, pp. 445-461, 2014.

[9] C. Zou, C. Zhou, Z. Wang, F. Tsung, “A self-starting control chart for linear profiles”. *Journal of Quality Technology*, vol. 39, no. 4, pp. 364-375, 2007.

[10] W. A. Jensen, J. B. Birch, W. H. Woodall, “Monitoring correlation within linear profiles using mixed models”. *Journal of Quality Technology*, vol. 40, no. 2, pp. 167–183, 2008.

[11] R. Noorossana, A. Amiri, P. Soleimani, “On the monitoring of autocorrelated linear profiles”. *Communications in Statistics-Theory and Methods*, vol. 37, no. 3, pp. 425–442, 2008.

[12] R. B. Kazemzadeh, R. Noorossana, A. Amiri, “Phase I monitoring of polynomial profiles”. *Communications in Statistics-Theory and Methods*, vol. 37, no.10, pp. 1671–1686, 2008.

[13] S. T. A. Niaki, M. Khedmati, M. E. Soleymanian. "Statistical monitoring of autocorrelated simple linear profiles based on principal components analysis." *Communications in Statistics-Theory and Methods,* vol. 44, no. 21, pp. 4454-4475, 2015.

[14] P. Soleimani, R. Noorossana, A. Amiri. "Simple linear profiles monitoring in the presence of within profile autocorrelation”. *Computers & Industrial Engineering*, vol. 57, no. 3, pp. 1015-1021, 2009.

[15] R. L. Brown, J. Durbin, J. M. Evans, “Techniques for testing the constancy of regression relationships over time”. Journal of the Royal Statistical Society. Series B (Methodological), vol. 37, no. 2, pp. 149-192, (1975).

[16] E. L. Lehmann, G. Casella, “Theory of point estimation”. *Wadsworth & Brooks/Cole Advanced Books & Software*, 1991, ch. 1, pp. 2-80.

[17] C. P. Quesenberry, “SPC Q charts for start-up processes and short or long runs”. *Journal of Quality Technology*, vol. 23, no. 3, pp. 213-224, 1991.

[18] M. T. Chao, S. W. Cheng, “Semicircle control chart for variables data”. *Quality Engineering*, vol. 8, no. 3, pp. 441-446, 1996.

[19] R. Ghashghaei, A. Amiri. "Sum of Squares Control Charts for Monitoring of Multivariate Multiple Linear Regression Profiles in Phase II. Published Online, *Quality and Reliability Engineering International,* 2016.

[20] H. Xie, “Contributions to Qualimetry”. Ph.D. thesis, University of Manitoba, Winnipeg, Canada, ch. 4, pp. 38-45, 1999.