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Capacity of channel with energy harvesting transmitter

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Abstract: The authors propose a new technique to model the arriving energy's knowledge (AEK) in the energy harvesting (EH) communication systems. They consider a batteryless EH transmitter with the AEK non-causally provided to it. They show that the capacity of the authors' model in general case, that is, discrete memoryless channel, can be derived through a binning scheme. This resembles a Gelfand–Pinsker type capacity formula which intuitively is the result of treating the energy as a channel state. Moreover, they investigate two especial cases of binary and Gaussian channels with the EH transmitters. In the binary symmetric channel with EH transmitter (BSCEH), their coding scheme leads to achieve the capacity of the classic binary symmetric channel (BSC). In the batteryless Gaussian channel with EH transmitter, the upper bound on the capacity is derived, and it is shown this bound is less than the capacity of the EH Gaussian channel with infinite battery.

1 Introduction

Energy harvesting (EH) is an encouraging technology for many wireless networking applications as it brings self-endurability and practically unlimited lifetime [1]. In applications where transmitter has EH capability, needed energy for sending the messages is harvested stochastically throughout the communication session. There is a considerable research interest growth in the EH communication systems from three main perspectives in the theoretical sense; the first line of works focuses on the packet scheduling policy's optimisation as well as the transmission completion time minimisation [2–10], whereas the second line derives the optimal power allocation through the achievable rate's optimisation [1, 11–14]. The last line of works considers the fundamental limits of the rates can be achieved by EH transmitters from an information-theoretic point of view [15–17]. In [16], a point-to-point communication channel is considered with an EH transmitter, in which the transmitter has a large enough battery allowing to save the energy for later use; the Gaussian channel with an EH transmitter (with the arrival sequence of energy of average P) is shown to have the same capacity with the classic Gaussian channel (i.e. a transmitter with an average power constraint P) with additive white Gaussian noise (AWGN) N of unit normal distribution $\mathcal{N}(0, 1)$; this means $C = (1/2)\log(1 + P)$, in which P represents the average of the arrival sequence of energy $\{e_1, e_2, \ldots, e_n\}$. The achievable part is proved by proposing two schemes; namely 'save-and-transmit' and 'best-effort-transmit' schemes [16]. The former scheme relies on sending zero code symbols initially (in a portion of the total block length, negligible as the block length increases) to save enough energy to make the fixed rate transmission possible in the remaining slots. In the later scheme, when the available energy is sufficient to send the code symbol, it is put to the channel, while a zero is put to the channel if there is not enough energy in the battery. In [17], the classic AWGN channel is considered where the channel input is constrained to the amplitude which stochastically varies independent of the message at each channel use. In this scenario, there is no battery to save the energy for the later use and also the arriving energy's knowledge (AEK) is 'causally' provided to the transmitter. Then, the transmitted sequence is treated as a function of the observed amplitude constraint sequence and the capacity of this channel is derived by Ozel and Ulukus [17], using the channel capacity under the amplitude constraint [18].

Now, the question is that without the capability of energy-saving, that is, with 'batteryless' EH transmitter, how one can use the 'non-causal' AEK. Is it possible to adapt the transmitted codewords to the instantaneous energy arrival (without saving), based on its before-hand (non-causal) knowledge?

In this paper, answering positively to this question, we find the optimal adaptation strategy in terms of the channel capacity. We model the stochastic behaviour of the arriving energy with a random variable (R.V.) which is 'non-causally' available at the transmitter as side information. Our setup is a point-to-point communication with an EH transmitter without any battery for saving the arriving energy. This may relate to state-dependent channels with non-causal channel state information at transmitter (CSIT). A batteryless EH transmitter with non-causal AEK wishes to transmit a message set $\{x_1, x_2, \ldots, x_n\}$ through the discrete memoryless channel (DMC). The 'binning' is used as the coding scheme for the achievability part which is shown to be the capacity achieving scheme, by providing the converse proof. Our derived capacity resembles a Gelfand–Pinsker type capacity formula which intuitively is the result of treating the energy as a channel state. Moreover, we investigate two especial cases of Gaussian and binary channels with the EH transmitters. In the binary symmetric channel with EH transmitter (BSCEH), our coding scheme leads to achieve the capacity of the classic binary symmetric channel (BSC) without any rate-loss. In the Gaussian case, the rate-loss incurred by energy shortage in the EH transmitter in these cases is computed; through comparisons to the existing results for the classic average energy constrained transmitter. In the Gaussian channel with EH transmitter with non-causal AEK at the transmitter, the rate-loss occurs because the transmitted codeword is amplitude constrained, whereas the channel noise is assumed to be average power restricted.

The rest of this paper is as follows. In Section 2, we mention the channel model and the notations. In Section 3, the main result on the channel and the proof are presented. Then two especial examples on BSCEH and Gaussian channel are presented in Section 4. Finally, this paper is concluded with some discussions in Section 6.

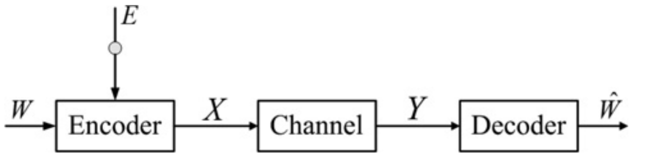

Fig. 1 *DMC with batteryless EH transmitter and stochastic energy arrival*

2 Channel model and preliminaries

First we explain the notation. $\mathcal{X}$ represents a finite alphabet with cardinality $|\mathcal{X}|$. The members of $\mathcal{X}^n$ are written as $x^n = \{x_1, x_2, \ldots, x_n\}$, where the subscripted and the superlative letters represent the components and the vectors, respectively. For the random vectors and the R.V.s, which are denoted by uppercase letters, a similar convention is applied.

As shown in Fig. 1, the transmitter wishes to transmit the message W through the channel by encoding the message to the sent signal X. The channel input X suffers the energy constraint as follows

$$x_i^2 \le e_i \tag{1}$$

which means that the power of the transmitted signal in each channel use cannot exceed the arrival energy at the transmission time. The sequence e^n is assumed to be known at the transmitter non-causally. The R.V. E represents the stochastic energy arrival. The channel is assumed to be DMC.

3 Main result: the capacity of the channel

The following theorem states our main result.

Theorem 1: For the DMC with an EH transmitter with the AEK at the transmitter non-causally, the capacity of the channel is equal to

$$C = \left\{ \begin{array}{l} \max_{p(u|e), p(x|u,e)} \{I(U;\ Y) - I(U;\ E)\}, \\ \text{s.t.} \quad x_i^2 \le e_i, \ \text{for } i \in \{1,\ 2,\ \ldots,\ n\}, \end{array} \right\} \tag{2}$$

where U is an auxiliary R.V. and we have $|\mathcal{U}| \le \min\{|\mathcal{X}||\mathcal{E}|,\ |\mathcal{Y}| + |\mathcal{E}| - 1\}$.

Remark 1: It is easy to show that the capacity of the DMC with EH transmitter in which the AEK available causally at the transmitter, can be derived by substituting $I(U;E) = 0$ in (2), noting the independency of E and U. Moreover, the maximisation is taken over $p(u)$ instead of $p(u|e)$.

Remark 2: For the AWGN channel with EH transmitter in which the arrival energy is known causally at the transmitter, the result is reduced to the one presented in [17].

Remark 3: Intuitively, it is clear that the capacity of the channel with non-causal AEK is an upper bound on the capacity of the channel with causal AEK. Moreover, the capacity in (2) is an achievable rate for the channel in which the transmitter has a limited or unlimited battery.

Proof: To prove Theorem 1, we follow a binning scheme similar to the approach proposed by Gelfand and Pinsker in [19]. The main difference lies in the 'amplitude' constraint on the transmitted codewords. The coding scheme is illustrated in Fig. 2. We use the non-causal AEK at the transmitter to produce the U^n codewords typical with the arriving energy sequence (E^n). Then, X^n is generated as a function of U^n and E^n and transmitted in the channel. At the receiver, a typicality decoder is utilised. The details follow.

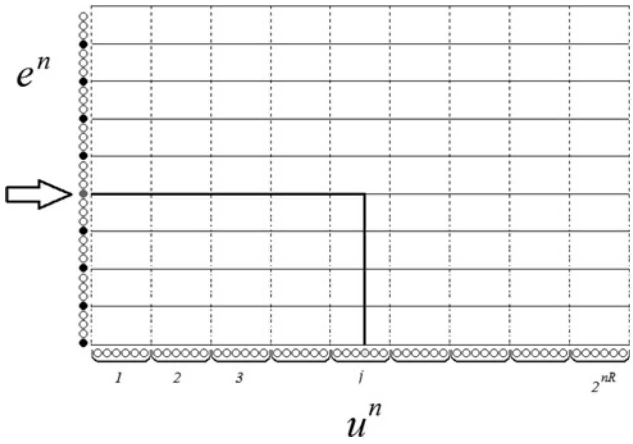

Fig. 2 *Coding scheme*

Each of the 2^{nR} bins contains $2^{nR'}$ sequences $u^n(l, m)$

Codebook generation: Fix $p(u|e)$ and $p(x|u,\ e)$ as the capacity achieving input distribution. Generate $2^{n(R+R')}$ independent sequence $u^n(l,\ m)$, $l \in \{1,\ \ldots,\ 2^{n(R+R')}\}$ according to the distribution $p(u^n) = \prod_{i=1}^n p(u_i)$, and partition these sequences into 2^{nR} bins each contains $2^{nR'}$ sequences. Index each bin by $m \in \{1, \ldots, 2^{nR}\}$.

Encoding: To send message $m \in \{1, \ldots, 2^{nR}\}$ given energy sequence e^n, the encoder tries to find l such that $(u^n(l,\ m),\ e^n) \in T_{\epsilon'}^{(n)}$ where $T_{\epsilon'}^{(n)}$ denotes the ϵ'-typical n-sequences $u^n(l,\ m)$ and e^n. If there is no such sequence, it chooses $l = 1$. Then, the transmitter sends $x_i = x^n(u_i(l, m), e_i)$ at time $i \in \{1, \ldots, n\}$.

Decoding: The receiver on receiving y^n, tries to find a unique sequence $u^n(l,\ m)$ such that $(u^n(l,\ m),\ y^n) \in T_{\epsilon}^{(n)}$ where $\epsilon > \epsilon'$. The transmitted message is estimated as $\hat{m}$. Otherwise an error is declared.

Analysis of the probability of error: Without loss of generality, we assume that $M = 1$ is transmitted. For $M = 1$, let L denote the index of chosen sequence U^n. For given $M = 1$ and E^n, an error is declared if one of the following events happen:

1. $\mathcal{E}_1$: At the encoder; for all $U^n(l, 1)$, $\{(U^n(l,\ 1),\ E^n) \notin T_{\epsilon'}^{(n)}\}$.
2. $\mathcal{E}_2$: At the decoder; there is no $U^n(L,\ m)$ such that $\{(U^n(L,\ m),\ Y^n) \in T_{\epsilon}^{(n)}\}$.
3. $\mathcal{E}_3$: At the decoder; there are some $U^n(l,\ m)$, $m \ne 1$ such that $\{(U^n(l,\ m),\ Y^n) \in T_{\epsilon}^{(n)}\}$.

Thus, we have the following upper bound on the error probability

$$P(\mathcal{E}) \le P(\mathcal{E}_1) + P(\mathcal{E}_1^c \cap \mathcal{E}_2) + P(\mathcal{E}_3) \tag{3}$$

Using the covering lemma [20, Sec. 3] if $R' > I(U;E) + \delta(\epsilon')$, $P(\mathcal{E}_1)$ tends to zero as $n \to \infty$. Then, we have

$$\mathcal{E}_1^c = \{(U^n(L,\ 1),\ E^n) \in T_{\epsilon'}^{(n)}\} = \{(U^n(L,\ 1),\ X^n,\ E^n) \in T_{\epsilon'}^{(n)}\} \tag{4}$$

and

$$Y^n | \{U^n(L,\ 1) = u^n,\ X^n = x^n,\ E^n = e^n\}$$
$$\sim \prod_{i=1}^n P_{Y|U,X,E}(y_i|u_i,\ x_i,\ e_i) = \prod_{i=1}^n P_{Y|X,E}(y_i|x_i,\ e_i) \tag{5}$$

Hence, using the asymptotic equipartition property properties [21], $P(\mathcal{E}_1^c \cap \mathcal{E}_2)$ tends to zero as $n \to \infty$. Finally, because that every $U^n(l,\ m)$, $m \ne 1$ is independent of Y^n and is distributed according to $\prod_{i=1}^n p(u_i)$, using the packing lemma [21], $P(\mathcal{E}_3)$ tends to zero as $n \to \infty$ if $R + R' < I(U;Y) - \delta(\epsilon)$. Thus, we derive that

$R < I(U;Y) - I(U;E) - \delta(\epsilon) - (\epsilon')$. This completes the proof of the achievability. □

Now, we consider an alternative framework, proposed by Yassaee *et al.* in [22], to prove our achievable rate.

For fixed $p(u|e), x(u,e)$, we assume that $(U^n, E^n, X^n, Y^n) \sim p(u,e,x,y)$. By performing two independent random binning $W(u^n) \in \{1, \ldots, 2^{nR}\}$ and $F(u^n) \in \{1, \ldots, 2^{n\tilde{R}}\}$ of the sequences in $\mathcal{U}^n$, deduce joint probability of mass function (pmf) as follows

$$\begin{aligned} &P(u^n, w, f, e^n, x^n, y^n, \hat{u}^n) \\ &= p(u^n, e^n)P(w, f|u^n)p(x^n|u^n, e^n)p(y^n|x^n, e^n)P^{\mathrm{SW}}(\hat{u}^n|y^n, f) \\ &= P(w, f, e^n)P(u^n|w, f, e^n)p(x^n|u^n, e^n) \\ &\quad \times p(y^n|x^n, e^n)P^{\mathrm{SW}}(\hat{u}^n|y^n, f) \end{aligned} \tag{6}$$

in which w is the message, f is the agreed on a priori codebook and e^n is the energy sequence which is forced by the nature, P denotes the R.V.'s pmf and P^{SW} denotes the used R.V.'s pmf at the decoder to employ the Slepian–Wolf decoder [21]. Now, we want w, f and e^n to be independent in order to obtain an operational protocol. Using [22, Theorem 1] leads us to have $R + \tilde{R} < H(U|E)$. For reliability and success decoding, [22, Lemma 1] gives us $\tilde{R} > H(U|Y)$. From these two conditions, we have $R < H(U/E) - H(U/Y)$ which completes the achievability of the rate.

To prove the converse, we identify U_i such that $U_i \to (X_i, E_i) \to Y_i$ forms a Markov chain. Thus

$$\begin{aligned} nR &\le I(W; Y^n) + n\epsilon_n = \sum_{i=1}^{n} I(W; Y_i|Y^{i-1}) + n\epsilon_n \\ &\le \sum_{i=1}^{n} I(W, Y^{i-1}; Y_i) + n\epsilon_n \\ &= \sum_{i=1}^{n} I(W, Y^{i-1}, E_{i+1}^n; Y_i) - \sum_{i=1}^{n} I(Y_i; E_{i+1}^n|W, Y^{i-1}) + n\epsilon_n \\ &\overset{(a)}{=} \sum_{i=1}^{n} I(W, Y^{i-1}, E_{i+1}^n; Y_i) - \sum_{i=1}^{n} I(Y^{i-1}; E_i|W, E_{i+1}^n) + n\epsilon_n \\ &\overset{(b)}{=} \sum_{i=1}^{n} I(W, Y^{i-1}, E_{i+1}^n; Y_i) - \sum_{i=1}^{n} I(W, Y^{i-1}, E_{i+1}^n; E_i) + n\epsilon_n \end{aligned} \tag{7}$$

where (a) follows by the Csiszar sum identity and (b) is because of the fact that (W, E_{i+1}^n) is independent of E_i. By substituting $U_i = (W, Y^{i-1}, E_{i+1}^n)$, we obtain the Markov chain as

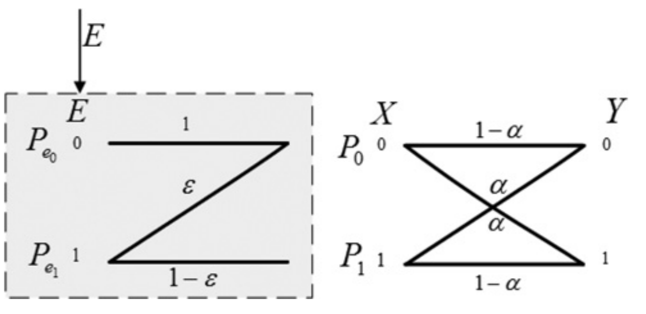

Fig. 3 *BSCEH and stochastic energy arrival with Bernoulli distribution*

When $E = 0$, that is, there is no energy to send the symbol $X = 1$, the transmitter must send symbol $X = 0$ to the receiver. In contrary, when the arriving energy is equal to $E = 1$, the transmitter can send both symbols $\{0, 1\}$ according to the probabilities $\{\varepsilon, 1-\varepsilon\}$ respectively

Table 1 Random function of $X(U, E)$

E	U	X
0	0	$\Pr\{X=0\}=1$
0	1	$\Pr\{X=0\}=1$
1	0	$\Pr\{X=0\}=a$, $\Pr\{X=1\}=1-a$
1	1	$\Pr\{X=0\}=b$, $\Pr\{X=1\}=1-b$

$U_i \to (X_i, E_i) \to Y_i$ for $i \in \{1, \ldots, n\}$ and we have

$$\begin{aligned} nR &\le \sum_{i=1}^{n} [I(U_i; Y_i) - I(U_i; E_i)] + n\epsilon_n \\ &\le \max_{p(u,x|e)} \{I(U; Y) - I(U; E)\} + n\epsilon_n \end{aligned} \tag{8}$$

The bound on the cardinality of U can be proved using the method in [20, Appendix 1]. We can show that it is sufficient to maximise (8) over $p(u|e)$ and x is a random function of (u, e). For fixed $p(e|u)$, the maximisation in (8) is only over $I(U;Y)$ which is convex in $p(y|u)$ for fixed $p(u)$.

Thus, $I(U;Y)$ is convex in $p(x|u, e)$ because for a fixed $p(u|e)$ we have

$$p(y|u) = \sum_{(x,e)} p(e|u)p(x|u, e)p(y|x, e) \tag{9}$$

which is linear in $p(x|u, e)$. This implies that the maximum is attained at an extreme point of the set of pmfs $p(x|u, e)$. The proof of Theorem 1 is completed. □

Remark 4: Considering that the output of the channel depends on x, which also depends on the energy e, our channel is similar to the state-dependent DMC in which the state of the channel is known non-causally at the transmitter [22].

4 Binary and Gaussian examples

In this section, the result of Theorem 1 is extended to two especial cases. First, in the binary channel, we find the appropriate auxiliary R.V. U and the function $X(U, E)$ for the coding scheme proposed in Theorem 1. Our result shows that the capacity of the BSCEH, with the non-causal AEK, is equal to the capacity of the BSC with average power constraint. Then, for the Gaussian channel, using a sphere packing technique, we find an outer bound on the capacity which is less than the capacity of the average power constrained Gaussian channel.

4.1 Binary channel with EH transmitter

In this section we consider the BSCEH. As the model in Fig. 3, we consider a BSC with parameter α with binary input X, where $\Pr\{X=0\} = P_0$. In this channel, the EH transmitter with probability P_{e_1}, harvests the arrival energy from the nature (i.e. $E = 1$), and thus it is able to send $\{0, 1\}$. However, when the energy cannot be harvested (i.e. $E = 0$ with probability $P_{e_0} = 1 - P_{e_1}$), the transmitter must send symbol $X = 0$. We determine the strategy of choosing X based on E in the following corollary, for this channel.

Corollary 1: The capacity of the BSCEH, shown in Fig. 3, in which the energy arrival R.V. E assumed to be known non-causally at the transmitter, when $(1/2) \le \alpha \le P_{e_1} \le 1$, is as follows

$$C_{\mathrm{BSCEH}} = 1 - H(\alpha) \tag{10}$$

where $H(t) = -t\log_2(t) - (1-t)\log_2(1-t)$.

Table 2 Values for a, b and β

a	b	β
$a_1 = 1$	$b_1 = \left(1 - \frac{1}{2\alpha}\right)$	$\beta_1 = 1 - \frac{\alpha}{P_{e_1}}$
$a_2 = 1 - \frac{1}{P_{e_1}}$	$b_2 = \left(1 - \frac{1}{2\alpha}\right)\left(1 - \frac{1}{P_{e_1}}\right)$	$\beta_1 = 1 - \frac{\alpha}{P_{e_1}}$

Proof: The proof can be deduced from Theorem 1. Let $U \in \{0, 1\}$ (a binary R.V.), independent of E, with the probability $\Pr\{U = 0\} = \beta$. We define X as a random function of U and E in Table 1.
As we can see, if $E = 0$, the channel input must be $X = 0$, but if $E = 1$ the channel input can be equal to $\{0, 1\}$ with the probabilities $\{a, 1 - a\}$ when $U = 0$ and $\{b, 1 - b\}$ when $U = 1$. Then, from (2) we have

$$\begin{aligned} C_{\text{BSCEH}} &= \max_{p(u),\,p(x|u,e)} \{I(U;\, Y) - I(U;\, E)\} \\ &\overset{(c)}{=} I(U;\, Y) = H(Y) - H(Y|U) \\ &= H(P_1(2\alpha - 1) + (1 - \alpha)) - \beta H[aP_{e_1}(1 - 2\alpha) \\ &\quad + (1 - \alpha - P_{e_1} + 2\alpha P_{e_1})] \\ &\quad - (1 - \beta)H[2b\alpha P_{e_1} + (\alpha - 2\alpha P_{e_1} + P_{e_1})] \end{aligned} \tag{11}$$

where (c) follows from the independence of the R.V.s U and E and we have

$$\begin{aligned} P_1 &= 1 - P_0 \\ &= P_{e_1}\{\beta(1 - a) + (1 - \beta)(1 - b)\} \\ &= P_{e_1}\{\beta(b - a) + (1 - b)\} \end{aligned} \tag{12}$$

Now, by defining $a = 1$, $b = (1 - (1/2\alpha))$, $\beta = 1 - (\alpha/P_{e_1})$ and substituting these values in (11) the proof is completed. Note that $0 \le b, \beta \le 1$. Thus, $(1/2) \le \alpha \le P_{e_1} \le 1$. □

Remark 5: It can be shown that the obtained values of a, b and β are not unique. For example, two choices for each parameter are shown in Table 2. This determines four choices to define X (as a function of E and U) and two choices for defining U as a binary R.V.. Note that a_2 only works when $P_{e_1} = 1$.

Remark 6: Interestingly, the capacity of the BSCEH with AEK non-causally at the transmitter, is equal to the capacity of the classic BSC, that is, $C_{\text{BSC}} = 1 - H(\alpha)$, in which the transmitter assumed to be average power constrained. When there is an infinite battery available at the transmitter, the problem reduces to an average power constrained one, such as the Gaussian case in [16]. This means that our proposed coding scheme helps to use the non-causal AEK to relief the absence of battery at the transmitter.

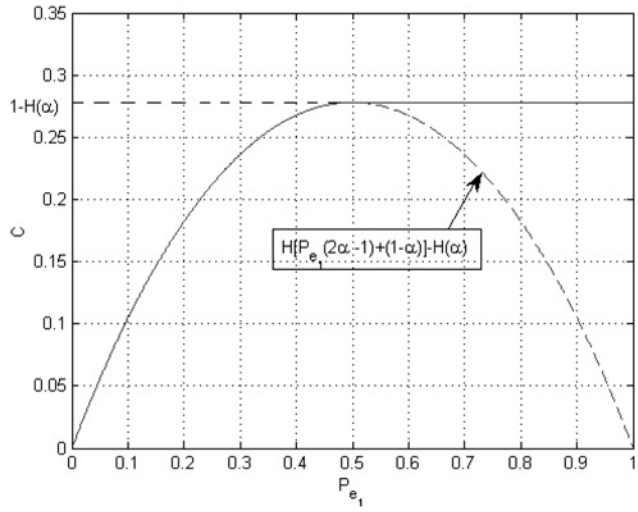

Fig. 4 *Capacity of the BSCEH with AEK non-causally at the transmitter*

If $0 \le P_{e_1} < (1/2)$, $C = H(P_{e_1}(2\alpha - 1) + (1 - \alpha)) - H(\alpha)$, otherwise, if $(1/2) \le P_{e_1} \le 1$, $C = 1 - H(\alpha)$

Remark 7: An alternative proof (for the BSCEH) can be obtained straightforwardly from [18], by calculating $I(X;Y)$. By considering the model in Fig. 3, the capacity of this channel can be calculated using the capacity of the classic BSC [21]

$$C_{\text{BSC}} = \max_{P_0} \{H(P_0(1 - 2\alpha) + \alpha)\} - H(\alpha) \tag{13}$$

By substituting the values of $P_0 = P_{e_0} + P_{e_1}\varepsilon$ and $P_1 = P_{e_1}(1 - \varepsilon)$, we have

$$C_{\text{BSCEH}} = \max_{\varepsilon} H[P_{e_1}(1 - 2\alpha)(\varepsilon - 1) + (1 - \alpha)] - H(\alpha) \tag{14}$$

To maximise (14), taking a derivative with respect to ε and setting to zero yields

$$P_{e_1}(1 - 2\alpha)\,\log_2\left[\frac{\alpha - P_{e_1}(1 - 2\alpha)(\varepsilon - 1)}{P_{e_1}(1 - 2\alpha)(\varepsilon - 1) + (1 - \alpha))}\right] = 0 \tag{15}$$

Therefore the optimum probability ε can be derived as $\varepsilon^* = 1 - (1/2P_{e_1})$. By substituting ε^* in (14), the capacity of the BSCEH is obtained.

We should note that since $0 \le \varepsilon \le 1$, for the probability P_{e_1} we have $(1/2) \le P_{e_1} \le 1$. It means that this strategy can be used if $(1/2) \le P_{e_1} \le 1$. Otherwise, if $0 \le P_{e_1} < (1/2)$, we should choose $\varepsilon = 0$. Thus, the capacity of the channel in this situation is equal to

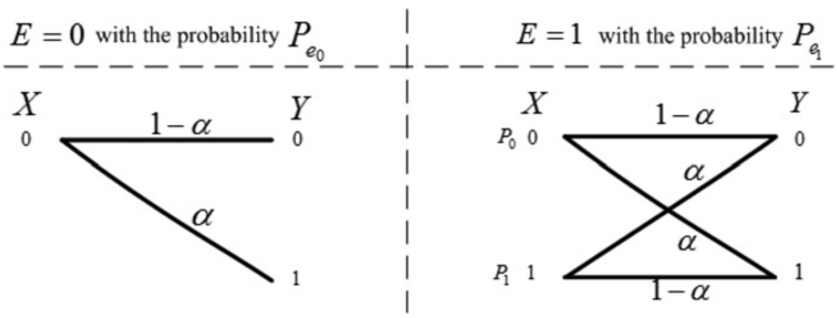

Fig. 5 *DBSC in which the transmitter is imposed to send $X = 0$ if $E = 0$*

Capacity of this channel in case $E = 0$, is equal to zero. If $E = 1$, it can transmit $X = 0, 1$ and the capacity of the channel in this case is equal to $P_{e_1}(1 - H(\alpha))$. Thus, the total capacity of the DBSC is $P_{e_1}(1 - H(\alpha))$

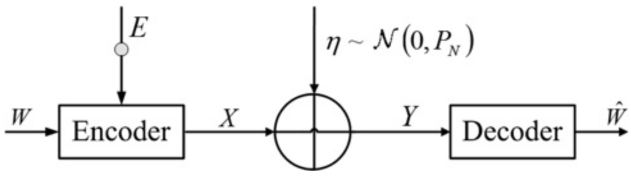

Fig. 6 *Gaussian channel with EH transmitter and stochastic energy arrival*

$H(P_{e_1}(2\alpha - 1) + (1 - \alpha)) - H(\alpha)$. The capacity of this channel is shown in Fig. 4.

Remark 8: Note that BSCEH is not equal to the model of decoupled BSC (DBSC) shown in Fig. 5. Later, the transmitter faces two different cases depending on the arriving energy sequence. If $E = 0$, the transmitter must send $X = 0$, but if $E = 1$, it can transmit $X = \{0, 1\}$. Thus, the capacity of the DBSC (Fig. 5) is equal to $P_{e_1}(1 - H(\alpha))$. There is an interesting intuition indicating this difference. In our setup, where the transmitter knows e^n non-causally, it can use this extra information to choose the best codebook. This codebook should contain the codewords with zero symbols for the slots where no-energy is available as much as possible. Hence, the capacity of the channel when there is no energy available to the transmitter is not zero. However, in DBSC, the codebook is designed only for the BSC without considering P_{e_1}.

4.2 Gaussian channel with EH transmitter

Now, we consider the Gaussian channel with EH transmitter. In this channel (Fig. 6), we wish to transmit the message W through the channel with zero-mean Gaussian noise with the variance P_N. It should be mentioned that the noise is average power constrained, that is, for the sequence of the noise $\eta^n = \{\eta_1, \eta_2, \ldots, \eta_n\}$, we have $(1/n)\sum_{i=1}^{n} \eta_i^2 \le P_N$.

The energy arrival is modelled as a R.V. E with arbitrary distribution and with variance P_E. The sequence of the energy arrival, that is, $e^n = \{e_1, e_2, \ldots, e_n\}$, is assumed to be known non-causally at the transmitter. Transmitted signal X is assumed to be amplitude power constrained, that is, $x_i^2 \le e_i$, or equivalently, $|x_i| \le \sqrt{e_i} = v_i$, for each transmission time $i \in \{1, 2, \ldots, n\}$. First, we state the following existing result.

Theorem 2 [18]*:* The capacity of the Gaussian channel with channel input X and probability density function F_X in the space of $\mathcal{F}_X$, in which the channel input assumed to be constrained to take values on $[-V, V]$, can be achieved as follows

$$C = \max_{F_X \in F_X} I(X;\, Y) \tag{16}$$

From this theorem and Theorem 1, we derive the following corollary which states an upper bound for the capacity region of the Gaussian channel with EH transmitter.

Corollary 2: An upper bound on the capacity of the Gaussian channel, shown in Fig. 6, with EH transmitter in which the energy arrival R.V. E assumed to be known non-causally at the transmitter, is as follows

$$\mathcal{R}_U = \frac{1}{2}\log_2\left[\frac{2}{\pi e}\left(1 + \frac{P_E}{P_N}\right)\right] \tag{17}$$

Remark 9: This bound on the capacity is less than the one given in [17] for the Gaussian point-to-point channel with EH transmitter and with large enough battery, whose capacity is equal to $(1/2)\log_2(1 + (P_E/P_N))$. The bound in [17] was achieved by two approaches. Obviously, their bound is a trivial upper bound for our model, since we do not have battery. Therefore we find a strictly tighter upper bound.

Proof: To prove the corollary, we use the approach taken by Shannon [23]. As an intuition, we want to use sphere packing in the space of the feasible sets of the input sequences. From (1), we know that we have

$$\begin{aligned} x_1^2 &\le e_1 \\ x_2^2 &\le e_2 \\ &\vdots \\ x_n^2 &\le e_n \end{aligned} \tag{18}$$

which compose a n-polyhedral n-dimensional space where in each face we have $-\sqrt{e_i} \le x_i \le \sqrt{e_i}$ for $i \in \{1, \ldots, n\}$. Our goal is to find the number of the hypersphere with radius $\sqrt{nP_N}$ contained in this polyhedral. Since the hyperplanes of this polyhedral are orthogonal, the volume of the polyhedral can be obtained as

$$V_{\text{Polyhedral}} = 2^n \prod_{i=1}^{n} \sqrt{e_i + P_N} \tag{19}$$

It should be mentioned that because of the fact that some spheres can be exactly centred on the surface of the n-polyhedral, each side of the n-polyhedral must be equal to $2\sqrt{e_i + P_N}$.
On the other hand, the volume of the hypersphere with radius $\sqrt{nP_N}$ can be obtained as

$$\begin{aligned} V_{\text{Hypersphere}} &= \frac{\pi^{n/2}}{\Gamma((n/2)+1)} \\ &= \begin{cases} \dfrac{\pi^{n/2}}{(n/2)!}(\sqrt{nP_N})^n, & \text{if } n \text{ is even} \\ \dfrac{2^{n+1}\pi^{(n-1)/2}((n+1)/2)}{(n+1)!}(\sqrt{nP_N})^n, & \text{if } n \text{ is odd} \end{cases} \end{aligned} \tag{20}$$

in which the $\Gamma((n/2) + 1)$ denotes the Gamma function. Now, we can calculate the ratio of the $V_{\text{Polyhedral}}$ to the $V_{\text{Hypershere}}$.

First, we consider the case where n is even

$$\frac{V_{\text{Polyhedral}}}{V_{\text{Hypersphere}}} = \frac{2^n \prod_{i=1}^{n}\sqrt{e_i + P_N}}{\pi^{n/2}(n/2)!(\sqrt{nP_N})^n} \tag{21}$$

Using the Stirling's formula as $n! \sim \sqrt{2\pi n}\, n^n \mathrm{e}^{-n}$ for $n \to \infty$, we have

$$\frac{V_{\text{Polyhedral}}}{V_{\text{Hypersphere}}} = \frac{2^n \prod_{i=1}^{n}\sqrt{e_i + P_N}}{\pi^{n/2}(\sqrt{nP_N})^n}\sqrt{\pi n}\left(\frac{n}{2}\right)^{n/2}\mathrm{e}^{-n/2} \tag{22}$$

which gives the number of hyperspheres can be packed in the polyhedral. Thus, the rate of this code is equal to

$$\begin{aligned} R &= \lim_{n\to\infty}\frac{1}{n}\log_2\left(\frac{V_{\text{Polyhedral}}}{V_{\text{Hypersphere}}}\right) \\ &= \lim_{n\to\infty}\frac{1}{n}\log_2\left(\frac{2^n \prod_{i=1}^{n}\sqrt{e_i + P_N}}{\pi^{n/2}(\sqrt{nP_N})^n}\sqrt{\pi n}\left(\frac{n}{2}\right)^{n/2}\mathrm{e}^{-n/2}\right) \\ &= \lim_{n\to\infty}\log_2\left(\frac{\sqrt{2}\prod_{i=1}^{n}\sqrt[n]{\sqrt{e_i + P_N}}}{\sqrt{\pi e}\sqrt{P_N}}\right) \\ &\quad + \lim_{n\to\infty}\log_2\left(\sqrt[2n]{\pi n}\right) \end{aligned} \tag{23}$$

The second term of (23) tends to zero as $n \to \infty$. Therefore we have

$$
\begin{aligned}
R &= \lim_{n\to\infty} \log_2\left(\frac{\sqrt{2}\prod_{i=1}^{n}\sqrt[n]{\sqrt{e_i+P_N}}}{\sqrt{\pi e}\sqrt{P_N}}\right) \\
&= \lim_{n\to\infty} \frac{1}{2}\log_2\left(\frac{\prod_{i=1}^{n}\sqrt[n]{e_i+P_N}}{\sqrt{P_N}}\right) + \frac{1}{2}\log_2\left(\frac{2}{\pi e}\right) \\
&\overset{(d)}{\leq} \frac{1}{2}\log_2\left(\frac{2}{\pi e}\frac{P_E+P_N}{P_N}\right)
\end{aligned} \tag{24}
$$

where (d) follows since the product of some variables with constant summation is less than the product of the average of the variables. This packing argument indicates that we cannot hope to send at rates greater than $\mathcal{R}_U$ with low probability of error. The results for the case of odd n can be derived similarly. □

5 Discussion and conclusions

In this paper, a point-to-point communication system with batteryless EH transmitter was considered in which the AEK is assumed to be available non-causally at the transmitter. The capacity of the channel was derived and using a binning type coding scheme the achievability of the capacity of the DMC with EH transmitter was proved. It is remarked that the derived capacity contains the capacity of the channel with causal AEK. The result was extended to the BSC and Gaussian channel with an EH transmitter. In the BSC case, the capacity of the channel is equal to the capacity of the classic BSC with average power constraint. In the Gaussian case, it was shown that there is a reduction in the capacity in comparison with the classic Gaussian channels with average power constraint.

To compare [16] with our paper, it should be mentioned that in [16] there is no assumption on AEK. The reason hides in using an infinite-size battery which eliminates the necessity of AEK in an infinite block length. However, in our model there is no battery assumption to save the energy to supply the transmitter. Thus, the AEK helps the transmitter to construct its message appropriate with the arriving energy. This capacity achieving coding scheme makes a reliable point-to-point communication between the parties possible, but this capacity is less than the capacity of a channel with an infinite-size battery equipped transmitter.

To compare [17] with our paper, in our scenario the energy arrival process is assumed to be known at the transmitter 'non-causally'. Hence, the transmitter can use this knowledge to construct its coding scheme. Our result contains the result of [17] as an especial case, described in the Remarks 1 and 2.

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