

A Novel Project Selection Scheduling Model

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Abstract In this paper, the optimal selection of portfolio of projects is addressed. The problem is examined at operational level where every project has several jobs with certain costs, profits and due dates. The duration of each job is affected by learning effect and its setup time is dependent on the sequence and processing times of the jobs prior to it. Any profit gained from the completion of a job is reinvested in the portfolio and the time horizon is assumed flexible. The objective is to maximize the total revenue of the selected projects. A mixed integer model is presented and analyzed to address this problem.

Keywords: Project selection scheduling, reinvestment strategy, learning effect, setup time, flexible time horizon

1 Introduction

Project selection scheduling problem (*PSSP*) is one of the most practical problems in project management, portfolio management, risk and investment management, and has drawn great attention in recent years (Jafarzadeh et al, 2015). The problem under study is a modification of *PSSP* and can be defined as follows. Several projects with same priorities are available for investment. Each project has several jobs. Projects can be interrupted, but if a job starts, it should be processed until its completion. The objective is to select a portfolio of projects and schedule their jobs within an optimal time horizon in order to maximize the total profit subject to the following conditions. Available budget and resources are limited and shared among the projects. Moreover, similar to Chen & Askin (2009), it is assumed that the amount of profit achieved from the completion of each job is added to the current budget to be used for the next investment. The planning horizon is defined in prior, but time slacks are assigned to this horizon so that, considering the objective, the best time horizon can be decided with addition of slacks. Any deviation

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from the default time horizon results in loss or gain depending on the fact that the planning is finished later or earlier than expected, respectively. Implementation of each job needs a certain amount of investment and resources which should be available at the start of that job. Each job has a due date, a percentage of learning effect and a past-sequence-dependent (*psd*) setup time. If due dates are not met, a separate cost is inflicted on the project at the end of its completion. If a job is part manual, part machine-based, then only the manual part undergoes learning. Learning effect refers to the fact that as the time goes on, the duration of a job decreases due to the increase in the operator's skill to perform that job (Peteghem & Vanhoucke, 2015). As the name indicates, *psd* setup time of a job has a variable value which is dependent on the sequence and processing times of its previous jobs. This type of setup time is mostly seen in industries such as chemical, textile, metallurgical, printed circuit board, and automobile manufacture (Amirian & Sahraeian, 2015). In this paper, a mixed integer model is proposed based on the model first presented in Belenky (2012) and later corrected by Jafarzadeh et al (2015). Their problem was limited to selection and scheduling of projects alone, while we also consider jobs of each project, their due dates, resource usages, setup times and their learning effects. Moreover penalties for tardiness of jobs are added to the objective. The idea is that breaking a project to its jobs and studying the problem at an operational level rather than a strategic level gives a more realistic view to selecting and scheduling of projects.

2 Problem Formulation

There are m projects available ($i=1,\dots,m$) with setup dependency parameters of b_i and each project has V_i jobs ($v=1,\dots,V_i$). Each job with duration of w'_{iv} , rate of machine time M_{iv} ($0 \leq M_{iv} \leq 1$) and learning parameter of a_{iv} ($0 \leq a_{iv} \leq 1$) needs r_{iv} resources to commence. A job needs to meet its due date D_{iv} , otherwise a penalty of β_{iv} is occurred. The achieved profit and cost requirement in time h for job v of project i which has started at time j are denoted as $d_{ij}(h)$ and $c_{ij}(h)$ respectively. Maximum available resource at time j is R_j and P is the *initial* available budget. The gain (loss) in each additional period j , due to finishing all projects sooner (later) than the default time horizon is denoted by $\alpha_j > 0$ ($\alpha_j < 0$). The default time horizon is T periods ($h, h', j=1,\dots,T$), the slack values $\lambda_{\max}, \lambda_{\min}$ show the upper and lower bounds of time horizon tolerance and we have $\hat{\lambda} = \max\{|\lambda_{\max}|, |\lambda_{\min}|\}$. The decision variable x_{ij} equals 1 if job v of project i starts at time j and 0 otherwise. Binary variable y_j counts the number of planning periods skewed from time T . The completion time of each job is Co_{iv} and the

numbers of executive time periods that planning is finished later or earlier than time T are λ^+ and λ^- , respectively. The state variables are the lateness L_{iv} and tardiness $TA_{iv} = \max(0, L_{iv})$ of each job. Additional state variables are *psd* setup time s_{ivj} and learning processing time $w_{ivj}(h')$ at time h' for job v of project i which has started at time j . Processing time under the effect of learning is formulated with the assumption that if a job is continued uninterrupted, then the operator becomes more experienced as the time goes on. Thus, if h' and j are the current time and the starting time of a job, respectively, then the longer a job is processed (i.e. the greater $h' - j$ is), the more affected its processing time by learning. Also it is assumed that learning has a meaningful effect if processing time of a job is higher than one. Equations (1) and (2) formulate the partial and complete learning processing times of each job. Similarly, setup times can be formulated according to equations (3) and (4). Since there is no job before the first job of each project, $s_{i|1j}$ for all projects and periods equal to zero.

$$w_{ivj}(h') = 1 / (M_{iv} + (1 - M_{iv}) \cdot (h' - j)^{a_{iv}}) \quad (1)$$

$$w_{iv} = \sum_{j=1}^{T+\lambda_{\max} - \sum_{f=v}^{V_i} w_{if}'} x_{ivj} \sum_{h'=j+1}^{w_{iv}'+j} w_{ivj}(h') \quad \forall i = 1, \dots, m; v = 1, \dots, V_i \quad (2)$$

$$s_{ivj} = b_i \sum_{f=1}^{v-1} w_{if} \sum_{t=1}^{j-1} x_{ift} \quad \forall i = 1, \dots, m; v = 2, \dots, V_i; j = 1, \dots, T + \lambda_{\max} \quad (3)$$

$$s_{iv} = \sum_{j=1}^{T+\lambda_{\max} - \sum_{f=v}^{V_i} w_{if}'} x_{ivj} \cdot s_{ivj} \quad \forall i = 1, \dots, m; v = 2, \dots, V_i \quad (4)$$

The objective function in equation (5) seeks to maximize the achieved profit at the end of the optimal planning horizon. It consists of five parts: the initial profit, the profit achieved by the jobs finishing at the last period, the profit extracted from the jobs finishing during the planning horizon, the loss occurred by finishing a job later than its due date and finally the penalty (gain) due to completing all selected projects after (before) the original planning horizon. Constraints (6) and (7) focus on the limitation on expenditure in each period while considering reinvestment strategy. Equation (8) emphasizes that each job of a project starts only once during the time horizon. Constraint (9) ensures that the resource usage of all jobs in a period cannot exceed the available resources in that particular period. Equations (10) to (13) formulate the scheduling part of the problem where the completion times of all jobs and their lateness values are achieved. Equations (14) states that each project should finish within the flexible time horizon i.e. the default time horizon

plus the positive and negative time slacks. Equations (15) and (16) count the number of periods that planning is finished sooner or later than the original time horizon, respectively. Equation (17) ensures that the start of each additional period should not exceed the maximum optimal time horizon. Equation (18) states that if a project is not finished early at time $j+1$, it means it wasn't finished at time j neither. Basically, this equation makes sure the numbers of periods that projects are finished earlier than planned are calculated correctly. Equations (19) and (20) are either/or equations and make sure that either λ^+ or λ^- gets a value higher than zero. Equation (21) limits the optimal slacks between the minimum and maximum slacks. Equation (22) explains the decision variables.

$$\begin{aligned} \max Z = & P + \sum_{i=1}^m \sum_{v=1}^{V_i} \sum_{j=1}^{T+\lambda_{\max} - \sum_{f=v}^{V_i} (w_{if} + s_{if}) - 1} d_{ivj}(T + \lambda_{\max}). x_{ivj} \\ & + \sum_{i=1}^m \sum_{v=1}^{V_i} \sum_{j=1}^{T+\lambda_{\max} - 1} \sum_{h=1}^{T+\lambda_{\max} - 1} (d_{ivj}(h) - c_{ivj}(h)). x_{ivj} - \sum_{i=1}^m \sum_{v=1}^{V_i} \beta_{iv} TA_{iv} + \sum_{j=T+\lambda_{\min}}^{T+\lambda_{\max}} \alpha_j \cdot y_j \quad (5) \end{aligned}$$

st.

$$\sum_{i=1}^m \sum_{v=1}^{V_i} c_{iv[1]}(1). x_{iv[1]} \leq P \quad (6)$$

$$\sum_{i=1}^m \sum_{v=1}^{V_i} \sum_{j=1}^k c_{ivj}(k). x_{ivj} \leq P + \sum_{i=1}^m \sum_{v=1}^{V_i} \sum_{j=1}^{k-1} \sum_{h=1}^{k-1} (d_{ivj}(h) - c_{ivj}(h)). x_{ivj} + \sum_{i=1}^m \sum_{v=1}^{V_i} \sum_{j=1}^{k-1} d_{ivj}(k). x_{ivj} \quad \forall k = 2, \dots, T + \lambda_{\max} \quad (7)$$

$$\sum_{j=1}^{T+\lambda_{\max}} x_{ivj} \leq 1 \quad \forall i = 1, \dots, m; v = 1, \dots, V_i \quad (8)$$

$$\sum_{i=1}^m \sum_{v=1}^{V_i} r_{iv} \cdot x_{ivj} \leq R_j \quad \forall j = 1, \dots, T + \lambda_{\max} \quad (9)$$

$$Co_{i[v+1]} \geq Co_{iv} + \sum_{j=1}^{T+\lambda_{\max} - \sum_{f=v+1}^{V_i} w_{if} - j + w'_{i[v+1]}} \sum_{h=j+1} x_{i[v+1]j} \cdot (w_{i[v+1]j}(h) + s_{i[v+1]j}) \quad \forall i = 1, \dots, m; v = 1, \dots, V_i - 1 \quad (10)$$

$$Co_{i[1]} \geq 1 + \sum_{j=1}^{T+\lambda_{\max} - \sum_{f=1}^{V_i} w_{if} - j + w'_{i[1]}} \sum_{h=j+1} x_{i[1]j} \cdot w_{i[1]j}(h) \quad \forall i = 1, \dots, m \quad (11)$$

$$Co_{iv} - D_{iv} \cdot \sum_{j=1}^{T+\lambda_{\max}} x_{ivj} = L_{iv} \quad \forall i = 1, \dots, m; v = 1, \dots, V_i \quad (12)$$

$$Co_{iv} \leq H \cdot \sum_{j=1}^{T+\lambda_{\max}} x_{ij} \quad \forall i=1, \dots, m; v=1, \dots, V_i \quad (13)$$

$$\sum_{j=1}^{T+\lambda_{\max}-\sum_{f=1}^{V_i} w'_{if}} j \cdot x_{i[1]j} + \sum_{v=1}^{V_i} \sum_{j=1}^{T+\lambda_{\max}-\sum_{f=v}^{V_i} w'_{if}} x_{ij} \cdot (w_{iv} + s_{iv}) \leq T + \lambda^+ - \lambda^- \quad \forall i=1, \dots, m \quad (14)$$

$$\sum_{j=T+\lambda_{\min}}^{T-1} y_j = \lambda^- \quad \& \quad \sum_{j=T+1}^{T+\lambda_{\max}} y_j = \lambda^+ \quad (15-16)$$

$$jy_j \leq T + \lambda^+ \quad j=T+1, \dots, T+\lambda_{\max} \quad \& \quad y_j \leq y_{j+1} \quad j=T+\lambda_{\min}, \dots, T-2 \quad (17-18)$$

$$\lambda^+ \leq (1-\delta)\hat{\lambda} \quad \& \quad \lambda^- \leq \delta\hat{\lambda} \quad \& \quad \lambda_{\min} \leq \lambda^+ - \lambda^- \leq \lambda_{\max} \quad (19-21)$$

$$\begin{aligned} & free(L_{iv}), Co_{iv} \geq 0 \quad \forall i=1, \dots, m; v=1, \dots, V_i, int(\lambda^+, \lambda^- \geq 0), \quad \delta \in \{0, 1\} \\ & y_j \in \{0, 1\} \quad \forall j=T+\lambda_{\min}, \dots, T+\lambda_{\max}, x_{ij} \in \{0, 1\} \quad \forall i=1, \dots, m; v=1, \dots, V_i; j=1, \dots, T+\lambda_{\max} \end{aligned} \quad (22)$$

3 Model's Analysis

In order to illustrate the performance of the proposed model, a numerical example is examined in details as follows. Suppose four projects are available to pursue and projects one to four have 3, 2, 5 and 4 jobs respectively. The management should decide which project to follow and how to schedule its jobs. The costs $c_{ij}(h)$ and profits $d_{ij}(h)$ of different jobs of the projects are summarized in Table 1. Note that any costs or profits that are not included in the table get the value of zero. The normal processing times, required resources and due date penalties for the jobs are summarized in Table 2. The default planning horizon has 6 periods (i.e. $T=6$) and the assigned time slacks are considered $\lambda_{\min}=-1, \lambda_{\max}=18$. Hence, the planning horizon would be flexible in range $[T-1, T+18]$ i.e. $[5, 24]$. The due dates are calculated as $D_{iv}=6-\sum_{f=v+1}^{V_i} w'_{if}$. The initial budget is set as 2000 ($P=2000$). The maximum renewable resources for each period are considered 30 ($R=30$). The lateness penalties for periods 7 to 24 are set as $\alpha_7=-10, \alpha_8=-11, \alpha_9=\alpha_{10}=\alpha_{11}=-12$ and $\alpha_{j=12, \dots, 24}=-13$. Also let $M=0.25, a=0.515, b=0.5$. One possible solution for this problem is shown in Table 3. Replacing the values for x_{ij} from Table 3 in equations (1) to (4) yield the processing and setup times under learning, shown in Table 4. As an example, according to equations (1) and (2),

$w_{41}' = 2$ reduces to 1.7565 as: $w_{41} = x_{411}(w_{411}(2) + w_{411}(3)) = 1 \times (1 + 0.7565)$. Similarly for setup times, according to equations (3) and (4), we have $s_{43} = x_{434} \times s_{434}$ where $s_{434} = b_4 \times (w_{41} \times x_{411} + w_{42} \times x_{423})$ which is: $0.5 \times (1.7565 \times 1 + 1 \times 1) = 1.3782$. Also the resource usage variations during the planning horizon are investigated in Table 5. Note that the used resources are all less than or equal to the maximum available resources (i.e. 30). The final scheduling of such a problem is illustrated in Figure 1. Now, consider project 4, where its second job needs $c_{42j}(h) = 2005$ to start. If the classic version of investing was considered where our only source of asset was the initial capital of $P = 2000$, then project 4 would not have been selected since one of its jobs needs more than the available budget of 2000. If we had forgone the selection of project 4, the total profit from selecting project 1 and 2 would have been 3307.5 according to equation (5). However, with reinvestment strategy, the profits of other projects accumulate to 2225 at the start of this job in time 3.6348. This makes it possible to select project 4 and the final profit increases to 3831.4214. On the other hand, if time horizon was fixed at $T = 6$, then project 3 would not have been chosen since its duration surpasses 6 periods. However with flexible time horizon strategy, planning horizon is increased from 6 to 24 which makes it possible to select project 3, and in turn, the profit has increased from 3831.4214 to 4160.1814.

Table 1 Costs and profits of different jobs of projects

Project	Costs	Profits
1	$c_{111}(1) = 350, c_{122}(2) = 800$ $c_{134}(4) = c_{134}(5) = 220$	$d_{134}(6) = 1200, d_{111}(2) = d_{122}(3) = 400$
2	$c_{211}(1) = 350, c_{222}(2) = 250$	$d_{211}(2) = d_{222}(3) = 750$
3	$c_{314}(4) = 350, c_{314}(6) = 450$ $c_{34[14]}(14) = c_{35[17]}(17) = 250$ $c_{314}(5) = c_{328}(8) = c_{33[10]}(10) = 250$	$d_{314}(7) = 1200, d_{328}(9) = d_{33[10]}(11) = 400$ $d_{34[14]}(15) = d_{35[17]}(18) = 400$
4	$c_{411}(1) = 350, c_{423}(3) = 2005$ $c_{411}(2) = c_{448}(8) = 250, c_{436}(6) = 150$	$d_{423}(4) = 2750$ $d_{411}(2) = d_{436}(7) = d_{448}(9) = 275$

Table 2 Normal durations, required resources and due date penalties of jobs

Project	Normal durations	Required resources	Due date penalties
1	$w'_{[1]v} = \{1, 1, 2\}$	$r_{[1]v} = \{10, 10, 4\}$	$\beta_{[1]v} = \{10, 10, 10\}$
2	$w'_{[2]v} = \{1, 1\}$	$r_{[2]v} = \{10, 10\}$	$\beta_{[2]v} = \{5, 5\}$
3	$w'_{[3]v} = \{3, 1, 1, 1, 1\}$	$r_{[3]v} = \{10, 10, 4, 5, 10\}$	$\beta_{[3]v} = \{3, 3, 3, 3, 3\}$
4	$w'_{[4]v} = \{2, 1, 1, 1\}$	$r_{[4]v} = \{10, 20, 4, 5\}$	$\beta_{[4]v} = \{2, 2, 2, 2\}$

Table 3 A feasible solution

Project	x	Co	L
1	$x_{111} = x_{122} = x_{134} = 1$	$Co_{11} = 2, Co_{12} = 3.5, Co_{13} = 6.25$	$L_{11} = -1, L_{12} = -0.5, L_{13} = 0.25$
2	$x_{211} = x_{222} = 1$	$Co_{21} = 2, Co_{22} = 3.5$	$L_{21} = -3, L_{22} = -2.5$
3	$x_{314} = x_{328} = x_{33[10]} = 1$	$Co_{31} = 7.02, Co_{32} = 9.22, Co_{33} = 11.92$	$L_{31} = 5.02, L_{32} = 6.22$
	$x_{34[14]} = x_{35[17]} = 1$	$Co_{34} = 15.11, Co_{35} = 18.81$	$L_{33} = 7.9, L_{34} = 10.11, L_{35} = 12.8$
4	$x_{411} = x_{423} = 1$	$Co_{41} = 2.7, Co_{42} = 4.6$	$L_{41} = -0.24, L_{42} = 0.63$
	$x_{436} = x_{448} = 1$	$Co_{43} = 7.01, Co_{44} = 9.89$	$L_{43} = 2.01, L_{44} = 3.89$
Other variables	$\lambda^+ = 12, \lambda^- = 0, y_{j=7, \dots, 18} = 1, \delta = 0$		

Table 4 Learning durations and *psd* setup times of different jobs

Project	Learning durations	Setup times
1	$w_{[1]v} = \{1, 1, 1.75\}$	$s_{[1]v} = \{0, 0.5, 1\}$
2	$w_{[2]v} = \{1, 1\}$	$s_{[2]v} = \{0, 0.5\}$
3	$w_{[3]v} = \{2.39, 1, 1, 1, 1\}$	$s_{[3]v} = \{0, 1.19, 1.69, 2.19, 2.69\}$
4	$w_{[4]v} = \{1.7565, 1, 1, 1\}$	$s_{[4]v} = \{0, 0.8783, 1.3782, 1.8782\}$

Table 5 Resource usage in different points in time

Time	Resource	Time	Resource	Time	Resource
1-2	30	4.63-6.01	14	9.22-9.89	5
2-2.5	10	6.01-6.25	18	9.89-10.92	0
2.5-2.75	30	6.25-7.01	14	10.92-11.92	4
2.75-3.5	20	7.01-7.02	10	11.92-14.11	0
3.5-3.63	0	7.02-8.22	0	14.11-15.11	5
3.63-4.5	20	8.22-8.89	10	15.11-17.81	0
4.5-4.63	24	8.89-9.22	15	17.81-18.81	10

In summary, adding setup times and learning effect result in a more realistic planning while the reinvestment strategy and flexible horizons provide an opportunity for increasing the final profit by enabling to include more projects in the portfolio.

4 Conclusion

In the current paper, a model on the integrated problem of project selection and scheduling is presented. First, past sequence dependent setup times and learning

effects are added to the jobs of each project to provide a more realistic view on scheduling. Second, reinvestment strategy is considered which makes it possible to include a project in the portfolio even if its cost is higher than the initial capital at hand. Third, time horizon is assumed to be flexible. This assumption weighs the benefit versus loss of adding another additional period to the default planning horizon. The effects of these strategies are investigated using a numerical example. Future works on the subject involve considering uncertain parameters in the model and developing appropriate simulation methods to tackle the problem.

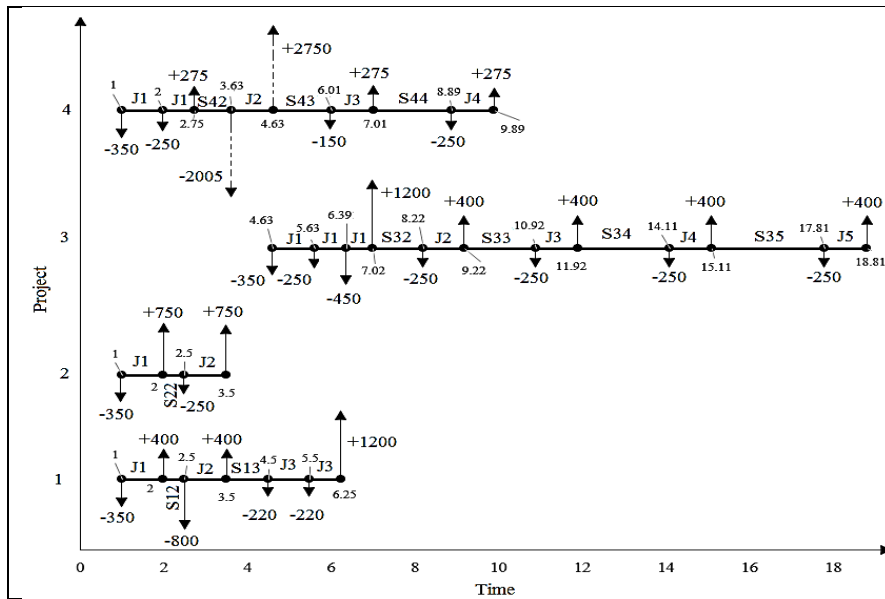


Fig. 1 Feasible schedule with *psd* setup time and learning effect

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